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Multicriteria energy policy investments and energy market clearance via integer programming

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Abstract The deregulation of energy markets has created a framework for policy making, still under evolution, which is much more complex than the previous one. As a consequence, new requirements need to be met, concerning both technical design and financial management. This framework renders the use of multicriteria techniques attractive. Here, the investments in suppliers, depending on the policy implemented, are formulated as an integer programming problem, which consists of different subproblems according to the assumptions made and the market's regulations. The equivalent relaxed problem is a mixed integer programming problem that can represent the clearance of the energy market by considering several criteria besides price and quantity. Nonlinearities are reformulated by inserting additional binary variables so that the solution algorithms are more effective and efficient in most realistic cases. The feasible solutions and the optimal solution that maximizes every time the market regulator's gain are obtained, after imposing some thresholds on the criteria used to evaluate the different energy technologies, thus creating a decision support system for the regulator.

Keywords Energy market · Policy · Multicriteria decision support · Optimization

1 Introduction

The structure of electricity energy markets has undergone many reforms, especially recently. The monopolies have given their place to liberalized markets, where many independent power producers and users can enter or exit at any time. These producers,

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acting in the energy markets as cooperators (Jia and Yokoyama 2003) or competitors (Geerli et al. 2001), aim to maximize their financial profit. This is achieved through their bidding strategy which can be modeled in many ways. The market's operator receives the bids and offers, prioritizes the producers according to the regulations and clears the transactions of the market at precise time intervals, usually daily.

The deregulation of the energy market has created a framework for policy making, still under development, which is much more complex than the previous one. The demand for higher penetration of renewable energy sources and the strict environmental terms, aiming at the reduction of gas emissions, have also contributed to this complexity (Celebi and Fuller 2007, 2012; Stoft Steven 2002; Denny and Dismukes 2002; Jia and Yokoyama 2003). As a consequence, new requirements need to be met, concerning both technical design and financial management.

In the deregulated energy market independent organisations who are responsible for the operation and regulation of the market are necessary. The operator and the regulator aim at maximum social welfare and must coordinate all types of energy production in order to make the best supply decision for the society. As far as the resources are concerned, there are various generation technologies available, each having its own advantages and drawbacks. Therefore, many kinds of energy resources and producers can be assumed, using variable methodologies/technologies such as Oil, Lignite, Natural gas, Photovoltaic cells, Wind, Hydropower, Biomass etc. Even imports from neighbors (Bulgaria, FYROM, Albania,, Turkey, Italy for the case of Greece) can be considered indirectly as a separate energy resource. Social welfare includes besides the satisfaction of the energy demands per se other aims as well. An Independent System Operator (ISO) is no longer interested in just the economic cost of energy production, since it is not sufficient for determining socially good choices. Until now, every decision was taken according to it; this cost however does not guarantee that the best economic decision will be the best for the society too. Citizens and thus governments are also interested in other aspects that influence the society and people's lives, like the environment, that should be taken into account. Therefore, there are multiple criteria to evaluate each resource: bidding price, quantity offered, reliability due to physical conditions (sun, wind, gas' pressure, international circumstances), flexibility (of resource to entry and exit the system in order to deal with emergencies), environmental impact (COx, NOx, SOx and other emissions), strategic characteristics (e.g. resource's importance for the country's energy autonomy), social characteristics (e.g. resource's contribution to a region's social policy, employment and growth), saturation of the energy sector (taking into consideration the saturation restrictions for each region) etc. For each above mentioned criterion, each resource is given a technically definable value (for quantitative criteria, such as chemical substances production) or an estimated value (for qualitative criteria) based on other considerations. These values, even if they are based on scientific methodologies, are possibly subject to slight errors which, however, should not influence the final decision of the system operator otherwise the impact of these errors must be assessed.

In conclusion, the regulator needs to decide which resources and producers will be licensed, subsidized and prioritized, so that the operator can receive their bids, clear and settle the transactions, known as market-clearing. It should also be evaluated what the risk of these decisions is. The responsibilities of the regulator and the operator are so closely related that they have to cooperate in order to decide the best energy policy for the society. For this reason, in this study it is assumed that there is one ISO who must decide which producers should be encouraged through subsidizing, licensing and prioritizing, and also clears the market. The suppliers must be properly selected so that the limits the ISO imposes on some of the criteria mentioned are satisfied. It should be taken into account that there may not exist a unique best decision; many different choices could be suggested, accompanied by relevant evaluative scales.

In order to address this multicriteria problem, the Pareto solution set is taken into account. To do that, weights are assigned to all these criteria and thus they can be incorporated into a unique function that represents the total gain of the ISO from economic, social, technical and environmental point of view according to its energy policy. This function is then optimized with use of integer and mixed-integer programming algorithms that guarantee convergence to the optimal solution. For different weights, different solutions are obtained that constitute the Pareto set of solutions. To find the best solution, algorithms based on branch-and-bound approach (Der-San Chen et al. 2010) are used. After formulating the problem using binary variables, the algorithm searches for a binary integer feasible solution, updates the best binary integer feasible solution found so far and verifies that no better integer feasible solution is possible. Using binary variables is also convenient for formulating numerous real case studies that extend our simple example as will be later discussed.

The selection of one solution among all the feasible Pareto ones, but also the importance given to each evaluation criterion (weight), rests on the system operator. Since the ISO represents the whole society and is interested in maximizing the social welfare, the energy policy decided becomes a political issue and thus, it could be influenced by many factors and stakeholders. Every time the ISO sets the policy, acting as the leader of the market, the other participants respond by playing a game among them so as to maximize their own profits based on this policy, acting as followers. The ISO needs to take into account these interactions, consequently the use of game theory notions and social choice theories is crucial to obtaining a robust solution that will be universally accepted. Such features can be incorporated to the mathematical problem treated in the present study.

