ENERGY OPTIMISATION OF THE WATER SUPPLY SYSTEM

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INTRODUCTION

The reduction of energy cost, as well as the restriction of the continuously increasing need of energy, constitute an important issue. Greece facing problems of overloading the energy distribution network, particularly during the summer months, has increased the effort for an optimal management of consumption and distribution. The critical situation of the management of water resources is intensified by the ineffective decisions, lack of planning and programming, the under-estimation of problems of water quality and quantity, and the delay of facing the environmental factors. In the area of Chania, as in the majority of provincial cities of Greece, the water needs are satisfied through an interlinked network of drillings, pumps and water tanks. The distribution of water into the tanks and in the final consumption takes place via a network of electrical pumps.

Our objective is the development of an energy management software, operates the pumps taking into consideration the optimisation of the energy cost at the Public Power Corporation of Greece (PPC) and the total energy consumption, based on the safety level limits of the tanks, the frequency of restarting the pumps and t he profile of demand of energy and water.

The basic characteristic of the proposed solution is the application of optimal control methods. The basic method of operation should finally materialise itself without the necessity of frequent human intervention.

In this paper we formulate the problem as a dynamic problem in discrete time, linear as regards the state and the control, with linear constrains on the state and control values. The cost is nonlinear in the case we examine. This not-linearity is due to the way of pricing of the Public Power Corporation of Greece (PPC), that associates the cost with the maximum value in time of the consumed energy, aiming at the standardisation of the requirements of consumers and the restrictions of demand peaks. Using a simple transformation we change equivalently the cost from nonlinear to linear with increased dimensions and thus we can resolve the problem using linear programming. This aims at avoiding the use of complicated not linear or stochastic optimisation methods[see 5]. This model is applied to various cases of requirements and finds the optimal solutions. Related work dealing with regulation of water pump systems has also been reported in references 1-5.

The model is easy to extend to implement more general cases with more pumps and other possible restrictions.

Knowledge area: Energy Management of Aquatic Resources – Optimal Control

Words keys: energy optimisation, management of aquatic resources, optimal control, linear and nonlinear optimisation, cost function,.

EQUIVALENT PROBLEM DESCRIPTION AS A PROBLEM OF LINEAR OPTIMAL PROBLEM

The state equation of our problem is:

$$x_{k+1} = a * x_k + b * u_k \qquad k = 0,1,2,\ldots..N \qquad (1)$$

The equation that describes the functional cost, that is to be minimised is:

$$J = f(u_o, \ldots u_N) \qquad (2)$$

Subject to the restrictions:

$$c_k * u_k + d_k * u_k \leq d_k \qquad (3)$$

a, b, c, d are constant matrices with specific dimensions and c_k, d_k vectors. x is the state ,u is the control and are vectors.

The cost function can be linear (see equation 8) or nonlinear (see equation 12), depending on the method we use to calculate the charge which we have to pay at the Public Power Corporation of Greece (PPC). This problem can be solved by using the Minimum Principle of Pontryagin for systems of discrete time or dynamic programming (see reference 6,8).

If the time duration N is short, then we can handle the problem directly by transforming it directly to a linear or nonlinear programme with faster and more direct results. This method is the one we use in our paper.

PRESENTATION OF OUR CASE STUDY

Visiting the area of Chania and collecting information from the installed SCADA system at the area we draw the network. By some simplifications of the network, we consider the existence of tanks of big quantity of water only in the areas:

- Ag.Iwannis
- Vante
- Agyia

The rest of the areas are presented only as consumptions that they consume quantities of water depending on the demands. The water paths in the network are shown at the Figure 1.

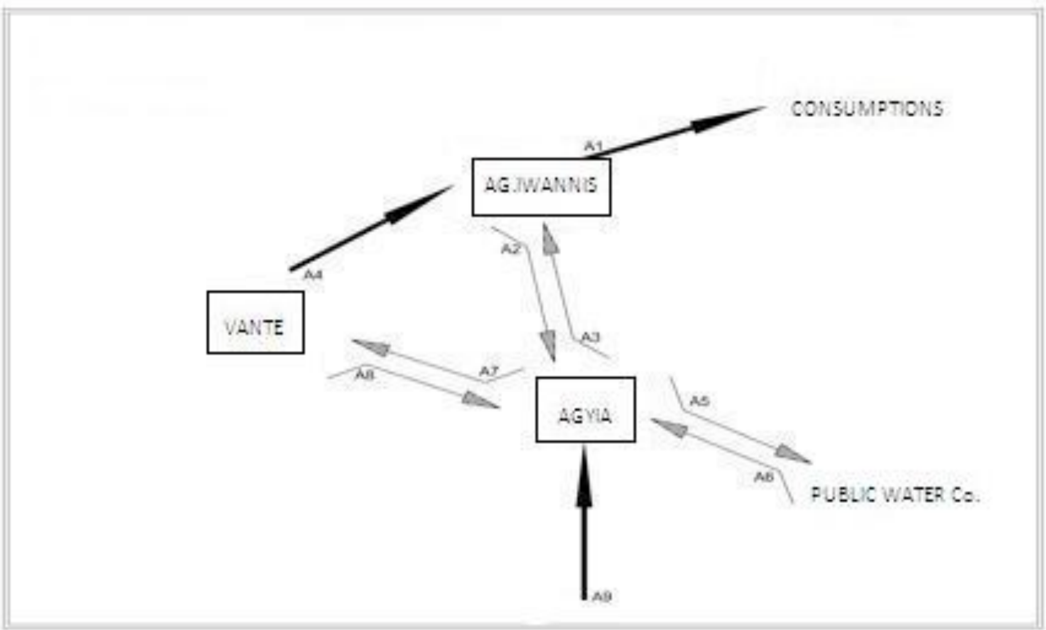


Figure 1

The examined system of Figure 1 is constituted by 3 tanks.The one at Vante will be symbolized as W, the one at Ag.Iwannis as V and the tank at Agyia as Q. The sampling of water level will be every L hours. The transfer water rate (m^3/h) to a water tank is symbolised as A1,A2… Ai ,where "i" is the number of the water paths (see Figure 1). There is always a water pump between any two tanks and in general there is always a pump between the arrival ant departure of a water link. On theses pumps there are switches that are symbolized as $DA1_\kappa, DA2_\kappa, DA3_\kappa ....DAi_\kappa$. We denote with L the hours between two checks and we denote k (k=24/L) the time levels when the sampling occurs.

