# Games among Long and Short Term Electricity Producers and Users

Nikolaos Chr. Kakogiannis*, Panagiotis Kontogiorgos, Elena Sarri and George P. Papavassilopoulos
Electrical and Computer Engineering, National Technical University of Athens, Athens, Greece
Iroon Polytechniou 9, Zografou 15773, Athens, Greece

**Abstract - We consider a system that consists of a major player infinite time horizon and minor players remaining in the system for overlapping and different finite time periods and with different quality features. We study how they interact among themselves (horizontal interaction), and with the major player respectively (vertical interaction), via their decisions /strategies and the impact of the length of the time interval that the minor players remain in the system. In this paper this time is various and we study how this feature influences the cost of the major player and the cost of the minor players. Our major player is the PPC (Public Power Corporation) and the minor players are the energy consumers (houses, industries etc.) or renewable energy producers (for example photovoltaic solar energy producers). We employ the basic game theoretic Nash solution. The optimal equilibrium (the decisions/strategies of the minor players and of the major player) will result in a dual target: global system minimum cost and individual best performance. Our game is dynamic, deterministic and evolves in discrete time. The equilibriums we consider satisfy the Dynamic Programming Principle and are thus "robust" in the presence of noise in the state equation.**

***Index Terms*— energy optimization cost, equilibrium, game theory, major player, minor players, Nash**

## I.INTRODUCTION

.

The work presented here is motivated by the game between a large Public Power Corporation referred to as the major player and the many small producers/consumers referred to as the minor ones. This game has many features worthy of investigation. We choose to address here the role of the time duration of the minor players which is small relative to the time horizon of the major player (PPC). We change the time period that the minor players remain in the system and we examine its influence on the cost of the major player. Similarly the influence on the costs of the minor players and the strategies of all the players can be studied. Clearly the other parameters involved in describing the time evolution and costs of the players influence the costs and strategies. Our target is to create a parametric model, which will take into consideration all these features, and thus we will be able by changing the time durations and the parameters to study their influence on the strategies and costs of the players. In many applications considered in the literature up to now, the "smaller" short term players are aggregated as a single long-term player and the impact of the parameter values on the strategies and costs is studied. In the present paper we intend to address explicitly the impact of the short term versus the long-term characters of the players, as well as the overlapping of the intervals of play of the short term players.

We study a deterministic version of the problem in discrete time. The Nash equilibrium is employed. We consider the LQ case and since we are interested in strategies that survive in a stochastic framework ([2], [5]) we use the principle of optimality to derive the solutions. (The formulation of the problem and the associated dynamic programming equations for the nonlinear case is straightforward but their solution is obviously much more demanding.) We derive the associated Ricatti equations. In order to provide some simple examples we consider the time invariant scalar case and thus we assume that all the matrices involved are time-invariant constants. We also assume that all minor players have the same cost function, act during different time periods but for the same duration T. This results to having to solve a system with T+1 equations. Now by changing the values of the parameters involved we can easily solve each time the system of the Riccati equations and find the optimal controls-decisions and costs in every case for each player.

The usefulness of this work for the case where a framework of liberalization of the energy market is considered, lies in providing an indicator as to the impact of having various lengths of contractual agreements concerning producers or consumers. If we assume that the major player is concerned more directly with the social welfare benefits, he is faced with the question-among many others-of whether he should favour giving licences to producers of smaller or bigger time duration. Similarly, he is faced with the question of whether he should provide consumers with smaller or bigger time duration electricity supply commitments. The answer to these questions depends obviously on the particular parameter values describing the time evolution and costs of the producers /consumers. Thus one has to study the interplay of parameters and time durations and their joined impact on costs. Clearly, besides the energy sector, such problems appear in other areas where market designs are sought.