Many studies until now have addressed the energy supply criteria. Many aspects should be taken into account except for the financial cost of the producers (Krupa and Jones 2013; Migheli 2012). Among them the reliability (Soleymani et al. 2008) and the environmental impacts of the energy production (Stephen and Anders 2013; Krupa and Jones 2013) are considered very important. Most studies implement game theory tools in order to model the interaction between suppliers and consumers (Reeves et al. 2005; Skoulidas et al. 2010, 2002). Here, a decision support framework for the ISO is created that includes all needed criteria in order to obtain the socially best solution for energy investments or supply, according to the desired energy policy. The ISO's decision is assumed to incorporate citizens' preferences, thus consumers do not interact directly with the model presented but they are also affected by the energy policy implemented.

In the next sections, various methods are described, based on which the numerous decision scenarios can be chosen and evaluated. The motivation and formulation of the different problems are presented in Sect. 2. Section 3 contains the methodology and the algorithms for solving each of the problems described. In Sect. 4, different examples are solved in order to make some observations. In Sect. 5, the sensitivity and robustness of the solutions are studied. Finally in Sect. 6 the conclusions are recapitulated and some future extensions are suggested.

2 Formulation of the problem

The purpose of this study is to help an ISO of an energy market to decide on the energy suppliers' selection by taking into consideration different kinds of criteria and its energy policy. The same criteria hold if the ISO is interested either in favoring some energy technologies or in clearing the energy market with a cost that reflects all factors considered important for the society.

First of all, the criteria that are capable of evaluating an energy resource from all important points of view need to be decided. The ISO is interested in setting some limits that these criteria should meet. After finding all feasible solutions of the problem, the ISO seeks the optimal solution that maximizes its objective function, which represents the social welfare based on its energy policy. That is why, ISO's energy policy is considered to be a weighting procedure. After assigning weights representing each criterion's importance, a unique social welfare function is created where the thresholds demanded by the ISO are considered to be constraints to the problem. This multicriteria decision support method is simple to be implemented and very effective since it can converge quickly to the optimal solution, depending on the problem's formulation.

2.1 Introductory example

The problem is, given some energy resources which are evaluated based on some criteria, to decide which ones of them should be selected for energy supply by the system operator. This decision can either refer to an investment strategy, meaning the ISO can give incentives for the allocation of energy resources, either to a clearance of the market mechanism, meaning the ISO takes the bids of the energy producers and settles the transactions according to the regulations. Table 1 provides an example of such a marking with eight resources (R1, R2, R3, R4, R5, R6, R7, R8) and nine criteria (Cr1: price per unit produced, Cr2: production capacity, Cr3: reliability, Cr4: flexibility, Cr5: COx emissions, Cr6: NOx emissions, Cr7: strategic characteristics, Cr8: social characteristics, Cr9: saturation). These criteria represent independent factors that influence the final decision and are at the same time sufficient and necessary for the ISO to decide. The values represent a theoretical model, but for a particular application, realistic values can be provided by an Independent System Operator.

Each resource is assigned with a marking column in which criteria 3, 4, 7, 8 and 9 have a maximum value of 100 (e.g. 80 stands for 80%). The value of criteria 5 and 6 stands for the produced quantity of COx and NOx respectively per unit produced and thus has a negative meaning. So, high values of criteria 2,3,4,7 and 8 are desired, whereas low values for criteria 1, 5, 6 and 9 are preferred. It is easily observed that taking into account only price and quantity offered resource 4 dominates, but this

Table 1 Resources 'values' table ($A = \left(a_{j,i}\right), j = 1, 2, \ldots, 9, i = 1, 2, \ldots, 8$)

Cr\R	R1	R2	R3	R4	R5	R6	R7	R8
Cr1	2	2.1	2.4	1.9	3	3.2	2.2	2.4
Cr2	115	80	90	150	20	15	110	80
Cr3	100	95	85	90	70	75	95	95
Cr4	80	90	60	80	85	80	75	70
Cr5	1	1.1	1.1	1	0.2	0.1	0.5	0.7
Cr6	1	0.8	0.9	1.2	0.3	0.5	0.2	0.3
Cr7	100	80	100	70	100	100	60	30
Cr8	70	65	85	90	75	85	65	95
Cr9	70	65	80	65	75	40	90	50

could change by introducing all the criteria. Moreover, resource 5 is better as far as environmental criteria Cr5 and Cr6 are concerned, but is inferior regarding the other criteria. Questions vary: Which resources should be chosen and in which way so that the demand is satisfied, the emissions do not exceed some limits, the total reliability is tolerable, the average consumer's economic cost is minimized etc. Furthermore, in any case, the expected stability or risk should be able to be determined, meaning how would this choice change and at which point if the values of Table 1 are modified.

It is assumed that the selection of resource R1 is indicated by $w_1 = 1$, whereas not selecting resource R1 is indicated by $w_1 = 0$. Similarly, the selection of resource Ri is indicated by $w_i = 1$ and the non-selection of resource Ri is indicated $w_i = 0$. This way, the final decision is reduced to a vector $\boldsymbol{w} = (w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8)$, where $w_i' s$ are binary variables (value 0 or 1). Each resource is evaluated based on criteria $\mathrm{Cr_j}$, $j \in R$. Based on these criteria, costs $\mathrm{J_j}$'s are formulated. $\mathrm{J_2}$ is the sum of the products of the quantity of each resource and the corresponding $\mathrm{w_i}$. Similarly we define $\mathrm{J_7}$ and $\mathrm{J_8}$. $\mathrm{J_1}$, $\mathrm{J_5}$ and $\mathrm{J_6}$, depend on quantities and are given by the linear sum of products of the corresponding quantities, criteria's values and $\mathrm{w_i}$. $\mathrm{J_3}$ and $\mathrm{J_4}$ are ratios given by the quotient of a similar product divided by quantity cost $\mathrm{J_2}$. $\mathrm{J_9}$ is also a quotient, a weighted sum of the saturacy of the selected producers.

