SUBSIDIZED POWER INTERRUPTION FOR REDUCING PEAKS IN ENERGY DEMAND: A STACKELBERG GAME

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ABSTRACT

The insertion of new technologies in energy markets has created a framework full of opportunities concerning better energy planning. This study addresses the use of Power Supply Interruption as a tool for reducing consumers' demand voluntarily, also known as Voluntary Load Curtailment (VLC) programs. In VLC, a fee similar to a subsidy is paid to a consumer so as not to use energy for a certain time period. We consider a power producer and a group of consumers with different characteristics concerning their energy consumption and the level of interruption they can accept. For several types of VLC schemes and fee compensations, this problem is formulated as a static Stackelberg game which is addressed with use of bilevel programming methods. We seek to find the interaction between the compensation offered and the best decision for each player. Some first results show that all players could benefit from a VLC program at the same time.

Index Terms— energy market, load curtailment, demand response, bilevel programming, Stackelberg game algorithms

1. INTRODUCTION

In deregulated energy markets, load control programs are implemented in order to affect consumers' demand. These programs use demand response in order to persuade consumers to reduce their energy consumption by using price signals or other incentives [1,2]. All these programs can be considered as part of the ancillary services offered in energy markets. As a result, consumers interact with the other players of the energy market and these interactions can be modeled with use of game theory and solved with various optimization methods [3].

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Voluntary Load Curtailment (VLC) programs are such a service, used as an incentive and not a punishment for the consumers in order to flatten the demand curve and cut off its peaks. In a VLC program, the ISO or a power producer considers it would be beneficial to reward the consumers if they reduce their energy demand. Therefore, a fee is announced for the consumers so as not to use an amount of energy in a certain time period, meaning that they could benefit from accepting this curtailment voluntarily. Based on the consumers' response, the player who offers the subsidy decides on the best VLC scheme. In game theory, this is known as the Leader-Follower or Stackelberg game [4]. In this case, the power supplier is the leader who decides on the subsidy offered while the consumers follow by responding to this. A static Stackelberg game is generally very complex and difficult to solve because of nonlinearities and nonconvexities that arise. The first algorithmic attempts to solve these kind of static Stackelberg problems were [5-12]. For recent surveys of algorithms in this area the interested reader can see [3,13,14].

VLC programs have started to be implemented during the last few years with success, especially in US [15]. It has been shown empirically that VLC programs can reduce demand [16], therefore they can be implemented either in high price periods or in system security emergencies. Until recently, such programs were aimed only at large-scale power consumers. However, the technological progress and the emersion of Smart Grids have made demand response programs even more important [17]. With use of smart meters, any consumer connected to the energy grid can participate in VLC programs. This new capability along with the insertion of new technologies creates a whole new framework that can be used for better energy planning, making the market operation even more economic and reliable by extending the positive effects.

In this paper, we model the problem assuming a power producer offers a VLC program to various consumers so as to study if and how all players can be benefited from it. Each player has his own characteristics, like the level of discomfort he can endure and the amount of energy he needs for his basic needs if he is a consumer, or his production function and supply cost if he is the producer. We seek to find the optimal response of the consumers at a certain time period and the profits for each player.

The paper is structured as follows. In Section 2, the description of the problem and the assumptions made are presented. In Section 3, the mathematical formulation of the problem is derived and in Section 4, two simple examples are solved. Finally, in section 5, we present the conclusions and future extensions of the research.

2. DESCRIPTION OF THE PROBLEM

The energy market has a supply and a demand function that represent the aggregate supply of all power producers and the aggregate demand of all consumers as functions of the quantity of energy. These functions are assumed to be linear for simplicity reasons without affecting the theoretical model. At the intersection of these functions is the energy market's equilibrium that results in the amount of energy demanded and its price. This equilibrium can be found before the actual clearing of the market takes place since the supply function results from the power producers' bids and the demand function is estimated by using historical data. When the amount of energy or/and the price are expected to be too high, there is need to interfere for economic and stability reasons in order to move the equilibrium lower. This can be done through VLC programs. Each consumer knows his own expected demand and this information is also known to the producer, along with the total estimated demand, because of the smart meters installed. This way the consumers cannot lie about their intentions so as to receive higher fees. Eventually, consumers will cede an amount of energy voluntarily if this is in their interest, which will be subtracted from the quantity of energy that they would request, thus changing the expected equilibrium in the real-time energy market.

