Interval Analysis to address Uncertainty in Multicriteria Energy Market Clearance

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***Abstract*--** **A serious problem in every complex decision making process is how to deal with uncertainty. In complex systems the uncertainty is usually addressed with the use of probabilistic models where information about the distribution of the uncertain parameters is available or derived. However, in many engineering problems such information is unknown. Evermore, quite often the decision is made once, rendering irrelevant the probabilistic framework. In this study a multicriteria methodology is used in order to model the clearance of the energy market. The proposed model deals with uncertainty as far as the desired energy policy and restrictions are concerned with use of interval analysis. This way, the robustness of the optimal solution for different energy policies, which is necessary in order to evaluate them, is studied, thus creating a decision support system for the market operator.**

***Index Terms*--** **Energy market, energy policy, multicriteria decision support, robustness, interval analysis**

I. Introduction

THE energy markets' structure has greatly changed due to their deregulation. There are many stakeholders now in an energy market that could possibly compete or cooperate in order to maximize their profits [7,9].

In liberalized markets there are also independent organizations that are responsible for the regulation. In energy markets, an independent system operator (ISO) is responsible for the clearance of the market. The ISO receives the bids of the power producers and clears the transactions of the market at precise time intervals. This way, the equilibrium of the market as far as the supply and demand are concerned is achieved, in an attempt to maximize the social welfare. However, the regulation of the market and the best supply decision is not simple. The demand for renewable and sustainable energy that subjects to certain environmental restrictions needs to be taken into consideration [2,3,5,15]. Moreover, there is a great variety of energy resources and technologies used for power production that need to be regulated and evaluated.

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Until now, the clearance of the market is performed according to the financial cost of the producers' bids. However, the importance of other factors such as reliability and environmental impact are taken into consideration in many recent studies [4,10,11,14]. There are various criteria that should be included in the supply decision process so that the clearance of the market corresponds to the socially best result [10].

In order to address this multicriteria problem, the weighting method is used [16]. This means that the different criteria are incorporated into one objective function and each one is assigned with a weighting factor that corresponds to the criterion's importance for the ISO. This is solved as a mixed-integer linear programming problem [6]. Different weight combinations lead to different optimal solutions that constitute the Pareto solution set.

The weight decision is a complex procedure since there are many stakeholders in a deregulated energy market and the ISO needs to make the optimal decision from all points of view. Moreover, slight modification of the ISO's preferences, therefore to the weighting factors, could result in different optimal solutions. It is thus necessary to study the sensitivity of certain parameters and the robustness of the optimal solution obtained.

Parameters' uncertainty in integer programming can be addressed with various methodologies such as fuzzy sets [1] and stochasticity [12]. In this study, interval analysis is used, which is suitable for engineering integer programming problems when some input parameters are not deterministic [13]. The reason that the interval analysis framework is suitable is that it does not require further information regarding the distribution information of the uncertain parameters. Moreover, in complex engineering problems of energy planning and operation, the decision maker is interested in the range of the potential optimal solution and how the decision variables interact with it and not in a probabilistic distribution of the possible solutions. Therefore, if the ISO uses intervals with upper and lower bounds, instead of specific values, in order to assign the weights to the criteria, he can obtain an interval as a solution to the optimization of his objective function. Furthermore, he can study which supplier choices are influenced within the weighting intervals that he sets.

Interval analysis has been used in many energy management and planning problems so as to address uncertainty of some parameters. The purpose of this study is to create a decision support tool for the ISO of an energy market that needs to perform the market clearance according to certain criteria. This tool is able to cope with the difficulties and uncertainties as far as the subjective importance of each criterion is concerned and how these affect the value of the objective function and the selection of the potential suppliers.

The paper is structured as follows. In Section 2 the mathematical formulation of the multicriteria market clearance problem is derived and in Section 3 the interval analysis framework is described and incorporated into the initial problem. In Section 4, a case study is presented and several examples are solved. Finally, in Section 5 we present the conclusions and the future extensions of the research.