Considering a specific vector **w**, the available energy is the quantity offered:

$$J_2(\boldsymbol{w}) = 115w_1 + 80w_2 + 90w_3 + 150w_4 + 20w_5 + 15w_6 + 110w_7 + 80w_8$$

$$J_2(\boldsymbol{w}) = \sum_{i=1}^{8} \alpha_{2,i} w_i$$

The cost of energy is derived from the price and the corresponding quantity offered:

$$J_1(\boldsymbol{w}) = 2 * 115w_1 + 2.1 * 80w_2 + 2.4 * 90w_3 + 1.9 * 150w_4 + 3 * 20w_5 + 3.2 * 15w_6 + 2.2 * 110w_7 + 2.4 * 80w_8$$

$$J_1(\boldsymbol{w}) = \sum_{i=1}^{8} \alpha_{1,i}\alpha_{2,i} w_i$$

The total reliability is the percentage of the reliable energy divided with the total quantity offered:

$$J_3(\boldsymbol{w}) = \frac{\sum_{i=1}^{8} \alpha_{3,i}\alpha_{2,i} w_i}{\sum_{i=1}^{8} \alpha_{2,i} w_i}$$

Likewise, the total flexibility is:

$$J_4(\boldsymbol{w}) = \frac{\sum_{i=1}^{8} \alpha_{4,i}\alpha_{2,i} w_i}{\sum_{i=1}^{8} \alpha_{2,i} w_i}$$

In order for costs J_3 and J_4 to be defined, at least one resource should be selected. If no resources are selected, then the problem is infeasible.

The total COx emissions are (given that units in assessment table are $tnCO_x$ for a unit of quantity):

$$J_5(\boldsymbol{w}) = \sum_{i=1}^{8} \alpha_{5,i}\alpha_{2,i} w_i$$

Similarly, the total NOx emissions are:

$$J_6(\boldsymbol{w}) = \sum_{i=1}^{8} \alpha_{6,i}\alpha_{2,i} w_i$$

Respective strategic and social characteristics are linked to the equivalent resources' selection, whereas saturation must also be divided by the number of resources selected in order to be calculated as a percentage. Consequently the rest of the costs are:

$$J_7(\boldsymbol{w}) = \sum_{i=1}^{8} \alpha_{7,i} w_i$$

$$J_8(\boldsymbol{w}) = \sum_{i=1}^{8} \alpha_{8,i} w_i$$

$$J_9(\boldsymbol{w}) = \frac{\sum_{i=1}^{8} \alpha_{9,i} w_i}{\sum_{i=1}^{8} w_i}$$

It is evident that costs J_1, J_5, J_6 and J_9 must be minimized, so they can be given an upper bound, whereas J_2, J_3, J_4, J_7 and J_8 must be maximized (these costs can be considered as gains), so they can be given a lower bound.

2.2 Formulation of various objectives

As already stated, the challenge is to make some of the costs J_j's to be as large as possible and some of them to be as small as possible. Demanding $J_j(\boldsymbol{w})$ to be large (small) possibly means that is should be greater (lesser) than a given J_j^* meaning $J_j(\boldsymbol{w}) \geq J_j^* J_j(\boldsymbol{w}) \leq J_j^*$. Thus, in order to select the most profitable resources, the decision-maker, in our case a system operator, sets some thresholds, namely the minimum and maximum values he desires for any number of costs $\mathrm{J_j}$. Such a formulation leads to addressing a satisfiability problem:

$$\begin{aligned}
&\text{Find} \quad \boldsymbol{w} = (w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8)\,, w_i = 0 \quad \text{or} \quad 1, \\
&\text{such that:} \\
&J_j\,(\boldsymbol{w}) \geq J_j^*, \quad \text{for some } j \in \{2, 3, 4, 7, 8\} \\
&J_j\,(\boldsymbol{w}) \leq J_j^*, \quad \text{for some } j \in \{1, 5, 6, 9\}
\end{aligned} \tag{1}$$

For problem (1), the values of J_j^* are given. Such problems could have multiple solutions. By changing the values of J_j^* different solutions are found, all having respective properties. It is probable that the demand for a large (small) $J_j(\boldsymbol{w})$ leads to solving a maximization (minimization) problem such as:

$$\begin{aligned}
&\max_w J_3\,(\boldsymbol{w}) \\
&\text{subject to}: J_j\,(\boldsymbol{w}) \geq J_j^* \; or \; J_j\,(\boldsymbol{w}) \leq J_j^*, \; \text{for some } j \in [1, 9]
\end{aligned} \tag{2}$$

This problem addresses the maximization of a certain cost under the restriction that some costs lay within certain admissible limits. These constraints need not include all of our costs and the respective criteria. Moreover, a cost $\mathrm{J_j}$ could be given an upper and lower bound at the same time. We have assumed that there are nine criteria and thus nine costs also. It is thus possible that two or more costs are to be optimized. For example:

$$\begin{aligned}
&\min_w \{-J_2\,(\boldsymbol{w})\,, J_5\,(\boldsymbol{w})\,, J_6\,(\boldsymbol{w})\} \\
&\text{subject to}: J_j\,(\boldsymbol{w}) \geq J_j^*, \; \text{for } j = 3, 4, 7, 8
\end{aligned}$$

It should be noted that a negative or positive sign is assigned depending of the goal of minimum or maximum $J_j(\boldsymbol{w})$. In this case, we seek the Pareto solutions, which can be found by creating a single cost. Thus, a weighting method, part of multiobjective mathematical programming (MMP), is used:

$$\begin{aligned}
&\min_w \{b_1\,(-J_2\,(\boldsymbol{w})) + b_2 J_5\,(\boldsymbol{w}) + b_3 J_6\,(w)\} \\
&\text{subject to}: J_j\,(\boldsymbol{w}) \geq J_j^*, j = 3, 4, 7, 8
\end{aligned}$$

where $b_1, b_2, b_3 \geq 0$ and for every $\boldsymbol{b} = (b_1, b_2, b_3)$ a Pareto efficient solution exists. The weights are a measure of the importance of each criterion. They can be given any positive number after pair comparisons among them, but for better understanding, after proper reformulation they should sum up to 1 implying percentages.