This paper is a continuation of [6] and for reasons of self – sufficiency we repeat the basic formulation and derivations from there in Sections II and III. In Section II we formulate the problem, we define the state equations and costs and we derive the Ricatti equations which characterize the Nash solutions for the major and minor players. In Section III we specialise to the scalar case which we are going to solve numerically for several parameter values using Matlab. In the fourth section, which constitutes the core contribution of the present paper, we present our numerical results for several values of the parameters and time interval lengths of the minor players and we discuss the results. In the fifth section we consider again the scalar version of the problem where all the small players are concatenated as one player with the same time duration as the major player and we

*Correspondence to: Nikolaos Kakogiannis, E-mail: nikos_kakogiannis@yahoo.com

derive the associated Nash equilibrium. Our aim is to compare the solution derived now with those we derived earlier where we took explicitly into account the short-term time overlapping character of the minor players. The sixth is a short section with conclusions.

## II. Mathematical Formulation

The state equation is:

$$x_{k+1} = Ax_k + B_0u_k + B_1u_{1k} + B_2u_{2k} + B_3u_{3k} + B_4u_{4k} + B_5u_{5k}, k = 0,1,2... \quad (1)$$

where $x_k$ is the state, $u_k$ is the control of the long term player (PPC), $u_{ik}$ is the control of the minor players (clients or producers) at the i-th year remaining in the system (i=1-5). For example, a player who enters the game at time k will use controls $u_{1k}$, $u_{2k+1}$, $u_{3k+2}$, $u_{4k+3}$, $u_{5k+4}$, corresponding to times k,k+1,k+2,k+3,k+4. A, $B_i$ are given matrices of appropriate dimensions. If the players are six the state equation is:

$$x_{k+1} = Ax_k + B_0u_k + B_1u_{1k} + B_2u_{2k} + B_3u_{3k} + B_4u_{4k} + B_5u_{5k} + B_6u_{6k}, k = 0,1,2....$$

and so on for 7 and 8 players.

The quadratic costs of the major player (J) and the minor players ($J_l$) who act in the interval between l and (l+5) are:

$$J = \sum_0^{\infty}\left(x_k^TQ_0x_k + u_k^TR_0u_k\right),$$

$$J_l = \sum_{k=0}^{4}\left(x_{k+l+1}^TQ_fx_{k+l+1} + u_{(k+1)(l+k)}^TR_fu_{(k+1)(l+k)}\right) + x_l^TQ_fx_l \quad (2)$$

The Q's are symmetric real non negative matrices and the R's are symmetric positive defined matrices which are known. In our case we consider A, B, Q, R constant. (References [1]-[4] contain the appropriate material needed for deriving the Nash solution of linear quadratic problems.) To clarify how we derive the solution we think as follows. Consider a minor player who starts his interaction with the system at time k=30 with control $u_{1,30}=L_1x_{30}$ when he begins to consume/produce energy. Next year his control is $u_{2,31}=L_2x_{31}$, the next $u_{3,32}=L_3x_{32}$, $u_{4,33}=L_4x_{33}$ and the fifth and last year $u_{5,34}=L_5x_{34}$. Notice that the $L_1$, $L_2$, $L_3$, $L_4$, $L_5$ are independent of the year the minor player enters the system.

As regards the long-term player for his optimal reaction we consider the state equation:

$$x_{k+1} = Ax_k + B_0u_k + (BL_1 + BL_2 + BL_3 + BL_4 + BL_5)x_k \quad (3)$$

and we use the Ricatti equation:

$$K = A_0^T\left(K - KB_0\left(B_0^TKB_0 + R\right)^{-1}B_0^TK\right)A_0 + Q_0 \quad (4)$$

Then the long term player's optimal reaction is:

$$u_k^* = u^* = L_0x, L_0 = -\left(B_0^TKB_0 + R\right)^{-1}B_0^TKA_0 \quad (5)$$

and his optimal cost is:

$$J^* = x_0^TKx_0 \quad (6)$$

To derive the equations that provide the $L_i$'s of the minor player we think as follows. We will examine for example the system with a major player (PPC) and minor players (producers or consumers) who enter the system and their remaining lasts for 5 years. Consider the minor player who enters the calendar year 30 (k=30). He sees the following system (7)-(13) where in this first equation (7) he acts as first year consumer/producer and the other-year consumers/producers act with the fixed laws $L_2x_{30}$, $L_3x_{30}$, $L_4x_{30}$, $L_5x_{30}$.