II. MATHEMATICAL FORMULATION

The ISO seeks the optimal supplier selection based on his energy policy. In order to decide, he takes into account many different criteria in which he also assigns different thresholds. His objective function incorporates all these criteria, each one assigned with a weighting factor representing its importance. His objective function represents the social welfare. From the set of feasible solutions that satisfy the constraints that he imposes, the optimal solution is the one that maximizes the social welfare.

Assuming there are n potential suppliers, each one of them is represented by S_i, $i \in R$. If x_i is a binary variable that declares the selection or not of supplier S_i, the decision of the ISO is a vector $\boldsymbol{X} = (x_1, x_2, \dots, x_n)$. Moreover, assuming the ISO takes into consideration m criteria, each one corresponding to a cost C_j, $j \in R$, the objective function and the constraints are based on the weights and limits imposed on costs $C_j(\boldsymbol{X})$.

Therefore, the optimization problem of the ISO is:

$$\max_{\boldsymbol{X}} \sum_{j=1}^{m} w_j C_j(\boldsymbol{X}) \tag{1a}$$

Subject to:

$$C_j(\boldsymbol{X}) \le C_{th,j}, for\ some\ j \in [1, m] \tag{1b}$$

$$w_j \ge 0\ \forall j \tag{1c}$$

$$\sum_{j=1}^{m} w_j = 1 \tag{1d}$$

where w_j the weight of criterion j in the objective function and $C_{th,j}$ the threshold imposed on the cost C_j by the ISO.

Attention must be given to the sign of $C_j(\boldsymbol{X})$ in the objective that depends on whether the cost C_j need to be minimized or maximized, e.g. the cost that corresponds to the total quantity of energy could be considered as gain and needs to be as high as possible while the cost that corresponds to the environmental impact needs to be as low as possible.

As far as the constraints are concerned, the ISO can impose upper or lower bounds on the costs of some or even all of the criteria. In case of lower bounds, the inequality in equation (1b) needs to be reversed. The weighting factors w_j correspond to the importance given to each criterion. They can be given any positive value and then they can easily be reformulated so that the sum up to 1. This way, they can be easily interpreted as percentages declaring importance.

If the ISO wants to eliminate a certain criterion and its cost from a decision, he can set its weight equal to 0. In any case, each combination of weights leads to different social welfare and the optimal solutions obtained are Pareto efficient. Thus, every weight combination represents a different energy policy which needs to be decided carefully since it can affect the final optimal decision of the ISO.

The formulated problem is a binary integer programming problem that can be solved using various algorithms based on branch-and-cut approach [6]. The difficulty however, lies on the selection of the right energy policy that determines the optimal solution. There are many ways to assign values to the weighting factors. In any case, the ISO needs to take into account all the stakeholders of the energy market, therefore this can be a difficult and challenging task.

III. INTERVAL ANALYSIS

When some of the parameters are not deterministic, algorithms of mixed integer programming problems may not be effective. Uncertain parameters can be expressed as intervals that correspond to the range that the values of these parameters can obtain. The advantage of this formulation in comparison with other methodologies, such as stochasticity, is that the distribution information of the unknown parameters is not required in order to solve the problem. This way, more realistic and general case studies can be solved.

In interval analysis, an unknown parameter $x^{\pm}$ is expressed as an interval with an upper bound x^{+} and a lower bound x^{-}. If the parameter is continuous it can get any values inside $[x^{-}, x^{+}]$. In case of integer parameters, the upper and lower limits are also integer numbers and the feasible values are the integers inside these limits. Specifically for binary parameters the only possible values are the lower limit 0 and the upper limit 1.

An interval linear programming problem can be formulated as:

$$\min f^{\pm} = C^{\pm} X^{\pm} \tag{2a}$$

Subject to:

$$A^{\pm} X^{\pm} \le B^{\pm} \tag{2b}$$

$$X^{\pm} \ge 0 \tag{2c}$$

where $A^{\pm}, B^{\pm}, C^{\pm}, X^{\pm}$ are proper matrices, with elements that belong to a set of interval numbers $R^{\pm}$.