The system operator is enabled to customize its choice criteria depending on the policy that is desirable at each time period. Specifically, this policy may concern the decision that minimizes or maximizes a certain cost $\mathrm{J_j}$, either linear or ratio, or the decision that minimizes/maximizes a combination of any number of these costs, even all, simultaneously. Apparently, we can have a variety of $J_j(\boldsymbol{w})'s$ in the objective function and in the constraints. Important is the general case where all $J_j(\boldsymbol{w})'s$ are in the objective cost function. For example:

$$\max_w \begin{pmatrix} -b_1J_1(\boldsymbol{w}) + b_2J_2(\boldsymbol{w}) + b_3J_3(\boldsymbol{w}) + b_4J_4(\boldsymbol{w}) - b_5J_5(\boldsymbol{w}) \\ -b_6J_6(\boldsymbol{w}) + b_7J_7(\boldsymbol{w}) + b_8J_8(\boldsymbol{w}) - b_9J_9(\boldsymbol{w}) \end{pmatrix}$$

$$b_j \geq 0, \quad \sum_{j=1}^{9} b_j = 1, j = 1, 2, \ldots, 9 \tag{3}$$

Any number of costs in the objective function can be achieved by setting the weighs of the rest equal to 0. Different $\boldsymbol{b} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9)$ give different solutions which are Pareto. This means that b_j's consist a vector **b** defining the energy policy of the ISO according to its preferences.

Problems (1), (2) and (3) are the three types of problems we address. By solving them, the ISO is capable of finding all the feasible solutions satisfying its constraints and then, based on its energy policy to find the best feasible solution or choose another among them.

As already stated, the solution of all the described problems is a vector $\boldsymbol{w} = (w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8)$ where $w_i's$ are binary variables, so we have to deal with a binary integer programming problem (Der-San Chen et al. 2010; Wolsey and Nemhauser 1999). In reality, the clearance of an energy market after producers' bidding is more complex since the offer or the demand of energy need not represent the maximum installed capacity. Additional binary variables can then be used to reformulate more general and realistic cases. If an energy resource can be selected for particular levels of its offered energy quantity, $\mathrm{w_i}$ would be a discrete variable which can be easily expressed with a number of binary variables. For example, if a resource Ri can be selected for every 25% of its maximum capacity, then $w_i = (0, 25x_1{+}0, 5x_2{+}0, 75x_3)$ where $\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3$ are binary variables.

Additionally, discrete variables which can take any positive integer value can also be represented by binary variables by using the powers of 2. This could be the case if a resource represents a set of grouped producers and any number of them can be selected, thus scaling the problem. For example, integers up to 63 can be expressed with six binary variables, as $I = x_1 + 2x_2 + 4x_3 + 8x_4 + 16x_5 + 32x_6$, where I is an integer in the set [0,63] and $\mathrm{x_i}$, $\mathrm{i} = 1, \ldots, 6$ are binary variables. Most algorithms can deal with integer values in linear programming problems without needing to reformulate them. The reformulation using binary variables, however, is crucial in linearizing the objectives and constraints of nonlinear cost functions.

In any of these cases, after the ISO has stated the exact problem he deals with, any integer programming problem can be transformed into a binary integer programming one which is easier to be solved. Of course, the larger the sets we want to represent or the feasible integer values, the more complex the problem becomes slowing down the convergence to the solution. Moreover, this way, products of binary variables may occur, which nevertheless can be easily substituted by single binary variables.

2.3 Solution assessment

It is apparent that besides problem (2) that usually has a unique solution, all other problems addressed have multiple solutions. Thus, a methodology in order to choose one among all the feasible solutions is needed. For problem (3) solutions depend on the selected b_j's; this is equivalent to decide who or how b_j's are selected. There are many ways to define and calculate the weights or choose one among the feasible solutions originating from negotiations, game theory and multiple criteria decision-making (MCDM). This decision rests on the system operator.

A very important question regarding any suggested solution is if this solution changes in case some parameters are modified a lot or even slightly. It is already stated that the values used in such a problem could be subjected to errors. Thus, if a suggested solution is approved but a slight modification of the parameters results in a drastically different solution, the initial solution cannot be considered reliable. On the other hand, it is possible that such a change in the solution demonstrates the importance of the specific parameter that caused it. In any of the two cases, it is indispensable to study the parameters and their percentage of modification that influence the suggested solution. Another motive for studying this phenomenon has to do with the possible intention of a producer or stakeholder managing such decisions to influence the final result in a way not noticed by an external observer. Consequently, the possibility of strategic gaming by some of the players should be studied. These considerations can be examined by an analysis of the solutions' sensitivity.

3 Methods of solution

Here we present the basic methodologies for solving the problems delineated. We describe briefly the theoretical algorithms used, the modifications introduced and give appropriate references.

3.1 Problem 1: Satisfiability

First of all, a satisfiability problem is considered, which means we are interested in finding all the feasible solutions of the problem. So all the possible decisions that satisfy the bounds given to the criteria are examined.

To calculate all the feasible solutions, complete enumeration of all the possible combinations is needed. This can be very demanding as far as computational resources are concerned (Schaefer 1978), but in this case, including only eight resources and nine criteria, it is quickly solved (milliseconds). By adding resources, the solution space and the problem's requirements are augmented. In order to downsize the problem and study only the best solutions if a large scale model is to be solved, modifications to the initial satisfiability problem can be made, like reducing the feasible solution space by tightening the constraints' bounds, or searching for all the feasible solutions, satisfying these constraints, which have small divergence from a previously found or guessed optimal solution e.g. 10 %. In any case, the satisfiability problem is very important because it could give the operator a first hint of the available solutions and can be used so as he can choose a solution from the feasible set by implementing any methodology he considers best.