The important element of a VLC program is the fee that will be paid to the consumers. This fee is a subsidy that operates as an incentive so as to persuade them to voluntarily reduce their energy demand. It can be offered by the ISO of the market that aims to maximize social welfare by ensuring network's stability or a power producer if he judges that it is in his own interest. This could happen if the producer needs to supply more energy than he can currently produce due to false predictions, accidents that result in failures etc. In that case, the producer has to buy some energy from others or to operate costly units in order to meet his obligations. A VLC program could be also used in order to postpone costly new investments. However, the ISO is financially neutral and can have no deficit, therefore he would need to introduce an extra cost to the consumers equal to the sum of money he will need for the subsidies. This scheme cannot easily motivate the consumers and this is why we assume that the fee is offered by power producers that are private organizations.

In order to model VLC we need to take into consideration the costs and gains that a load curtailment introduces to each player. As far as a consumer is concerned, except for the money he pays for the quantity of energy demanded, any load curtailment introduces a fee gain and a comfort cost to him because he won't be able to use that amount of energy when he initially intended. Comfort cost and fee gain may be either fixed or associated, linearly or nonlinearly, with the quantity of energy ceded by the consumer. Comfort cost's parameters depend on the consumer, thus differentiating the consumers and their best response to each VLC scheme, whereas the fee parameters are provided by the producer and are assumed to be the same for all type of consumers. In order for the consumer to participate in a VLC program voluntarily, the subsidy he receives should be greater than his comfort cost.

As far as the producer is concerned, he receives money for the quantity of energy supplied. This quantity is separated into the quantity that can be currently produced by him and the quantity that is acquired from other resources at a higher cost. These quantities have their own cost functions. The load curtailed introduces an extra cost which is the fee paid to the consumers but the producer expects to be benefited in the end, because this load would have otherwise to be supplied and would cost a lot. In order to implement the VLC program requesting the consumers' permission to curtail load, the producer should first examine what his expected gain from it will be.

Consequently, for any VLC scheme, the producer and each consumer seek to benefit by optimizing their own objective functions. The producer, however, has an optimization problem depending on the consumers' response to his fee. Therefore, the consumers' optimization problems act as constraints for the producer and the problem becomes a Stackelberg problem with the producer as the leader.

3. MATHEMATICAL FORMULATION

In our problem we assume that the market consists of a power producer and n consumers, $n = 1,2, ...$ If q represents the quantity of energy, the supply function is $f_s = b_1 q, b_1 \geq 0$ and the aggregated demand function of the n consumers is expected to be $f_d = a_2 - b_2 q$ with $a_2, b_2 \geq 0$. For the supply to meet demand $f_s = f_d$ must hold. The resulting expected equilibrium values for price and quantity of energy are $p^* = \frac{b_1 a_2}{b_1 + b_2}$ and $q^* = \frac{a_2}{b_1 + b_2}$ respectively.

After a VLC program is implemented, consumer i wants to minimize his cost. Therefore his problem is $\min^{i}_{q_{d,i}, q_{c,i}} \left(p q_{d,i} + C_i - F_i \right)$, where $q_{d,i}$ is the quantity of energy consumer i demands (the total energy finally demanded is $q_d = \sum_i q_{d,i}$), p is its price, $C_i = c_{1,i} q_{c,i}^{n_i}$ is his comfort cost and $F = r_1 q_{c,i}^m$ is his fee gain, where $c_{1,i}, n_i, r_1, m \geq 0$ and $q_{c,i}$ is the load curtailed to the consumer i. For this load, $q_{c,i} = q_i^* - q_{d,i}$ holds, where q_i^* is his expected demand. Each consumer has his own basic needs and preferences, so there is a certain amount of energy $q_{min,i}$ he won't accept to cede, thus $q_{d,i} \geq q_{min,i} \geq 0$. Price p is obtained from the clearing of the market, so $p = b_1 q_d$.