$$x_{k+1} = (A + B_0L_0 + BL_2 + BL_3 + BL_4 + BL_5)x_k + Bu_{1,k} = A_1x_k + Bu_{1,k} \quad (7)$$

Similarly when he is at the second year it sees the following system

$$0 \; x_{k+2} = (A + B_0L_0 + BL_1 + BL_3 + BL_4 + BL_5)x_{k+1} + Bu_{2,k+1} = A_2x_{k+1} + Bu_{2,k+1} \quad (8)$$

and the other-year producers/consumers act with the fixed laws $L_1$, $L_3$, $L_4$, $L_5$ and so on.

Thus the whole system of equations that the small players who entered the calendar year k=30 and their staying last five years see, is:

$$x_{k+1} = (A + B_0L_0 + BL_2 + BL_3 + BL_4 + BL_5)x_k + Bu_{1,k} = A_1x_k + Bu_{1,k} \quad (9)$$

$$x_{k+2} = (A + B_0L_0 + BL_1 + BL_3 + BL_4 + BL_5)x_{k+1} + Bu_{2,k+1} = A_2x_{k+1} + Bu_{2,k+1} \quad (10)$$

$$x_{k+3} = (A + B_0L_0 + BL_1 + BL_2 + BL_4 + BL_5)x_{k+2} + Bu_{3,k+2} = A_3x_{k+2} + Bu_{3,k+2} \quad (11)$$

$$x_{k+4} = (A + B_0L_0 + BL_1 + BL_2 + BL_3 + BL_5)x_{k+3} + Bu_{4,k+3} = A_4x_{k+3} + Bu_{4,k+3} \quad (12)$$

$$x_{k+5} = (A + B_0L_0 + BL_1 + BL_2 + BL_3 + BL_4)x_{k+4} + Bu_{5,k+4} = A_5x_{k+4} + Bu_{5,k+4} \quad (13)$$

For this system of equations and the cost

$$J_{30} = \sum_{k=0}^{4}\left(x_{k+30+1}^TQ_fx_{k+0+1} + u_{(k+1)(30+k)}^TR_fu_{(k+1)(30+k)}\right) + x_{30}^TQ_fx_{30} \quad (14)$$

we derive the optimal policy by employing the Ricatti equations. The $L_i$'s are given by the following system of equations.

$$L_1 = -\left(B^T K_2 B + R\right)^{-1} B^T K_2 A_1 \qquad (15)$$

$$K_1 = A_1^T \left(K_2 - K_2 B\left(B^T K_2 B + R\right)^{-1} B^T K_2\right) A_1 + Q_f \qquad (16)$$

$$L_2 = -\left(B^T K_3 B + R\right)^{-1} B^T K_3 A_2 \qquad (17)$$

$$K_2 = A_2^T \left(K_3 - K_3 B\left(B^T K_3 B + R\right)^{-1} B^T K_3\right) A_2 + Q_f \qquad (18)$$

$$L_3 = -\left(B^T K_4 B + R\right)^{-1} B^T K_4 A_3 \qquad (19)$$

$$K_3 = A_3^T \left(K_4 - K_4 B\left(B^T K_4 B + R\right)^{-1} B^T K_4\right) A_3 + Q_f \qquad (20)$$

$$L_4 = -\left(B^T K_5 B + R\right)^{-1} B^T K_5 A_4 \qquad (21)$$

$$K_4 = A_4^T \left(K_5 - K_5 B\left(B^T K_5 B + R\right)^{-1} B^T K_5\right) A_4 + Q_f \qquad (22)$$

$$L_5 = -\left(B^T K_6 B + R\right)^{-1} B^T K_6 A_5 \qquad (23)$$

$$K_5 = A_5^T \left(K_6 - K_6 B\left(B^T K_6 B + R\right)^{-1} B^T K_6\right) A_5 + Q_f \qquad (24)$$

$$K_6 = Q_f \qquad (25)$$

The total cost of a small player who entered the system at year 30 is:

$$J_{30}^* = x_{30}^T K_1 x_{30} \qquad (26)$$

Notice that we consider linear no memory strategies. We know that may exist other solutions, which are not necessarily linear and may have memory. We know nonetheless (Selten and [2]) that these solutions disappear in the presence of noise.