3.2 Problem 2a: Linear integer programming

After solving the satisfiability problem, we address the problem of finding the solution that minimizes or maximizes a certain cost $\mathrm{J_j}$. This means that there is a binary integer programming problem, where the objective function is linear (nonlinear functions will be addressed using fractional programming in the next subsection) and it subjects to a set of linear constraints. In this case, all costs are linear except for $\mathrm{J_3}$, $\mathrm{J_4}$ and $\mathrm{J_9}$.

Efficient algorithms that solve large scale and mixed integer programming problems, as the ones in the next sections, use the branch-and-cut approach (Der-San Chen et al. 2010). There are various techniques for creating a better formulation of a problem and implementing cuts, which however must be used with prudence.

The problem to be solved is:

$$\begin{aligned}&\min_{\boldsymbol{w}} f^T \boldsymbol{w} \text{ or } \max_{\boldsymbol{w}} f^T \boldsymbol{w}\\ &\text{Subject to:} \boldsymbol{A}_1 \boldsymbol{w} \leq \boldsymbol{c}_1, \boldsymbol{A}_2 \boldsymbol{w} \geq \boldsymbol{c}_2\end{aligned}$$

where **A**'s and **c**'s vectors of constants and **w** a vector of binary variables as already stated.

Constraints of the form $\frac{\sum_i a_i w_i}{\sum_i d_i w_i} \leq c$ can easily be transformed as $\sum_i (a_i - c d_i) w_i \leq 0$ since the denominators in our case study are always positive. The same hold also for the constraints of the form $\frac{\sum_i a_i w_i}{\sum_i d_i w_i} \geq c$.

3.3 Problem 2b: Fractional integer programming

In the case of nonlinear costs more complex algorithms need to be used which must be selected carefully in order to converge and be efficient at the same time. However, nonlinear functions in this example (deriving from Cr3, Cr4 and Cr9) can be easily reformulated using techniques of fractional programming since they are quotients of linear functions (Charnes and Cooper 1962; Li 1993; Schaible Siegfried and Shi Jianming 2002; Siegfried 1982).

The processing of such costs, which are ratios, in the constraints has already been addressed. If such a cost is the objective function, then this is a single ratio problem, which is a case of fractional programming.

Problem:

$$\min \frac{p_i + \sum_j p_{ij} x_j}{q_i + \sum_j q_{ij} x_j}$$

Subject to:

$$q_i + \sum_j q_{ij} x_j > 0 \ i = 1, \ldots, m$$

$$r_k + \sum_j r_{kj} x_j \leq 0 \ k = 1, \ldots, h$$

$$x_j = \{0, 1\} \ \ j = 1, \ldots, n$$

By letting y_i equal to $1/q_i + \sum_j q_{ij} x_j$, the problem becomes:

$$\min \sum_i \left(p_i y_i + \sum_j p_{ij} y_i x_j \right)$$

Subject to:

$$q_i y_i + \sum_j q_{ij} y_i x_j = 1 \ i = 1, \ldots, m$$

$$r_k + \sum_j r_{kj} x_j \leq 0 \ k = 1, \ldots, h$$

$$x_j = \{0, 1\} \ y_i \geq 0 \ j = 1, \ldots, n$$

The polynomial mixed terms in the transformation $z = xy$ can be represented by four linear inequalities: $y - z \leq K - Kx, \ z \leq y, \ z \leq Kx, \ z \geq 0$ where $K > y$ and x is a binary variable.

Then, a mixed integer programming problem (MIP) with binary and positive continuous variables needs to be solved (Li 1993; Siegfried 1982).

A MIP solver could also be used if some of the original resources' variables were to be treated as binary and some others as linear, meaning that some resources should be completely selected or not at all but some others have the possibility to offer any percentage of their installed capacity. Here, the reformulations of discrete and integer variables presented in Sect. 2.1 can be used. The main advantage of using this methodology is that any given objective function that consists of linear functions and quotients can be transformed into a linear one by inserting more constraints.

3.4 Problem 3: Multiobjective integer programming

Finally, the more general case is addressed, where the objective function consists of multiple costs, even all of them. Multicriteria analysis for decision support in energy markets and specifically in the clearance of the market has gained attention the last years since energy production has expanded and new technologies must be assessed globally.

Multiobjective mathematical programming (MMP) is an extension of traditional mathematical programming theory. In this case, the problem to be solved is:

$$\begin{gathered} Max\ or\ Min\left\{f_1(x), f_2(x), \ldots, f_n(x)\right\} \\ \text{Subject to: } x \in S \end{gathered}$$

where x is the vector of the decision variables, $f_1, f_2, \ldots, f_n$ are the objective functions (linear or nonlinear) to be optimized and S is the set of feasible solutions.

Many objective functions need to be optimized at the same time, so there is not an optimal solution in the usual sense, since the objective functions are conflicting. Therefore, the goal is to find the set of efficient solutions, which contains all the solutions that are not dominated by any other solutions. These are called Pareto optimal solutions. MMP is part of the MCDP framework which is the basis for solving multi-criteria problems and consists of various methodologies (Doumpos and Zopounidis 2002; Zopounidis and Doumpos 2002). This framework helps the decision-maker participate actively in the decision process and solve complex realistic problems.