Therefore, for every consumer i, $i = 1, \dots, n$, the problem is:

$$\min_{q_{d,i}, q_{c,i}}{}^{\mathrm{i}} \left(p q_{d,i} + c_{1,i} q_{c,i}^{n_i} - r_1 q_{c,i}^m \right)$$

subject to:

$$c_{1,i} q_{c,i}^{n_i} < r_1 q_{c,i}^m$$
$$q_{c,i} = q_i^* - q_{d,i}$$
$$q_{d,i} \geq q_{min,i}$$

and $p = b_1 q_d = b_1 \sum_i q_{d,i}$ is a joint constraint for all the consumers. This problem is a nonlinear Nash game among the consumers. The interaction of the consumers' variables which we observe in the constraints sets is an example of a generalized Nash equilibrium [18].

On the other hand, by implementing a VLC program, the power producer also wants to minimize his cost. The supplied energy is q_s. Moreover, $q_s = q_{pr} + q_A$, where q_{pr} is the quantity of energy that the producer currently produces on his own and q_A the quantity of energy that he has to acquire by other means. The cost function of the produced energy is $C_{pr}(q_{pr}) = c_3 q_{pr}$, $c_3 \geq 0$ and is assumed to be linear. The cost function of the energy acquired elsewhere is assumed to be nonlinear and is given by $C_A(q_A) = M(q_{max} - q_{pr}) + c_4 q_A^2$, where $c_4 \geq 0$ and M is a very large positive number ensuring the producer will resort to this alternative only if he can't currently produce the energy on his own. The total fee paid to the consumers is $F = \sum_i F_i$. So, the producer's problem is $\min_{q_{pr}, q_A, q_s, q_{c,i} \forall i} \left(C_{pr}(q_{pr}) + C_A(q_A) - p q_s + F \right)$ and by substituting the costs and gains: $\min_{q_{pr}, q_A, q_s, q_{c,i} \forall i} \left(c_3 q_{pr} + M(q_{max} - q_{pr}) + c_4 q_A^2 - p q_s + \sum_i r_1 q_{c,i}^m \right)$.

Of course, $q_{pr}, q_A, q_s \geq 0$ should hold. Moreover, q_{pr} has an upper limit q_{max} equal to a percentage of the installed capacity q_{in}. So, $q_{pr} \leq q_{max}$, where $q_{max} = a_3 q_{in}$ and $0 \leq a_3 \leq 1$. For the supplier, the energy finally supplied along with the energy q_c totally curtailed, $q_c = \sum_i q_{c,i}$, should be equal to the energy that would be demanded if the VLC program was not implemented. Therefore, $q_s + q_c = q^*$ or $q_{pr} + q_A + q_c = q^*$. The clearing of the market is common so $p = b_1 q_s$ and $q_s = q_d$ hold.

In the end, the producer optimizes his objective function based on the decisions of the consumers who evaluate his fee proposal. The bilevel problem with the producer as the leader is:

$$\min_{q_{pr}, q_A, q_s, q_{c,i} \forall i} \left(c_3 q_{pr} + M(q_{max} - q_{pr}) + c_4 q_A^2 - p q_s + \sum_i r_1 q_{c,i}^m \right)$$

subject to:

$$\left.\begin{aligned}
& q_{pr}, q_A, q_s \geq 0 \\
& q_{pr} \leq q_{max} \\
& q_s = q_{pr} + q_A \\
& q_s + q_c = q^* \\
& q_d = \textstyle\sum_i q_{d,i} \\
& q_c = \textstyle\sum_i q_{c,i} \\
& \min{}^{\mathrm{i}}_{q_{d,i}, q_{c,i}} \left(p q_{d,i} + c_{1,i} q_{c,i}^{n_i} - r_1 q_{c,i}^m \right) \\
& \text{subject to:} \\
& c_{1,i} q_{c,i}^{n_i} < r_1 q_{c,i}^m \\
& q_{d,i} \geq q_{min,i} \\
& q_{c,i} = q_i^* - q_{d,i}
\end{aligned}\right\} \forall i, i = 1, \dots, n$$

with $q_s = q_d$ and $p = b_1 q_d$ as joint constraints.