## III. Scalar Form

Here we consider the scalar case and study the costs of the players by changing the parameters.

In this case we consider the matrices A, B, Q, R as constant scalars α, b, q, r. We take the R's and the B's to be equal to 1. So the system of the matrix equations becomes:

$$x_{k+1} = ax_k + v_k + L_1 x_k + L_2 x_k + L_3 x_k + L_4 x_k + L_5 x_k \qquad (27)$$

$$x_{k+1} = (a + L_1 + L_2 + L_3 + L_4 + L_5)x_k + v_k = \bar{a}x_k + v_k \qquad (28)$$

$$\bar{a} = a + L_1 + L_2 + L_3 + L_4 + L_5 \qquad (29)$$

$$u_k^* = L_0 x \qquad (30)$$

$$L_0 = -(K+I)^{-1} K\bar{a} \qquad (31)$$

$$K = \bar{a}^T \left(K - K\left(K+I\right)^{-1} K\right)\bar{a} + q_0 \qquad (32)$$

$$J = \sum_{k=0}^{\infty} \left(q_0 x_k^2 + v_k^2\right) \qquad (33)$$

$$x_{k+1} = (a + L_0 + L_2 + L_3 + L_4 + L_5)x_k + u_{1k} = a_1 x_k + u_{1k} \qquad (34)$$

$$x_{k+2} = (a + L_0 + L_1 + L_3 + L_4 + L_5)x_{k+1} + u_{2,k+1} = a_2 x_{k+1} + u_{2,k} \qquad (35)$$

$$x_{k+3} = (a + L_0 + L_1 + L_2 + L_4 + L_5)x_{k+2} + u_{3,k+2} = a_3 x_{k+2} + u_{3,} \qquad (36)$$

$$x_{k+4} = (a + L_0 + L_1 + L_2 + L_3 + L_5)x_{k+3} + u_{4,k+3} = a_4 x_{k+3} + u_{4,} \qquad (37)$$

$$x_{k+5} = (a + L_0 + L_1 + L_2 + L_3 + L_4)x_{k+4} + u_{5,k+4} = a_5 x_{k+4} + u_{5,} \qquad (38)$$

$$L_1 = -(K_2 + I)^{-1} K_2 a_1 \qquad (39)$$

$$K_1 = a_1(K_2 - K_2(K_2 + I)^{-1} K_2)a_1 + q_f \qquad (40)$$

$$L_2 = -(K_3 + I)^{-1} K_3 a_2 \qquad (41)$$

$$K_2 = a_2(K_3 - K_3(K_3 + I)^{-1} K_3)a_2 + q_f \qquad (42)$$

$$L_3 = -(K_4 + I)^{-1} K_4 a_3 \qquad (43)$$

$$K_3 = a_3(K_4 - K_4(K_4 + I)^{-1} K_4)a_3 + q_f \qquad (44)$$

$$L_4 = -(K_5 + I)^{-1} K_5 a_4 \qquad (45)$$

$$K_4 = a_4(K_5 - K_5(K_5 + I)^{-1} K_5)a_4 + q_f \qquad (46)$$

$$L_5 = -(K_6 + I)^{-1} K_6 a_5 \qquad (47)$$

$$K_5 = a_5(K_6 - K_6(K_6 + I)^{-1} K_6)a_5 + q_f \qquad (48)$$

$$K_6 = q_f \qquad (49)$$

(The K's and L's are scalar's)