PARAMETRIZATION – MODELING OF THE SYSTEM

The quantity of water (m^3) in each tank is denoted as W_k, V_k, Q_k, It holds:

$$W_{K+1} = W_K + L*A7*DA7_K - L*A8*DA8_K - L*A4*DA4_K$$
$$V_{K+1} = V_K + L*A4*DA4_K + L*A3*DA3_K - L*A2*DA2_K - L*A1*DA1_K$$
$$Q_{K+1} = Q_K + L*A2*DA2_K + L*A8*DA8_K + L*A9*DA9_K + L*A6*DA6_K -$$
$$-L*A7*DA87_K - L*A5*DA5_K - L*A3*DA3_K \qquad (4)$$

These are the state - equations describing the evolution of the state (W_k, V_k, Q_k) of the system.

At our system the tank water supplies A_k are fixed and there are 9 supplies: A1 – A9 and at different time levels there different quantities of water in each tank.

The tanks have initial quantity of water: Wo,Vo,Qo respectively.

Also according to the geometrical specifications of the tanks and the demand profile of each area, there is a minimum and a maximum limit of water in each tank.

Thus, it has to hold:

$$W_{\min} \leq W_1, W_2, W_3, W_4 \leq W_{\max}$$
$$V_{\min} \leq V_1, V_2, V_3, V_4 \leq V_{\max}$$
$$Q_{\min} \leq Q_1, Q_2, Q_3, Q_4 \leq Q_{\max} \quad (5)$$

The basic target of our paper is the determination of the variables D ij (D ij: pump switch of water path i at period time j). We consider that these variables are analog, and their value is between 0-1 depending of the necessity for power of each pump.
In our study we consider the cost of electricity R(euro/KWh) is a constant.
According to the Figure 1 the analytic parametric equations of the three tanks are:

$$W_{K+1} = W_K + L*A7*DA7_K - L*A8*DA8_K - L*A4*DA4_K \quad (6a)$$
$$V_{K+1} = V_K + L*A4*DA4_K + L*A3*DA3_K - L*A2*DA2_K - L*A1*DA1_K \quad (6b)$$
$$Q_{K+1} = Q_K + L*A2*DA2_K + L*A8*DA8_K + L*A9*DA9_K + L*A6*DA6_K - -L*A7*DA87_K - L*A5*DA5_K - L*A3*DA3_K \quad (6c)$$
(6)

where
- t: time sampling rate
- Dij: power switch of the pumps (0-1)
- Ak: tank water supplies
- k: time levels

As we have mentioned our control variables are the switches DAij and we use the notation Xi(X1,X2,…Xn) for each of the switches.
So the equations (6) become:

$$W_{K+1} = W_k + A7*Xi - A8*Xj - A4*Xn \quad (7a)$$
$$V_{K+1} = V_k + A4*Xb + A3*Xx - A2*Xv - A1*Xc \quad (7b)$$
$$Q_{K+1} = Q_K + A2*Xe + A8*Xp + A9*Xf + A6*Xu - -A7*Xj - A5*Xn - A3*Xm \quad (7c)$$
(7)

The technical specifications of the pumps are:
- J: electrical force of the pump (kw)
- R: Euro per kWh (Euro / kwh)

METHOD OF SYSTEM OPTIMISATION

Case 1: Linear cost
Here we consider the price of electricity as fixed and that cost of electricity(euro per kWh) includes only the total used KWh's. Thus we want to minimise the cost function f (x):

$$f(x) = \sum_{k=0}^{3} L*R*J(DA1_k + DA2_k + \ldots\ldots + DA9_k) \quad (8)$$

The pumps switches X_{ij} can use a percentage, from 0% to 100%, of their power depending on the system demand:

$$0 \leq X_{ij} \leq 1 \quad (9)$$

We can achieve this by using switches with power steps between 0% and 100% of the maximum power.
Minimising (8) subject to (5) and (9) is a linear programming problem.
The next step, after having written the descriptive equations, the cost function and the range value limits of the variables, is to use linear programming in the Matlab environment as it appears below: The system is:

$$A*X \leq b$$
$$LBnb \leq X \leq UBnb$$
$$\cos t\_function\_f(x)$$

These matrices describe the system including the water requirements and the ability for water supply o the pumps. Our aim is to study the nonlinear case.

Case 2: Nonlinear Cost
Now we present how the PPC calculates the cost of the consumed energy, whereby it also attempts to reduse the peak demands. Including all the others restrictions and conditions we create the equation that is our cost function.
As it was reported above there are 3 tanks: W, V, Q.