The methods for solving MMP problems are classified according to when the decision-maker expresses his preferences into three categories (Hwang and Masud 1979): a priori, interactive and generation or a posteriori methods. The weighting method used in this study is one of the most widely used generation methods, meaning the Pareto optimal solutions are generated and then the decision maker selects one of them. For this purpose, these costs are summed and a weight factor b_j is assigned to every one of them according to its importance for the ISO. This way results the total gain J of the ISO which is his objective function that needs to be maximized. Caution is needed regarding the signs used, depending on minimization or maximization of the objective function. Weight factor for each cost is very important and can influence the optimal solution. Clearly, the more important a cost becomes, the more are favored the resources with high (low) values of this cost if we want to maximize (minimize) the objective function and vice versa.

The purpose of this report is to find the solution of a formulated problem, thus arbitrary values for each b_j are used and we arrive at an optimal solution. However, there are many ways to decide on the weight factors as already stated in Sect. 2.3, depending on the stakeholders and their influence on the ISO. If the ISO wants to examine various combinations of weights, the different optimal solutions result into the efficient set of Pareto optimal solutions from which he will choose one just as for the satisfiability problem, since this set of optimal solutions is a subset of all the feasible solutions derived from the costs' limits.

Weights that equal 0 are not taken into consideration. In any case, weights after proper formulation should sum up to 1 so that they are easily interpreted, but this is not necessary. Different weights could also be assigned to each cost depending on the resource but this would mean that we assess each resource differently raising equality and justice issues. This case extends the previous examples, so if all the used criteria in the objective function are linear it results into a binary integer programming problem extending that in Sect. 4.2, whereas for every ratio in the objective function the linearization technique described in Sect. 4.3 is used. The same hold also for the constraints.

4 Examples

Here, the solutions of some cases are presented so as to demonstrate the algorithms. The introductory example consists of eight resources and nine criteria but this is not a restriction as the scalability of the algorithms is clear.

4.1 Examples of problem 1

The tighter the limits imposed on the costs, the smaller the feasible set of solutions satisfying them. If it is small enough, then the operator could even make a decision intuitively. Moreover, if the limits exceed certain values there may exist no feasible solutions and thus the problem would be infeasible. Here, three examples are cited which differ only in the lower bound of the second criterion (the supply must always fulfill demand).

Example 4.1.1

- For $J_1 < 10{,}000$, $\mathbf{J_2} > \mathbf{300}$, $J_3 > 30$, J4>30, $J_5 < 10{,}00$, $J_6 < 10{,}00$, $J_7 > 50$, $J_8 > 50$, $J_9 < 60$ the feasible solutions appear in Table 2.

Example 4.1.2

- For $J_1 < 10{,}000$, $\mathbf{J_2} > \mathbf{400}$, $J_3 > 30$, $J_4 > 30$, $J_5 < 10{,}00$, $J_6 < 10{,}00$, $J_7 > 50$, $J_8 > 50$, $J_9 < 60$ the feasible solutions are in Table 3.

Table 2 Results of Example 4.1.1

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
0	0	1	1	0	1	0	1	741	335	89.179	72.239	306.50	292.50	300	355	58.75
0	1	0	1	0	0	0	1	645	310	92.581	80.000	294.00	268.00	180	250	60.00
0	1	0	1	0	1	0	1	693	325	91.769	80.000	295.50	275.50	280	335	55.00
0	1	0	1	1	1	0	1	753	345	90.507	80.290	299.50	281.50	380	410	59.00
0	1	1	1	0	1	0	1	909	415	90.301	75.663	394.50	356.50	380	420	60.00
1	0	0	1	0	1	0	1	755	360	93.681	77.778	322.50	326.50	300	340	56.25
1	0	0	1	1	1	0	1	815	380	92.434	78.158	326.50	332.50	400	415	60.00
1	0	1	0	0	1	0	1	686	300	92.917	71.333	271.50	227.50	330	335	60.00
1	1	0	0	1	1	0	1	698	310	94.274	80.323	264.50	216.50	410	390	60.00
1	1	0	1	0	1	0	0	731	360	93.681	82.222	354.50	366.50	350	310	60.00
1	1	0	1	0	1	0	1	923	440	93.920	80.000	410.50	390.50	380	405	58.00

Table 3 Results of Example 4.1.2

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
0	1	1	1	0	1	0	1	909	415	90.301	75.663	394.50	356.50	380	420	60.00
1	1	0	1	0	1	0	1	923	440	93.920	80.000	410.50	390.50	380	405	58.00

Example 4.1.3

- For $J_1 < 10{,}000$, $\mathbf{J_2 > 500}$, $J_3 > 30$, $J_4 > 30$, $J_5 < 10{,}00$, $J_6 < 10{,}00$, $J_7 > 50$, $J_8 > 50$, $J_9 < 60$ there are no feasible solutions, which is also apparent from the previous examples since the greatest value of J_2 in the feasible solution set was 440, which, therefore, is the value that maximizes J_2 under these constraints.

It is easily observed that, according to the constraints, some resources are selected more often than others. Some of them may always be selected. Respectively, some resources, like resource R7 in this case, could never be selected. This is indicative of the stability and robustness of the solutions as well as of their sensitivity to slight changes of the constraints. This subject will be addressed extensively in the next Section.

These observations would be made even more easily and clearly by ranking the feasible solutions according to the cost the ISO is more interested in. For example, ranking the solutions of Table 2 with respect to J_2, results in the following Table 4:

Here, it is observed that resources R4 and R6 are more often selected, in contrast with resources R3 and R5, for big values of J_2. In conclusion, it is extracted that the results regarding resources R4, R6, R7, R8 are more robust in comparison with resources R3 and R5, whereas resources R1 and R2 do not have an obvious selection pattern.