After some transformations we created the following scalar equations (five for the short term players and one for the long term player) where the $x_i$'s, stand for the $K_i$'s, i=0,1,...5:

$$x_5 = q_f \qquad (50)$$

$$x_4 = q_f + A^2(x_5 + x_5^2) \qquad (51)$$

$$x_3 = q_f + A^2(x_4 + x_4^2) \qquad (52)$$

$$x_2 = q_f + A^2(x_3 + x_3^2) \qquad (53)$$

$$x_1 = q_f + A^2(x_2 + x_2^2) \qquad (54)$$

$$x_0 = q_0 + A^2(x_0 + x_0^2) \qquad (55)$$

$$A = \frac{a}{1 + S + x_0} \qquad (56)$$

$$S = x_1 + x_2 + x_3 + x_4 + x_5 \qquad (57)$$

$$x_0 = \frac{a}{A} - 1 - S \qquad (58)$$

Substituting $x_0$ from (58) into (55) we obtain the equivalent

$$F(A) = (\alpha - A - AS)(1 - A^2) - Aq_0 - A(a - A - AS)^2 \qquad (59)$$

If the time horizon of the minor players is 7 (or 6 or 8 or…), the formulae (15)-(25),(39)-(49),(50)-(57) will have to be modified appropriately. For example, if 7 is the time horizon, the equations (51)-(55) remain the same but we will also have similar formulae for x6,x7 ,and S in (57) will be given by:

$$S' = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

$$A = \frac{a}{1 + S' + x_0}$$

Since S can be calculated explicitly as a function of A by using recursively (50)-(54) we conclude that F(A) is a function of A, a , $q_0$, $q_f$ which has to be solved for its roots. Solving F(A) =0 we find the A and immediately the $x_i$'s from (50)-(54), (58). If F(A) =0 has many solutions A, then there exist many different $x_i$'s, i.e. the game has many solutions. Of course we seek solutions A of F(A) =0 which are smaller than 1 in magnitude so that the close-loop system is stable. (Notice that A is the closed loop matrix).

Notice also that in order that (55) has a real solution $x_0$ it must hold:

$$A < \frac{1}{\sqrt{q_0} + \sqrt{q_0 + 1}} \quad (60)$$

It must be $x_1 > x_2 > x_3 > x_4 > x_5$, and all the $x_i$'s positive. Also A and α have the same sign (see (56)) and that is why in the numerical examples we take α positive.

Equation (59) is very important because it constitutes the centrepiece of the numerical solution. A graphical representation of F(A) can be achieved as follows: For a given value of A we calculate $x_5, x_4, x_3, \ldots, x_0$ from (50)-(55) and S from (57). Knowing S we calculate the value F(A).We seek the roots of F(A) =0,with $|A|<1$. If the root is unique the game has a unique solution. If there are many then it has many solutions. This can happen as the Figures 5 and 7 indicate.

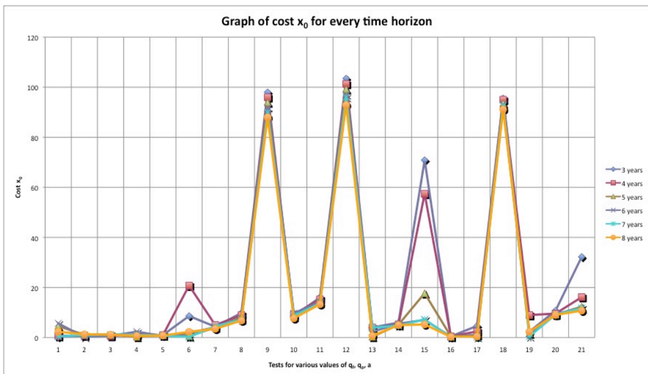
It would be of interest to try to derive the analogue of (59) for the matrix case since it would facilitate greatly the numerical solution for this case, but we have not achieved that. It seems unlikely that such a formula (in matrix form of course) will become easily available.

## IV. Results

Our next step is, by using the Matlab to solve these equations for several values of α, $q_0$, $q_f$.