- The sampling rate is every 6 hours: Kmax=4 , k=1,2,3,4
- The water flows m^3/h are symbolized as A1...A9
- The switches (per hour): αi(k): X1,X2,.......,X36 (i: 1-9, κ: 1-4)
- Time step of simulation: L=6h
- Number of Pumps: i=1-9, n=9
- Electrical Force of Pumps: Pi(k)
- αi(k)= % percentage of using power of pump at the moment k --> Xj, j=1-36

Total Force:

$$Pc = [\sum_{k=1}^{4}\sum_{i=1}^{9} 20 * a_i(k)] * 6 = 120 * \sum_{k=1}^{4}\sum_{i=1}^{9} a_i(k)$$

$$P\max = \max_k [\sum_{i=1}^{n} a_i(k) Pi] \qquad (11)$$

The system operation cost expressed by the equation (12) which reflects the tariff of PPC without taking into consideration, for now, the **cos(x)** of the pumps:

$$J = 400 * C2 * \max_k [\sum_{i=1}^{n} a_i(k) * Pi] + C3 * \max(0, \sum_{k=1}^{k\max} (\sum_{i=1}^{n} a_i(k) * Pi - 400)) \qquad (12)$$

Where:

$$Z2 = C3 * \max(0, \sum_{k=1}^{k\max} (\sum_{i=1}^{n} a_i(k) * Pi - 400))$$

$$Z1 = 400 * C2 * \max_k [\sum_{i=1}^{n} a_i(k) * Pi] \qquad (13)$$

So the cost (2) is **J = Z1+Z2** (14) subject to:

$$Z1 \ge a1(1) * P + a2(1) * P + ...+ a9(1) * P$$
$$Z1 \ge a1(2) * P + a2(2) * P + ...+ a9(2) * P$$
$$Z1 \ge a1(3) * P + a2(3) * P + ...+ a9(3) * P$$
$$Z1 \ge a1(4) * P + a2(4) * P + ...+ a9(4) * P \qquad (15)$$

$$Z2 \ge 0$$

$$Z2 \ge -4 * 400 + 120 * \sum_{i=1}^{36} Xi$$

It's obvious that minimising (12) subject to (15) equivalently we convert the problem from nonliner form to linear by introducing the new variables Z1,Z2 (see (13)), which is not a big burden at all.

There are 38 variables in our system. The 36 are associated with the 9 pumps and with the percentage of operation of each pump per 6 hours (L=6) [a (k) (i: 1-9, k: 1-4)] while the 2 other are the variables Z1 and Z2 from the cost function.

We have thus formulated our problem as

$$A * X \le b$$
$$LBnb \le X \le UBnb$$
$$\cos t\_function\_f(x)$$

We can use linear programming in the Matlab environment.

APPLICATIONS

Taking into consideration the system, the requirements for water supply and irrigation of the Chania area we present 5 operation scenarios of our system and we present the proper percentage of operation of each pump every 6h in a 24h operation.
We also have these datas for our system:

- Time sampling rate: 6 hours (L=6)
- Transfer water rates:

A1=950 m^3/h	A4=550 m^3/h	A7=750 m^3/h
A2=700 m^3/h	A5=600 m^3/h	A8=650 m^3/h
A3=850 m^3/h	A6=650 m^3/h	A9=950 m^3/h

- At different time levels there are different quantities of water in each tank. More specifically:

Time	**Symbolism of Tanks**
6:00	W1,V1,Q1
12:00	W2,V2,Q2
18:00	W3,V3,Q3
24:00	W4,V4,Q4

- Initial water quantities in the tanks: Wo = 1500 m^3, Vo = 2500 m^3, Qo = 3500 m^3 respectively
- Minimum and maximum safety levels:

$$W_{\min} = 1000m^3, W_{\max} = 5500m^3,$$
$$V_{\min} = 2000m^3, V_{\max} = 6500m^3,$$
$$Q_{\min} = 3000m^3, Q_{\max} = 8000m^3$$

The technical specifications of the pumps are:

- J: electrical force of the pump (kw) - 20 kW
- R: Euro per kWh (Euro / kwh) – 0,4 € / kWh

The 5 scenarios are:

1. **Scenario 1:** The pump 1 that supplies the main consumers of the system has a minimum point of operation at 80% and the pump 9 that draws water from the earth has minimum operation at 40%. The remaining pumps function from 0 to 100%.
2. **Scenario 2:** The pump 1 has the minimum point of operation at 80% and the pump 9 at 40%.The remaining pumps operate from 10% to 100%.
3. **Scenario 3:** The pump 1 has the minimum point of operation at 80% and the pump 9 at 40%.The pumps 5,6 can be inactive and the rest of the pumps have minimum operation at 40%.
4. **Scenario 4:** The pump 1 has the minimum point of operation at 100% and the pump 9 at 40%. The pumps 5,6 can be inactive and the remaining pumps have minimum operation at 40%.
5. **Scenario 5:** The pump operates at 100% and the pump 9 has minimum operation at 40%. The remaining pumps function from 40% until 100% while pumps 5,6 function as minimum 30 %.

In the following tables we show the results of optimising for the 5 scenarios and their associated costs.

- **Scenario 1**

Pump	6h scale	**1° (6:00)**	**2° (12:00)**	**3° (18:00)**	**4° (24:00)**
	1	80%	80%	80%	80%
	2	0%	0%	0%	0%
	3	65.09%	86,79%	83,45%	81,15%
	4	48,48%	0%	0%	0%
	5	0%	0%	0%	0%
	6	0%	0%	0%	0%
	7	24,44%	0%	0%	0%
	8	0%	0%	0%	0%
	9	40%	42,74%	46,08%	48,38%
Factors Z1/Z2		Z1=41,9059		Z2=0	
COST		41,9059			

- **Scenario 2**

Pump	6h scale	**1º (6:00)**	**2º (12:00)**	**3º (18:00)**	**4º (24:00)**
	1	80%	80%	80%	80%
	2	10%	10%	10%	10%
	3	67,30%	91,98%	90,08%	85,50%
	4	40,98%	10%	10%	10%
	5	10%	10%	10%	10%
	6	10%	10%	10%	10%
	7	45,60%	10%	10%	10%
	8	10%	10%	10%	10%
	9	40%	40,92%	42,83%	47,40%
Factors Z1/Z2		Z1=54,5807		Z2=0	
COST		54,5807			