Thus, we have a static Stackelberg game with nonlinear costs. As mentioned, the first algorithmic attempts to solve these kind of static Stackelberg problems were [5-12]. For recent surveys of algorithms in this area the interested reader can also see [3,13,14].

4. RESULTS

Two simple examples with one producer as the leader and two consumers as followers are presented, so as to show that all players could be benefited at the same time from the implementation of a VLC program. The examples were solved using GAMS software. We assume that $b_1 = 10, a_1 = 1, a_2 = 132$, so $p^* = 120, q^* = 12$. It is assumed that the two consumers have the same expected energy demand, $q_1^* = q_2^* = 6$.

For the first example, the parameters have the following values:

$$c_3 = 5, q_{max} = 7, c_4 = 50, r_1 = 3, m = 1$$
$$c_{1,1} = 2, c_{1,2} = 1.5, n_1 = n_2 = 1, q_{min,1} = 4, q_{min,2} = 3.5$$

This means that the fee and comfort costs functions are linear and the two consumers have different comfort cost parameters and minimum energy quantities needed. The results are:

$$q_{pr} = 7, q_A = 0.5, q_s = q_d = 7.5, p = 75$$
$$q_{c,1} = 2, q_{d,1} = 4, q_{c,2} = 2.5, q_{d,2} = 3.5$$

the producer's cost is -501.5, the first consumer's cost is 298 and the second consumer's cost is 264. The values of the costs if the VLC program was not implemented would be -155, 720 and 720 respectively.

For the second example, the parameters have the following values:

$$c_3 = 5, q_{max} = 7, c_4 = 50, r_1 = 8, m = 1$$

$$c_{1,1} = 6, c_{1,2} = 5, n_1 = 1, n_2 = 2, q_{min,1} = 4, q_{min,2} = 3.5$$

This means that in this case, the comfort cost of the second consumer becomes nonlinear. The comfort cost parameters are again different and the minimum energy quantities needed are as in the first example. The results are:

$$q_{pr} = 7, q_A = 1.4, q_s = q_d = 8.4, p = 84$$

$$q_{c,1} = 2, q_{d,1} = 4, q_{c,2} = 1.6, q_{d,2} = 4.4$$

the producer's cost is -543.8, the first consumer's cost is 332 and the second consumer's cost is 369.6. The values of the costs if the VLC program was not implemented would be -155, 720 and 720 respectively.

From these two examples some basic conclusions can be drawn. First of all, the consumers are motivated to reduce their demand depending on the fee they will get. This reduction could include all the quantity they don't consider important or a part of it. Moreover, the best response also depends on the comfort cost function, so it is different for each category of consumers and this is something the producer should take into consideration. Furthermore, the producer is benefited too by a demand reduction since he won't have to supply very expensive energy. Therefore, if the producer can currently satisfy all the expected demand he doesn't need to implement a VLC program. In any case, the result could be a lower equilibrium with less demand and at a lower price too.

Much attention must be drawn to the fact that consumers may be motivated to act strategically endangering the network's stability. This effect can however be prevented by monitoring the energy consumption so that the producers can forecast each consumer's demand based on historical data.

5. CONCLUSIONS

The problem of subsidizing the consumers so as to reduce their energy demand was described and formulated as a static Stackelberg game. The solution of such a game is very difficult because of the structure of the problem and there are various algorithms attempting to find the global minimum which are constantly evolving.

However, load curtailment is a way of reducing the energy demand in case of emergency or high prices. Producers and consumers can both be benefited from the implementation of a VLC program. Smart grids offer new possibilities concerning each consumer's consumption control and monitoring, making the participation in such a program possible for all consumers. Therefore, the advantages multiply and power interruption could be considered as an incentive mechanism. Extensions of this study could be the case with many leaders and a dynamic setup of the game.

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