In order to solve this problem, two submodels must be formulated and solved [8]. If the objective function needs to be maximized, a submodel corresponding to f^{+} is formulated and based on its solutions another submodel corresponding to f^{-} is then formulated and solved. If the goal is to minimize the objective function, the model corresponding to f^{-} is formulated and solved first.

Assuming that $b^{\pm}$ are positive, the first submodel is:

$$\max f^{+} = \sum_{j=1}^{k_1} c_j^{+} x_j^{+} + \sum_{j=k_1+1}^{n} c_j^{+} x_j^{-} \tag{3a}$$

Subject to:

$$\sum_{j=1}^{k_1} |a_{ij}|^{-} Sign(a_{ij}^{-})\, x_j^{+}/b_i^{+} + \sum_{j=k_1+1}^{n} |a_{ij}|^{+} Sign(a_{ij}^{+})\, x_j^{-}/b_i^{-} \le 1, \forall i \quad (3b)$$

$$x_j^{\pm} \ge 0, \forall j \quad (3c)$$

where $x_j^{\pm}$ for $j = 1, \dots, k_1$ are continuous or discrete variables with positive cost coefficients and for $j = k_1 + 1, \dots, n$ are continuous or discrete variables with negative cost coefficients.

After solving the first submodel and obtaining the optimal values $x_{j,opt}^{+}$ for $j = 1, \dots, k_1$ and $x_{j,opt}^{-}$ for $j = k_1 + 1, \dots, n$, the second submodel is:

$$\max f^{-} = \sum_{j=1}^{k_1} c_j^{-} x_j^{-} + \sum_{j=k_1+1}^{n} c_j^{-} x_j^{+} \quad (4a)$$

Subject to:

$$\sum_{j=1}^{k_1} |a_{ij}|^{+} Sign(a_{ij}^{+})\, x_j^{-}/b_i^{-} + \sum_{j=k_1+1}^{n} |a_{ij}|^{-} Sign(a_{ij}^{-})\, x_j^{+}/b_i^{+} \le 1, \forall i \quad (4b)$$

$$x_j^{\pm} \ge 0, \forall j \quad (4c)$$

$$x_j^{-} \le x_{j,opt}^{+}, j = 1,2,\dots,k_1 \quad (4d)$$

$$x_j^{+} \ge x_{j,opt}^{-}, j = k_1 + 1, k_1 + 2, \dots, n \quad (4e)$$

After solving the two submodels we obtain the final solutions f_{opt}^{-}, f_{opt}^{+} and $x_{j,opt}^{-}, x_{j,opt}^{+}\ for\ j = 1, \dots, n$.

IV. CASE STUDY

A. Description of the problem

In order to demonstrate the methodology a case study is solved. We assume that there are eight potential power producers in the energy market, each one representing a different power production technology such as oil, gas, wind turbine, photovoltaic, biomass etc. It is assumed that the ISO evaluates these producers based on six criteria and gives them a marking that corresponds to each criterion. These criteria are Cr_1 for pricing per unit, Cr_2 for quantity produced, Cr_3 for CO_2 emissions per unit, Cr_4 for other emissions per unit, Cr_5 for strategic characteristics (%) and Cr_6 for social characteristics (%). The markings that correspond to each producer S_i for every criterion Cr_j are given in Table 1.

TABLE I
PRODUCER MARKINGS

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
CR_1	0.17	0.5	0.24	0.39	0.3	0.15	0.22	0.14
CR_2	40	95	150	65	100	80	120	50
CR_3	1	0.4	1.3	0.5	1.1	1.2	0.8	1
CR_4	1.8	0.4	1	0.6	0.6	1.2	0.9	1.2
CR_5	90	78	95	75	70	98	92	77
CR_6	85	75	95	80	85	92	79	82

Some of these criteria are quantitative but the ones that are qualitative are generally more difficult to be given a marking. However, a slight modification of a marking could result to a different solution, so this is another case that interval parameters could be used. In this study, it is assumed that the markings are deterministic and uncertainty exists in the weighting procedure and in the limits imposed by the ISO on the constraints of the problems.