- **Scenario 3**

Pump	6h scale	**1º (6:00)**	**2º (12:00)**	**3º (18:00)**	**4º (24:00)**
	1	80%	80%	80%	80%
	2	40%	40%	40%	40%
	3	77,08%	99,47%	97,25%	92,69%
	4	54,82%	40%	40%	40%
	5	12,53%	0%	0%	0%
	6	0%	0%	0%	0%
	7	100%	49,04%	50,99%	55,72%
	8	40%	40%	40%	40%
	9	40%	41,10%	41,37%	41,20%
Factors Z1/Z2		Z1=77,9220		Z2=335,9078	
COST		413,8298			

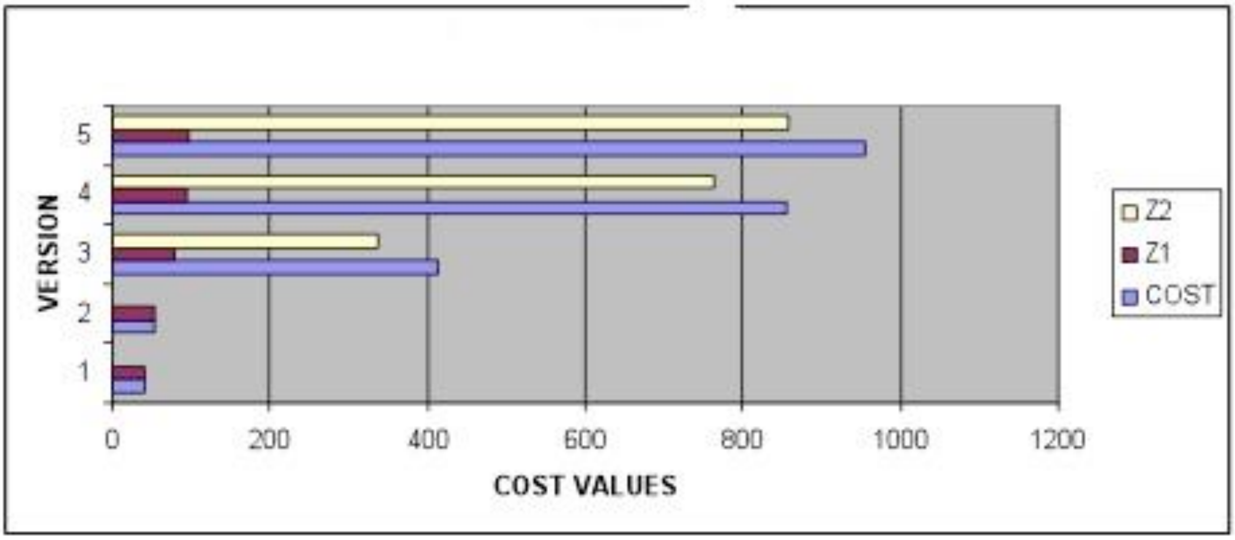
- **Scenario 4**

Pump	6h scale	**1º (6:00)**	**2º (12:00)**	**3º (18:00)**	**4º (24:00)**
	1	100%	100%	100%	100%
	2	40%	40%	40%	40%
	3	100%	100%	100%	100%
	4	100%	46,97%	54,90%	59,34%
	5	45%	0%	0%	0%
	6	0%	0%	0%	0%
	7	100%	85,76%	66,93%	66,41%
	8	40%	40%	40%	40%
	9	40%	55,09%	66%	62,07%
Factors Z1/Z2		Z1=93,5654		Z2=762,1774	
COST		855,7428			

- **Scenario 5**

Pump	6h scale	**1º (6:00)**	**2º (12:00)**	**3º (18:00)**	**4º (24:00)**
	1	100%	100%	100%	100%
	2	40%	40%	40%	40%
	3	100%	100%	100%	100%
	4	100%	47%	54,76%	59,45%
	5	55,83%	10%	10%	10%
	6	10%	10%	10%	10%
	7	100%	86,14%	66,54%	66,44%
	8	40%	40%	40%	40%
	9	40%	54,16%	66%	61,42%
Factors Z1/Z2		Z1=97,4601		Z2=857,2826	
COST		954,7201			

The schematic presentation of the cost of the 5 versions and the factors Z1 and Z2 as we have analyzed above are:



DISCUSSION

Comment 1:
Notice that the daily cost, as it was expected, increases as we increase the number of the pumps that we place in operation according to the water requirements. Also, the pumps 5,6 (that connect the two basic tanks without intermediate consumptions), remain inactive and they are activated only when one of the 2 main tanks needs further quantities of water in order to cover further requirements.
Comment 2:
Our study case was the area of Chania. We developed our model based on a typical day of the year and on typical water demands for water consumption. Our next step is to attempt to increase the sampling period (from 6 hours to 2 hours) and make a statistical analysis of the scada data. Then we can create average demands and their variance for each year period in order to calculate more robust decisions.
Conclusion:
After that our aim is to transform our problem from analogue to integer (the pumps have O/I switches) and finally we will build a full parametric model with user friendly interface.

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ENERGY OPTIMISATION OF THE WATER SUPPLY SYSTEM OF AN AREA. APPLICATION AT THE AREA OF CHANIA (CRETE)

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Outline

- Aim
- Our study case
- Parameterization – Modeling of the system
- Method of System Optimization
- Performance Evaluation
- Conclusions - Future Work

Aim

- Reduction of energy cost
- Increasing need of energy
- Greece facing problems of overloading the energy distribution network - summer months.
- Chania - drillings, pumps and water tanks via a network of electrical pumps.