Similar observations about the stability of the ISO's choices can be made for every ranking that the ISO is interested in. For example, if the ISO wants to follow an environmental friendly policy, he can make his decisions after ranking the feasible solutions in respect to the summation of J_5 and J_6, which represent gas emissions. All suppliers interact with the market and this means that the ISO's and other producers' decisions affect their own strategies and investments. Each supplier wants to maximize his gains, thus the market-clearing result and his response will constantly change until an equilibrium is achieved. For every policy followed the market will move to a different equilibrium, since the producers compete each other based every time on the ISO's energy policy. Thus, the methodology followed in order to select some of the

Table 4 Results of Example 4.1.3

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
1	1	0	1	0	1	0	1	923	440	93.920	80.000	410.50	390.50	380	405	58.00
0	1	1	1	0	1	0	1	909	415	90.301	75.663	394.50	356.50	380	420	60.00
1	0	0	1	1	1	0	1	815	380	92.434	78.158	326.50	332.50	400	415	60.00
1	0	0	1	0	1	0	1	755	360	93.681	77.778	322.50	326.50	300	340	56.25
1	1	0	1	0	1	0	0	731	360	93.681	82.222	354.50	366.50	350	310	60.00
0	1	0	1	1	1	0	1	753	345	90.507	80.290	299.50	281.50	380	410	59.00
0	0	1	1	0	1	0	1	741	335	89.179	72.239	306.50	292.50	300	355	58.75
0	1	0	1	0	1	0	1	693	325	91.769	80.000	295.50	275.50	280	335	55.00
0	1	0	1	0	0	0	1	645	310	92.581	80.000	294.00	268.00	180	250	60.00
1	1	0	0	1	1	0	1	698	310	94.274	80.323	264.50	216.50	410	390	60.00
1	0	1	0	0	1	0	1	686	300	92.917	71.333	271.50	227.50	330	335	60.00

resources is very demanding as far as the ISO is concerned, since an energy policy must have been decided and a game theory study of how this policy will affect the market should be taken into consideration.

4.2 Examples of problem 2

This is the simplest form of our problem. In any case, the only things needed for implementing the algorithm are the final linear objective function and the linear constraints which all depend on some binary variables. All optimization problems in this and the next sections are solved using GAMS software. The algorithm used provides the possibility to seek a solution close to the optimal for quicker convergence (GAMS Development Corporation 2012). In these examples the problems are not very large scale sized, so the optimal solution is seeked.

Example 4.2.1 In this example costs J_1, J_2 and J_3, after proper reformulation according to Sect. 3.3, are optimized given some limits for the other costs that act as constraints. The results are presented in Table 5.

Apparently, the more we tighten the bounds of the costs, the more we limit the feasible region of the objective function and its optimal value deteriorates.

4.3 Examples of Problem 3

These examples of MMP present the optimal solution if the weights are already decided, or the efficient set of Pareto optimal solutions if the ISO wants to examine various weighting schemes. In the problem studied, costs J_3, J_4, J_9 are nonlinear and must be modified according to Sect. 3.3. In the end, there is a MIP problem with binary variables w_i and positive variables z_{ij}, $i = 1, \ldots, 8$ and $j = 1, 2$.

Example 4.3.1 In this example, the optimal solution is seeked when the ISO has already decided which energy policy to implement. Therefore, he has to decide only which producers to select so as to optimize his objective function. For different constraints and weights' values the results are presented in Table 6.

There is the possibility to set either upper, or lower bounds, or both to any number of costs as constraints. There may even no bounds be imposed on these costs (set aside the

Table 5 Results of Example 4.2.1

Decision								Cost optimized	Criteria constraints								
w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	J_j	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
0	1	1	1	0	0	0	1	$\min J_1 = 861$	–	400	30	30	1,000	1,000	50	50	70
1	1	0	1	0	1	1	0	$\max J_2 = 470$	1,000	–	30	30	1,000	1,000	50	50	70
1	0	0	1	1	0	0	0	$\max J_2 = 285$	600	–	30	30	1,000	1,000	50	50	70
1	1	0	0	0	0	0	1	$\max J_3 = 97.091$	1,000	200	–	30	1,000	1,000	50	50	70
1	1	0	0	0	1	1	1	$\max J_3 = 95.688$	1,000	400	–	30	1,000	1,000	50	50	70

Table 6 Results of Example 4.3.1

Decision								Total gain	Weights									Constraints
w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	J	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	J_j
1	1	1	0	1	1	1	0	165.351	0.1	0.1	0.1	0.1	0.05	0.05	0.4	0.05	0.05	$J_1 < 10,00$
1	1	1	0	1	1	1	0	291.601	0.05	0.05	0.05	0.05	0.05	0.05	0.6	0.05	0.05	$J_1 < 10,00$
0	1	1	0	1	1	0	1	229.804	0.05	0.05	0.05	0.05	0.05	0.05	0.6	0.05	0.05	$J_1 < 10,00$, $J2 < 300$
0	0	0	0	1	1	0	1	63.822	0.05	0.05	0.6	0.05	0.05	0.05	0.05	0.05	0.05	$J_1 < 10,00$
1	1	1	0	1	1	1	0	90.173	0.05	0.05	0.5	0.05	0.05	0.05	0.15	0.05	0.05	$J_1 < 10,00$
0	0	0	0	0	1	0	0	−135	0.6	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	–

physical constraints of the problem). It is observed that slight changes in the weights may change the optimal solution but some other bigger changes may leave it intact. It is also observed that, as expected, the tighter the constraints become, the worse the value of the objective function for the optimal solution is. Sometimes the best value may even be negative, meaning the operator doesn't have profit at all. This phenomenon could happen if the values of the costs that reduce total gain (J_1, J_5, J_6, J_9) outweigh the rest of the costs that increase total gain. All these remarks have also to do with stability issues concerning the solutions and several examples must be solved to understand how each cost, each weight and each resource assessment affects the optimal solution.

Example 4.3.2 Here, an example of an efficient set is presented, given that $J_1 < 1{,}000, J_2 > 400, J_3 > 30, J_4 > 30, J_5 < 1{,}000, J_6 < 1{,}000, J_7 > 50, J_8 > 50, J_9 < 70$. The possible weights' combinations are of course infinite, but we assume that the ISO considers all criteria equivalent and of low importance, except one that is the most important among them. For such an assumption, the indicative values are considered to be 6 and 0.05 for the most important criterion and the rest of the criteria respectively, but other values are also possible. Perhaps the ISO hasn't yet decided which is the most important criterion for him. In that case, he finds a set of solutions, each of them being optimal when the respective weight combination is selected. Results appear in Table 7.