From these markings and criteria, the corresponding costs of the ISO according to his decision vector $\boldsymbol{X}$ are obtained:

$$C_1(\boldsymbol{X}) = \sum_{i=1}^{5} \alpha_{1,i}\alpha_{2,i}x_i \quad (5)$$

$$C_2(\boldsymbol{X}) = \sum_{i=1}^{5} \alpha_{2,i}x_i \quad (6)$$

$$C_3(\boldsymbol{X}) = \sum_{i=1}^{5} \alpha_{5,i}\alpha_{2,i}x_i \quad (7)$$

$$C_4(\boldsymbol{X}) = \sum_{i=1}^{5} \alpha_{6,i}\alpha_{2,i}x_i \quad (8)$$

$$C_5(\boldsymbol{X}) = \sum_{i=1}^{5} \alpha_{7,i}x_i \quad (9)$$

$$C_6(\boldsymbol{X}) = \sum_{i=1}^{5} \alpha_{8,i}x_i \quad (10)$$

Costs that result from financial cost and gas emissions should be minimized, therefore they must be incorporated in the objective function with a negative sign, whereas the rest that represent positive notions are given a positive sign. Therefore, the optimization problem of the ISO is:

$$\max_{\boldsymbol{X}} \begin{pmatrix} -w_1C_1(\boldsymbol{X}) + w_2C_2(\boldsymbol{X}) - w_3C_3(\boldsymbol{X}) \\ -w_4C_4(\boldsymbol{X}) + w_5C_5(\boldsymbol{X}) + w_6C_6(\boldsymbol{X}) \end{pmatrix} \quad (11)$$

Subject to any constraints he wants to impose on specific costs.

The problem could also include some costs that are not linear but then linearization techniques would be necessary in order to reformulate it. Finally, the problem should become a mixed integer linear programming problem (MILP) and the interval analysis framework that was described can be implemented.

As mentioned, the difficulty of the ISO lies mainly in determining the weighting factors w_j but also the thresholds $C_{th,j}$ for some criteria. Therefore, every uncertain weight and threshold can be expressed as an interval parameter $w_j^{\pm} = [w_j^{-}, w_j^{+}]$ and $C_{th,j}^{\pm} = [C_{th,j}^{-}, C_{th,j}^{+}]$ respectively.

The final optimization problem of the ISO becomes:

$$\max_{\boldsymbol{X}} \sum_{j=1}^{6} w_j^{\pm} C_j(\boldsymbol{X}^{\pm}) \quad (12a)$$

Subject to:

$$C_j(\boldsymbol{X}^{\pm}) \le C_{th,j}^{\pm}, for\ some\ j \in \{1,2,\dots,6\} \quad (12b)$$

$$w_j^{\pm}, x_j^{\pm} \ge 0\ \forall j \quad (12c)$$

where vector $\boldsymbol{X}^{\pm} = (x_1^{\pm}, x_2^{\pm}, x_3^{\pm}, x_4^{\pm}, x_5^{\pm}, x_6^{\pm}, x_7^{\pm}, x_8^{\pm})$ represents the supply decision of the ISO regarding the eight available power producers.

Since the weights are expressed as intervals, their summation is also an interval and can't be equal to 1. However, this condition is not necessary in order to solve the problem because the weighting factors express the relative importance of each criterion and the initial assumption was used just to resemble percentages.

The reformulated MILP problem is solved with the two stage method presented in the previous section that requires the creation of two submodels for f_{opt}^{-} and f_{opt}^{+}.

B. Example with interval weights

In the first example, we assume that the ISO has decided on the exact thresholds he will impose on the costs as constraints, but he is uncertain about some of the weighting factors.