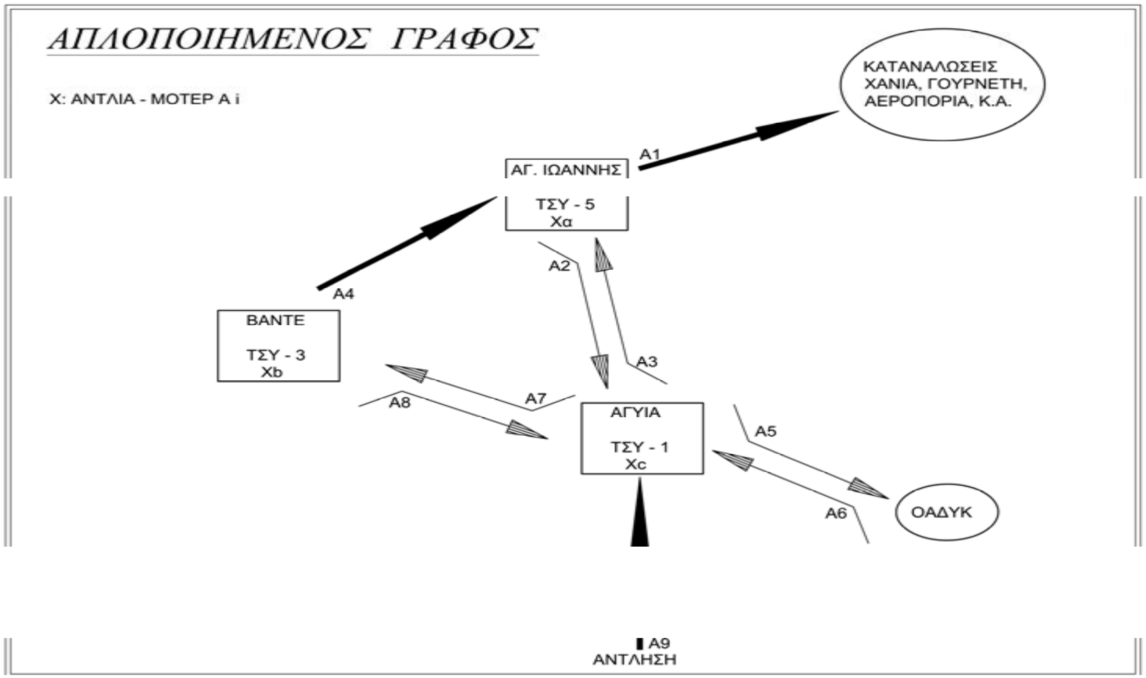
- Objective: development of energy management modelling, solution and software - operation of pumps *taking into consideration the optimisation of the energy cost at the National Electrical Company and the total energy consumption*, based on:
- *1. The **safety level limits** of the tanks.*
- *2. The **frequency of restarting** the pumps.*
- *3. The **profile of demand** of energy and water.*

- *Application of optimal control methods.*
- *No frequent human intervention.*
- We formulate the problem as a *dynamic problem in discrete time, linear as regards the state and the control, with linear constrains on the state and control values. The* ***cost is nonlinear*** *in the case we examine.*

- This not-linearity is due to the way of pricing of the National Electrical Company, that associates the cost with the **maximum** value in time of the consumed energy, aiming at the standardisation of the requirements of consumers and the **restrictions of demand peaks**.
- Using a simple transformation **we *change equivalently the cost from nonlinear to linear with increased dimensions*** and thus we can resolve the problem using linear programming.

Our study case

- Visiting the area of Chania and collecting information from the installed SCADA system at the area we draw the network:



- 3 tanks :
  -Ag.Iwannis
  -Vante
  -Agyia
- Sampling of water level every 6 hours.
- The transfer water rate (m^3 / h) to a water tank is symbolised as A1,A2… A9
- On these pumps there are switches that are symbolized as $\Delta A1_{\kappa}, \Delta A2_{\kappa}, ...\Delta A3_{\kappa}$
- With k we denote the time levels when the sampling occurs.

Parameterization – Modeling of the system

- The examined system is constituted by 3 tanks.
- The sampling of water level will be every 6 hours.
- The quantity of water in each tank is denoted as It holds:
-

$$W_{k+1} = W_k + 6 * A7 * \Delta A7_\kappa - 6 * A8 * \Delta A8_\kappa - 6 * A4 * \Delta A4_\kappa$$

$$V_{\kappa+1} = V_k + 6 * A4 * \Delta A4_\kappa + 6 * A3 * \Delta A3_\kappa - 6 * A2 * \Delta A2_\kappa - 6 * A1 * \Delta A1_\kappa$$

$$Q_{k+1} = Q_k + 6 * A2 * \Delta A2_\kappa + 6 * A8 * \Delta A8_K + 6 * A9 * \Delta A9_\kappa + 6 * A6 * \Delta A6_\kappa$$

$$-6 * A7 * \Delta A7_K - 6 * A5 * \Delta A5_\kappa - 6 * A3 * \Delta A3_\kappa$$

- According to the geometrical specifications of the tanks and the demand profile of each area, there is a minimum and a maximum limit of water in each tank:

Wmin=1000	Vmin=2000	Qmin=3000
Wmax=5500	Vmax=6500	Qmax=8000

- Thus, it has to hold:

$$1000 \leq W1, W2, W3, W4 \leq 5500$$

$$2000 \leq V1, V2, V3, V4 \leq 6500$$

$$3000 \leq Q1, Q2, Q3, Q4 \leq 8000$$

- The analytic parametric equations of each tank are:

$$W1 = 1500 + 6 * A7 * \Delta A7_0 - 6 * A8 * \Delta A8_0 - 6 * A4 * \Delta A4_0$$
$$W2 = W1 + 6 * A7 * \Delta A7_1 - 6 * A8 * \Delta A8_1 - 6 * A4 * \Delta A4_1$$
$$W3 = W2 + 6 * A7 * \Delta A7_2 - 6 * A8 * \Delta A8_2 - 6 * A4 * \Delta A4_2$$
$$W4 = W3 + 6 * A7 * \Delta A7_3 - 6 * A8 * \Delta A8_3 - 6 * A4 * \Delta A4_3$$

$$V1 = 2500 + 6 * A4 * \Delta A4_0 + 6 * A3 * \Delta A3_0 - 6 * A2 * \Delta A2_0 - 6 * A1 * \Delta A1_0$$
$$V2 = V1 + 6 * A4 * \Delta A4_1 + 6 * A3 * \Delta A3_1 - 6 * A2 * \Delta A2_1 - 6 * A1 * \Delta A1_1$$
$$V3 = V2 + 6 * A4 * \Delta A4_2 + 6 * A3 * \Delta A3_2 - 6 * A2 * \Delta A2_2 - 6 * A1 * \Delta A1_2$$
$$V4 = V3 + 6 * A4 * \Delta A4_3 + 6 * A3 * \Delta A3_3 - 6 * A2 * \Delta A2_3 - 6 * A1 * \Delta A1_3$$

- And

$$Q1 = Q_0 + 6 * A2 * \Delta A2_0 + 6 * A8 * \Delta A8_0 + 6 * A9 * \Delta A9_0 + 6 * A6 * \Delta A6_0 -$$
$$-6 * A7 * \Delta A7_0 - 6 * A5 * \Delta A5_0 - 6 * A3 * \Delta A3_0$$
$$Q2 = Q1 + 6 * A2 * \Delta A2_1 + 6 * A8 * \Delta A8_1 + 6 * A9 * \Delta A9_1 + 6 * A6 * \Delta A6_1 -$$
$$-6 * A7 * \Delta A7_1 - 6 * A5 * \Delta A5_1 - 6 * A3 * \Delta A3_1$$
$$Q3 = Q2 + 6 * A2 * \Delta A2_2 + 6 * A8 * \Delta A8_2 + 6 * A9 * \Delta A9_2 + 6 * A6 * \Delta A6_2 -$$
$$-6 * A7 * \Delta A7_2 - 6 * A5 * \Delta A5_2 - 6 * A3 * \Delta A3_2$$
$$Q4 = Q3 + 6 * A2 * \Delta A2_3 + 6 * A8 * \Delta A8_3 + 6 * A9 * \Delta A9_3 + 6 * A6 * \Delta A6_3 -$$
$$-6 * A7 * \Delta A7_3 - 6 * A5 * \Delta A5_3 - 6 * A3 * \Delta A3_3$$

- As we have mentioned our variables are the switches ΔAij. We introduce the following notation:

ΔA1o - X1	ΔA11 - X5	ΔA12 - X9	ΔA13 - X13
ΔA2o - X2	ΔA21 – X6	ΔA22 – X10	ΔA23 – X14
ΔA3o – X3	ΔA31 – X7	ΔA32 – X11	ΔA33 – X15
ΔA4o – X4	ΔA41 – X8	ΔA42 – X12	ΔA43 – X16
ΔA5o - X17	ΔA51 - X18	ΔA52 - X19	ΔA53 - X20
ΔA6o - X21	ΔA61 - X22	ΔA62 - X23	ΔA63 - X24
ΔA7o - X25	ΔA71 - X26	ΔA72 - X27	ΔA73 - X28
ΔA8o - X29	ΔA81 - X30	ΔA82 - X31	ΔA83 - X32
ΔA9o - X33	ΔA91 - X34	ΔA92 - X35	ΔA93 - X36

Method of System Optimization

- Let us present how the National Electrical Company (ΔΕΗ) calculates the cost of the consumed energy, whereby it also attempts to discourage the peak demands.
- There are 3 tanks: W, V, Q.

  -The sampling rate is every 6 hours: Kmax=4 , k=1,2,3,4

  -The water flows are symbolized as A1…A9

  -The pumps are symbolized as ΔA1,ΔA2,…,ΔA9 (n=9)

  -The switches (per hour): αi(k): X1,X2,…….,X36 (i: 1-9, κ: 1-4)

  -Time step of simulation: Δt=6h

  -Number of Pumps: i=1-9, n=9

-Electrical Force of Pumps: Pi(k) =20kW (stable)

-αi(k)= % percentage of using power of pump at the moment k --> Xj, j=1-36

- Total Force:

$$Pc = [\sum_{k=1}^{4} \sum_{i=1}^{9} 20 * a_i(k)] * 6 = 120 * \sum_{k=1}^{4} \sum_{i=1}^{9} a_i(k)$$

$$P\max = \max_k [\sum_{i=1}^{n} a_i(k) P_i]$$

- The system operation cost equation which reflects the tariff of National Electrical Company:

$$\cos t = 400 * C2 * \max_{k}(\sum_{i=1}^{n} a_i(k) * Pi) + C3 * \max(0, \sum_{k=1}^{k\max} (\sum_{i=1}^{n} a_i(k) * Pi - 400))$$