Solving the satisfiability problem, there are 31 feasible solutions that satisfy the criteria limits of this example. For each combination of weights presented, we have an optimal solution among the feasible ones. Based on these solutions, the ISO can then choose which solution is preferable and which energy policy to implement. It is observed that some energy resources are more often selected than others and that some optimal solutions are the same even if the weights are different. Of course, even in that case, the respective optimal value of profit J is different. In this example, when the importance of costs that should be minimized (meaning they cause cost in contrast with the rest that cause gain) is high, total profit is negative. This result could perhaps be improved if the demand for energy was less.

Table 7 Results of Example 4.3.2

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	J	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
0	1	1	1	0	0	0	1	−5,147,281	6	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
1	0	0	1	0	1	1	1	2,777,506	0.05	6	0.05	0.05	0.05	0.05	0.05	0.05	0.05
1	1	0	0	0	1	1	1	561,006	0.05	0.05	6	0.05	0.05	0.05	0.05	0.05	0.05
1	1	0	0	1	1	1	1	467,395	0.05	0.05	0.05	6	0.05	0.05	0.05	0.05	0.05
1	1	0	0	0	1	1	1	−1,885,559	0.05	0.05	0.05	0.05	6	0.05	0.05	0.05	0.05
1	1	0	0	1	0	1	1	−1,384,046	0.05	0.05	0.05	0.05	0.05	6	0.05	0.05	0.05
1	1	1	0	1	1	1	0	3,207,601	0.05	0.05	0.05	0.05	0.05	0.05	6	0.05	0.05
0	1	1	1	1	1	0	1	2,934,398	0.05	0.05	0.05	0.05	0.05	0.05	0.05	6	0.05
1	1	0	1	0	1	0	1	−364,254	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	6

5 Sensitivity/robustness

As in any optimization problem, one is very interested in the system's robustness and its sensitivity to changes. Sensitivity analysis considers the change of the optimal solution depending on changes of continuous parameters of the problem. However, integer values cannot be addressed with the classical sense of sensitivity. In an integer programming problem, as mentioned in the end of Sect. 4.1, solutions' robustness results from the consistency of the selections meaning that a robust system should generally approve or eliminate the same resources for small deviations of the value of the objective function. The main interest is in the optimal solution, but if the satisfiability problem is solved, ranking and studying its solutions will help in observing the consistency of the values of w_i (example in the end of Example 4.1.2). To avoid complexity, concentration can be limited to the ranked feasible solutions with small divergence from the optimal one.

In this sense, it is obvious that robustness is important when assigning values to the weights b_j, since their value can be any real number (between 0 and 1 if they sum up to 1). Moreover, the conventional indicator of sensitivity concerns the values of the criteria Cr_j for each resource. Many of these values may include measurement errors as far as quantitative criteria are concerned. It may be also difficult to quantify them accurately if the criteria are qualitative. Sometimes they can even change unexpectedly (for example, a power plant failure may drop the quantity offered, whereas a new investment may raise the social characteristics of the resource). It should be studied how slight variations may influence the optimal solution in order to conclude which choice is the best. Naturally, some values should be more easily amenable to variations than others.

The simplest example of our theoretical model will be examined again, that of the maximization of a linear function, e.g. J_2. At example 4.2.1, it was calculated that the solution that maximizes J_2 such that $J_1 < 1{,}000$, $J_3 > 30$, $J_4 > 30$, $J_5 < 1{,}000$, $J_6 < 1{,}000$, $J_7 > 50$, $J_8 > 50$, $J_9 < 70$ is:

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	J_2
1	1	0	1	0	1	1	0	470

If resource's R4 price (Cr1) raises from 1.9 to 2.1, the optimal solution becomes:

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	J_2
1	0	0	1	0	0	1	1	455

However, if resource's R4 social characteristics' value (Cr8) drops from 90 to 60, the optimal solution remains intact:

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	J_2
1	1	0	1	0	1	1	0	470

This is because the constraints we have imposed are loose as far as social characteristics are concerned, whereas they are tight for pricing. Thus, the sensitivity of the result depends also on the general formulation of the problem.

6 Conclusions and extensions

In the upcoming Liberalized Energy Markets, an Independent System Operator will have to decide which energy resources are more profitable based not only on their price, but also on many other criteria aiming at maximum social welfare. This study provides a multicriteria approach and a decision support framework is created.

To regulate the market, the ISO needs to decide and modify when needed the importance of these costs before including them in one function to be optimized. The other option is to find all the feasible solutions before choosing one among them. It is studied how this objective function and the whole problem with its constraints is formulated and solved with use of integer programming. It is also studied how the insertion of binary variables can reduce the complexity of the problem and also help in solving cases where nonlinearities occur. Many more general or realistic cases can be addressed with the methodology and formulations that are presented. A case that should be studied is that of a resource representing an energy producer who uses many kinds of energy technologies and for this reason his marking values are more complex to calculate.

The results are based on the introductory example and demonstrate how the strict bounds of the costs used, limit the feasible solution space and thus affect the optimal solution. The evaluation of the resources by the ISO should also be as accurate as possible because even slight modifications could alter the optimal solution depending on the problem's formulation.

Finally, the energy policy implemented, that is the weights of the different costs in the objective function or the ISO's preferences in the satisfiability problem, is very important. This means that further study is needed on how the ISO will choose the values of these weights and which methodology is more suitable. The existence of many market participants, like multiple energy resources and producers, many categories of end-users, municipality authorities and even civil society organizations, lead also to game theory notions and methodologies.

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