In this example, we assume that the ISO is uncertain about the weights w_1, w_5 and has decided on the rest. Assuming that $w_1^{\pm} = [0.1,0.5], w_2 = 0.3, w_3 = w_4 = 0.1$ $w_5^{\pm} = [0.2,0.4], w_6 = 0.2$ and the ISO imposes the constraint $C_1(\boldsymbol{X}) \leq 60$ so that the price will not exceed a certain limit. The results obtained after solving the two subproblems are:

$$f_{opt}^{\pm} = [138.10,297.98]$$
$$x_{1,opt}^{\pm} = [1,1]\,, x_{5,opt}^{\pm} = [0,0]$$
$$x_{2,opt}^{\pm} = [0,0]\,, x_{6,opt}^{\pm} = [1,1]$$
$$x_{3,opt}^{\pm} = [0,0]\,, x_{7,opt}^{\pm} = [1,1]$$
$$x_{4,opt}^{\pm} = [0,0]\,, x_{8,opt}^{\pm} = [1,1]$$

We observe that the final selection of the suppliers is robust and is not affected by this uncertainty. The optimal value of the objective function may vary within a certain limit since different weighting factors lead to different optimal values even for the same choice.

Attention must be given to the fact that the coefficients $c_j^{\pm}$ mentioned in Section 3 do not correspond to the coefficients $w_j^{\pm}$ of the objective function. We must first reformulate the objective function in order to find the coefficients of $x_j^{\pm}$'s.

In the previous example, all the coefficients of $x_j^{\pm}$ were positive and the constraints deterministic, therefore the problem generally is expected to have robust enough solutions. However, large uncertainty regarding multiple weight parameters is expected to decrease the system's robustness.

C. Examples with uncertainty in the constraints

When there is uncertainty in the constraints, the solutions' robustness should also be examined. Assuming that $w_1 = 0.5$ and the ISO gives the same importance equal to 0.2 to the rest of the criteria and is uncertain as far as the demand is concerned with $C_2(\boldsymbol{X}) \geq C_{th,2}^{\pm}$ and $C_{th,2}^{\pm} = [100,150]$, the results are:

$$f_{opt}^{\pm} = [123.49,123.49]$$
$$x_{1,opt}^{\pm} = [1,1]\,, x_{5,opt}^{\pm} = [1,1]$$
$$x_{2,opt}^{\pm} = [1,1]\,, x_{6,opt}^{\pm} = [1,1]$$
$$x_{3,opt}^{\pm} = [0,0]\,, x_{7,opt}^{\pm} = [1,1]$$
$$x_{4,opt}^{\pm} = [1,1]\,, x_{8,opt}^{\pm} = [1,1]$$

We observe that this is also a robust solution, meaning that this interval does not affect the optimal maximum value of the objective function and the optimal selection.

If however the uncertain constraint was the price, with $C_1(\boldsymbol{X}) \geq C_{th,1}^{\pm}$ and $C_{th,1}^{\pm} = [50,100]$, the results would be:

$$f_{opt}^{\pm} = [62.27,100.37]$$
$$x_{1,opt}^{\pm} = [1,1]\,, x_{5,opt}^{\pm} = [0,0]$$
$$x_{2,opt}^{\pm} = [0,1]\,, x_{6,opt}^{\pm} = [0,1]$$
$$x_{3,opt}^{\pm} = [0,0]\,, x_{7,opt}^{\pm} = [0,0]$$
$$x_{4,opt}^{\pm} = [1,1]\,, x_{8,opt}^{\pm} = [1,1]$$

In this example we observe that this uncertainty in the constraints affects the optimal solution. The optimal value of the objective function has a certain range depending on the limit imposed, since the weight parameters are considered deterministic. Moreover, the selection or not of some suppliers is robust, whereas the selection of some other suppliers depends on the uncertain limit imposed. More precisely, suppliers S_1, S_4, S_8 are surely selected and suppliers S_1, S_4, S_8 are surely not selected. Supplier's S_2 and S_6 selection is not robust. This could mean that if initially the ISO wants to be strict as far as his budget is concerned, they should be eliminated, but if in the end the available money are close to the upper bound of $C_{th,1}^{\pm}$ they should be selected.