- Where:

$$Z2 = C3 * \max(0, \sum_{k=1}^{k\max} (\sum_{i=1}^{n} a_i(k) * Pi - 400))$$

$$Z1 = 400 * C2 * \max_{k}(\sum_{i=1}^{n} a_i(k) * Pi)$$

- The restrictions for Z1,Z2 are:

$$Z1 \geq a1(1) * P + a2(1) * P + ... + a9(1) * P$$
$$Z1 \geq a1(2) * P + a2(2) * P + ... + a9(2) * P$$
$$Z1 \geq a1(3) * P + a2(3) * P + ... + a9(3) * P$$
$$Z1 \geq a1(4) * P + a2(4) * P + ... + a9(4) * P$$

and:

$$Z2 \geq 0$$
$$Z2 \geq -4 * 400 + 120 * \sum_{i=1}^{36} Xi$$

- The initial system with the new restrictions is :

$$X = \begin{bmatrix} X1 \\ . \\ . \\ X38 \end{bmatrix}$$

$$\begin{bmatrix} \left\langle A \middle| \begin{matrix} 0\_0 \\ .\_. \\ .\_. \\ 0\_0 \end{matrix} \right\rangle \end{bmatrix} \begin{bmatrix} X1 \\ . \\ . \\ . \\ X36 \\ X37 \\ X38 \end{bmatrix} \leq b \rightarrow Ax \leq b$$

- The matrix with the 6 equations describing the restrictions is:

$$\begin{bmatrix} PX:1,2,3,4,17,21,25,29,33|-1\_0 \\ PX:5,6,7,8,18,22,26,30,34|-1\_0 \\ PX:9,10,11,12,19,23,27,31,35|-1\_0 \\ PX:13,14,15,16,20,24,28,32,36|-1\_0 \\ 0...0|0\_-1 \\ 120.....................................120|0\_-1 \end{bmatrix}_{A\pi\varepsilon\rho=6x38} \begin{bmatrix} X1 \\ . \\ . \\ . \\ . \\ X38 \end{bmatrix}_{38x1} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4*400 \end{bmatrix}_{6x1}$$

- The expression of the total system is:

$$\begin{bmatrix} A_{24\chi 36} & \begin{vmatrix} 0\_0 \\ \ldots\ldots \\ \ldots\ldots \\ 0\_0 \end{vmatrix} \\ A\pi\varepsilon\rho_{6\chi 38} & \end{bmatrix} \begin{bmatrix} X1 \\ . \\ . \\ . \\ X38 \end{bmatrix} \leq \begin{bmatrix} b1 \\ . \\ . \\ b24 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1600 \end{bmatrix}_{30x1}$$

- The system cost function is:

$$f(x) = 400 * C2 * X37 + C3 * X38 = 21,88 * X37 + 0,03623 * X38$$

We have thus formulated our problem as:

$$A * X \leq b$$

$$LBnb \leq X \leq UBnb$$

$$\cos t\_function\_f(x)$$

and now we can use linear programming in the Matlab environment.

- Our aim is to transform our system from nonlinear to linear form. This can be achieved by introducing the two new variables Z1,Z2 ,which is not a big burden at all.

Performance Evaluation

- Taking into consideration the system, the requirements for water supply and irrigation of the Chania area, we present 5 operation scenarios of our system and we present the proper percentage of operation of each pump per 6h on a 24h operation basis. The 5 scenarios are:

- **Scenario 1:** The pump 1 that supplies the main consumers of the system has a minimum point of operation at 80% and the pump 9 that draws water from the earth has minimum operation at 40%. The remaining pumps function from 0 to 100%.
- **Scenario 2:** The pump 1 has the minimum point of operation at 80% and the pump 9 at 40%.The remaining pumps operate from 10% to 100%.
- **Scenario 3:** The pump 1 has the minimum point of operation at 80% and the pump 9 at 40%.The pumps 5,6 can be inactive and the rest of the pumps have minimum operation at 40%.
- **Scenario 4:** The pump 1 has the minimum point of operation at 100% and the pump 9 at 40%. The pumps 5,6 can be inactive and the remaining pumps have minimum operation at 40%.
- **Scenario 5:** The pump operates at 100% and the pump 9 has minimum operation at 40%. The remaining pumps function from 40% until 100% while pumps 5,6 function as minimum 30 %.

- In the following tables we show the results of optimising for the 5 scenarios and their associated costs.

- **Scenario 1**

Pump	6h scale	1º (6:00)	2º (12:00)	3º (18:00)	4º (24:00)
1		80%	80%	80%	80%
2		0%	0%	0%	0%
3		65.09%	86,79%	83,45%	81,15%
4		48,48%	0%	0%	0%
5		0%	0%	0%	0%
6		0%	0%	0%	0%
7		24,44%	0%	0%	0%
8		0%	0%	0%	0%
9		40%	42,74%	46,08%	48,38%
Factors Z1/Z2		Z1=41,9059		Z2=0	
COST		41,9059			

- **Scenario 2**

Pump	6h scale	1º (6:00)	2º (12:00)	3º (18:00)	4º (24:00)
1		80%	80%	80%	80%
2		10%	10%	10%	10%
3		67,30%	91,98%	90,08%	85,50%
4		40,98%	10%	10%	10%
5		10%	10%	10%	10%
6		10%	10%	10%	10%
7		45,60%	10%	10%	10%
8		10%	10%	10%	10%
9		40%	40,92%	42,83%	47,40%
Factors Z1/Z2		Z1=54,5807		Z2=0	

- **Scenario 3**

Pump	6h scale	1º (6:00)	2º (12:00)	3º (18:00)	4º (24:00)
1		80%	80%	80%	80%
2		40%	40%	40%	40%
3		77,08%	99,47%	97,25%	92,69%
4		54,82%	40%	40%	40%
5		12,53%	0%	0%	0%
6		0%	0%	0%	0%
7		100%	49,04%	50,99%	55,72%
8		40%	40%	40%	40%
9		40%	41,10%	41,37%	41,20%
Factors Z1/Z2		Z1=77,9220		Z2=335,9078	
COST		413,8298			