D. Examples with interval weights and uncertainty in the constraints

The most difficult decision for the ISO is when he is uncertain about some of the weight parameters and regarding the thresholds of some constraints at the same time. However, this is a more realistic case and therefore the robustness of the optimal solution must be assessed.

In the next example, we assume that the weight decisions of the ISO are:

$$w_1^{\pm} = [3,4], w_2^{\pm} = [2,3]$$
$$w_3 = w_4 = 0.5$$
$$w_5^{\pm} = w_6^{\pm} = [2.4]$$

and the constraints he wants to set are:

$$C_1(\boldsymbol{X}) \leq C_{th,1}^{\pm} \text{ and } C_{th,1}^{\pm} = [100,200]$$
$$C_2(\boldsymbol{X}) \geq C_{th,2}^{\pm} \text{ and } C_{th,1}^{\pm} = [50,100]$$
$$C_3(\boldsymbol{X}) \leq C_{th,3}^{\pm} \text{ and } C_{th,1}^{\pm} = [200,250]$$

This means that he hasn't decided exactly on the importance of the criteria regarding price, quantity, strategic and social characteristics of the suppliers and he wants to limit the financial cost and the CO_2 emissions meeting at the same time the demand but he is not certain about the exact limits he will impose. The results are:

$$f_{opt}^{\pm} = [1077.8,3038.14]$$

$$x_{1,opt}^{\pm} = [1,1]\,, x_{5,opt}^{\pm} = [0,0]$$
$$x_{2,opt}^{\pm} = [0,1]\,, x_{6,opt}^{\pm} = [1,1]$$
$$x_{3,opt}^{\pm} = [0,0]\,, x_{7,opt}^{\pm} = [0,0]$$
$$x_{4,opt}^{\pm} = [0,0]\,, x_{8,opt}^{\pm} = [1,1]$$

We observe that the range of the optimal value of the objective function is larger since the values of the weight parameters are also greater and the objective function is scalable. As far as the robustness of the solution is concerned, only the selection of supplier S_2 is not sure.

In the next example it is assumed that the degree of uncertainty is greater as far as the weights and constraints' thresholds are concerned. Assuming that the weight decisions of the ISO are:

$$w_1^{\pm} = [3,6], w_2^{\pm} = [2,5]$$
$$w_3 = w_4 = 0.5$$
$$w_5^{\pm} = w_6^{\pm} = [2,6]$$

and the constraints he wants to set are:

$C_1(\boldsymbol{X}) \leq C_{th,1}^{\pm}$ and $C_{th,1}^{\pm} = [100,400]$
$C_2(\boldsymbol{X}) \geq C_{th,2}^{\pm}$ and $C_{th,1}^{\pm} = [50,250]$
$C_3(\boldsymbol{X}) \leq C_{th,3}^{\pm}$ and $C_{th,1}^{\pm} = [200,400]$

The results are:

$$f_{opt}^{\pm} = [1079.35,7510.1]$$
$$x_{1,opt}^{\pm} = [1,1]\,, x_{5,opt}^{\pm} = [0,0]$$
$$x_{2,opt}^{\pm} = [1,1]\,, x_{6,opt}^{\pm} = [0,1]$$
$$x_{3,opt}^{\pm} = [0,0]\,, x_{7,opt}^{\pm} = [0,1]$$
$$x_{4,opt}^{\pm} = [1,1]\,, x_{8,opt}^{\pm} = [1,1]$$

We observe that since the lower limits of the intervals haven't changed, the lower limit of the optimal objective function hasn't changed a lot. However its range has increased a lot because the range of the intervals, therefore the uncertainty has increased a lot too. Moreover, we observe that the optimal supplier selection has also changed as far as which suppliers will be surely selected or not and the solution is less robust since the uncertainty concerns two suppliers, S_6 and S_7. Consequently, the larger the uncertainty as far as the number of uncertain parameters and their range is, the more the optimal solution is affected.

Therefore, the proposed methodology provides the ISO with the necessary information in order to study the robustness of the optimal solution and how his decision should change depending on the circumstances. If, however, the ISO had assumed a Gaussian distribution for the uncertain parameters and the problem was solved for values of the uncertain parameters equal to the average of their upper and lower limits, which would be the most probable scenario, the optimal decision would be:

$$x_{1,opt} = 1\,, x_{2,opt} = 1, x_{3,opt} = 0\,, x_{4,opt} = 1$$
$$x_{5,opt} = 0\,, x_{6,opt} = 1, x_{7,opt} = 0\,, x_{8,opt} = 1$$

but the ISO wouldn't have any other information as far as the robustness of the solution is concerned. In the optimistic scenario with loose constraints supplier S_7 should also be selected, whether in the pessimistic scenario with strict constraints supplier S_6 shouldn't be selected. Therefore, the interval analysis framework provides a decision support tool much more useful for the ISO. Moreover, the advantage of the interval approach instead of a best and worst case scenarios is that the range of the optimal value of the objective function corresponds to a set of feasible and robust solutions for the decision variables respectively. Therefore, the different potential decisions can be better assessed and the decision making procedure is facilitated.

V. CONCLUSIONS

The deregulation of the energy markets and the new requirements that need to be met by the system operators have created a new framework as far as energy planning and decision making is concerned. One of their tasks is the energy market clearance and the selection of the power producers that are going to supply energy.

This task should nowadays be treated as multicriteria problem since social welfare involves low financial cost but other factors as well. However, the importance and desired limits for each of these criteria can't always be decided accurately. That is why a decision support tool was created, based on MILP, where interval analysis is used in order to address uncertainty regarding these parameters which is practical for engineering problems since the distribution information of the uncertain parameters is usually not needed but also difficult to find. Furthermore, this approach is superior to finding the best and worst case scenario providing ISO with a set of feasible and robust solutions.

Further research could include the use of the proposed model in order to study the robustness of the solutions with uncertain suppliers' markings and the insertion of nonlinear criteria after proper reformulation.

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VII. Biographies

Panagiotis Kontogiorgos received the Diploma from the School of Electrical and Computer Engineering of National Technical University of Athens where he is currently a Ph.D. candidate. His interests include game theory, mathematical programming and optimization with applications on the energy sector.

Michael N. Vrahatis was born in Kalamata, Greece, on April 27, 1955. He received his Ph.D. in Mathematics in 1982 from the University of Patras, Greece. His work includes topological degree and fixed point theory, systems of nonlinear algebraic and transcendental equations, nonlinear dynamics and complexity, numerical and intelligent optimization, data mining and unsupervised clustering, intelligent cryptography, intelligent music, bio–inspired computing, biomedical informatics as well as computational and swarm intelligence. He is a professor in the Department of Mathematics at the University of Patras since 2000. He was also a visiting research fellow and a visiting professor to several universities and he serves/served as honorary editor–in–chief, honorary advisor, editor–in–chief, associate editor–in–chief, managing guest editor, special issue editor or member of the editorial board of seventeen international scientific journals.

George P. Papavassilopoulos received the Diploma from the Department of Mechanical and Electrical Engineering, National Technical University of Athens, Greece, in 1975 and the M.S. and Ph.D. degrees from the Department of Electrical Engineering, University of Illinois at Urbana-Champaign, in 1977 and 1979, respectively. He was a Professor in the Department of EE-Systems, University of Southern California, Los Angeles, from 1979 to 2000 when he joined the Department of Electrical and Computer Engineering, NTUA, as Professor. He has conducted research in dynamic and stochastic game theory, optimization and control theory, parallel algorithms, Markovian learning, BMIs for robust control. He is also interested in applications in biomedical engineering, energy and telecommunication policies.