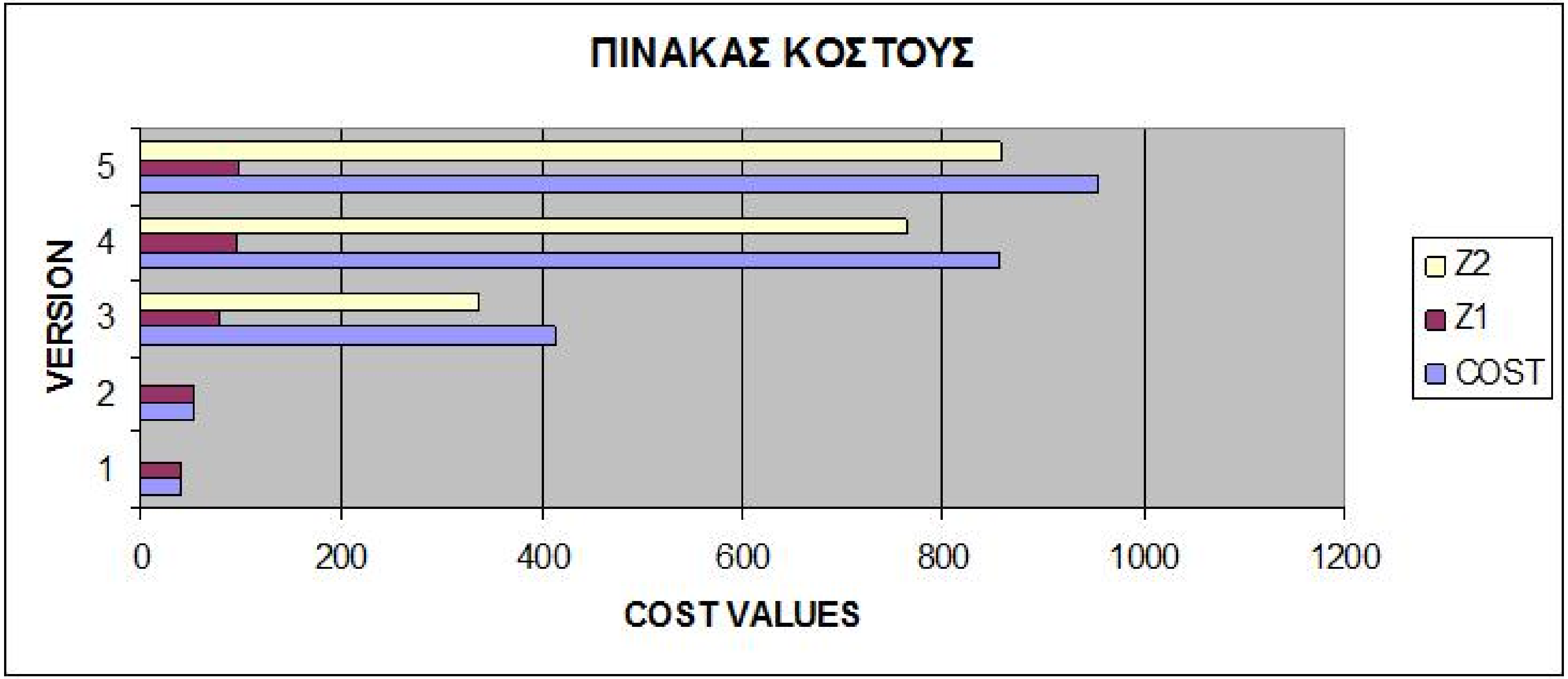
- **Scenario 4**

Pump	6h scale	1º (6:00)	2º (12:00)	3º (18:00)	4º (24:00)
1		100%	100%	100%	100%
2		40%	40%	40%	40%
3		100%	100%	100%	100%
4		100%	46,97%	54,90%	59,34%
5		45%	0%	0%	0%
6		0%	0%	0%	0%
7		100%	85,76%	66,93%	66,41%
8		40%	40%	40%	40%
9		40%	55,09%	66%	62,07%
Factors Z1/Z2		Z1=93,5654		Z2=762,1774	
COST		855,7428			

- **Scenario 5**

Pump	6h scale	1º (6:00)	2º (12:00)	3º (18:00)	4º (24:00)
1		100%	100%	100%	100%
2		40%	40%	40%	40%
3		100%	100%	100%	100%
4		100%	47%	54,76%	59,45%
5		55,83%	10%	10%	10%
6		10%	10%	10%	10%
7		100%	86,14%	66,54%	66,44%
8		40%	40%	40%	40%
9		40%	54,16%	66%	61,42%
Factors Z1/Z2		Z1=97,4601		Z2=857,2826	
COST		954,7201			

- The schematic presentation of the cost of the 5 versions and the factors Z1 and Z2 as we have analyzed above are:



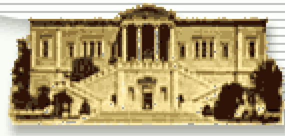
Conclusions

- Notice that the daily cost, as it was expected, increases as we increase the number of the pumps that we place in operation according to the water requirements. Also, the pumps 5,6 (that connect the two basic tanks without intermediate consumptions), remain inactive and they are activated only when one of the 2 main tanks needs further quantities of water in order to cover further requirements.
- Our study case was the area of Chania. We developed our model based on a typical day of the year and on typical demands for water

Future work

- Our next step is to attempt to **increase the sampling period** (from 6 hours to 2 hours) and make a statistical analysis of the scada data.
- Then we can create average demands and their variance for each year period in order to calculate **more robust decisions**.
- After that our aim is to transform our problem **from analogue to integer** (the pumps have O/I switches) and finally we will build **a full parametric model with user friendly interface.**

ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ



Thank you for your attention

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