We present some results of the numerical experiments in Tables I-VI which are presented in the Appendix and in more compact form in Plot 1.

PLOT 1




For each triplet of values of α, $q_0$, $q_f$, we solve first the equation F(A)=0 (see (59)). Knowing the solution A of (59) we use this value to calculate the $x_i$'s.

Notice that F(0)=α>0, F(1)<0 and thus F(A)=0 always has a solution in (0,1). Since multiple solutions of F(A) =0 implies many solutions of the game, we provide some plots of F(A), see Appendix. It appears in our experiments that F(A) =0 has usually a unique solution but there are cases such as those shown in Figures 5 and 7 where there are two solutions .

We experimented with several values of α, $q_0$, $q_f$ and we tried to combine cases with α stable (0<α<1), α unstable (α>1), α small/large, $q_0$ small/large, $q_f$ small / large.

By observing the values $x_0$, $x_1$ we draw some conclusions about the optimal costs of the PPC (J) and the cost of the consumer/producer ($J_l$) which are proportional to $x_0$ and $x_1$ respectively. At the 5-year system when the system is unstable (α=10) we notice that when $q_f>q_0$ then $J_l>J$ and respectively when $q_f<q_0$ then $J_l<J$. When $q_f=q$ and they have small prices $J_l>>J$. While the $q_f$, $q_0$ get bigger prices and still holds $q_f=q_0$ then $J_l$ gets closer to J until they become almost equal. So we conclude that in unstable system and with small values for $q_f$, $q_0$ the cost of PPC is much bigger than consumers'/producers' and the long term player is essentially more sensitive.

In a more stable system (α=0.1) we notice that the players interchange roles and the consumers/producers are more sensitive with bigger cost.

As we observe the rest tables, the system shows similar behaviour. When the system is unstable (α=10) and $q_f>q_0$ then $J_l>J$ and respectively when $q_f<q_0$ then $J_l<J$. When $q_f=q$, for small prices $J_l>>J$ and while they get bigger the $J_l$ gets closer to J until they become almost equal (for small time horizon this becomes more difficult and that is why for 3 year duration J decreases but in the end is still bigger than $J_l$). When the system is stable (a=0,1) the players interchange roles and minor players have bigger costs.

The main point however is to study the costs as the time horizon increases. We notice that the unstable system, when $q_f>q_0$, has great cost for the minor players ($J_l$) which gets smaller as the time horizon increases. On the other hand, the cost of the major player (J) is very big for small time horizon and small values of $q_0$ at the same time. In these particular cases (3 and 4 years) as show the plots in the Appendix there are more feasible solutions of F(A)=0, so there also other sets of x and costs which however present the same behaviour (big costs J). In the rest of the cases it is small with fluctuations. When $q_f<q_0$, then $J_l<<J$ but J tends to decrease as the time horizon increases. When $q_f=q$ happens the same.

When the system is stable, $J_l$ is practically stable as the time horizon increases whereas J has some fluctuations but tends to decrease.

The results show that the stable system has smaller costs than the unstable for all the players. Additionally, the costs are bigger for small time horizons especially for the major player. The major player has generally smaller cost than the minor players when the system is stable (except for when $q_f<q_0$) or when $q_f>q_0$.

## V. CONCATENATED PLAYER

Our purpose in this section is to compare the solutions obtained in sections II, III for example for time horizon 5 years with the solution that would result if we were to group all the small duration players together. In this case we would have two long term players. We work out only the scalar case.

By considering all the short term players as one player with a cost equal to the sum of the costs of all the players

$$J_f = \sum_{k=0}^{\infty}(5q_f x_k^2 + 5v_k^2) = \frac{1}{5}\sum_{k=0}^{\infty}(25q_f x_k^2 + (5v_k)^2) \qquad (61)$$

, and a state equation

$$x_{k+1} = ax_k + u_k + 5v_k \qquad (62)$$

where $5v_k$ is the control of this concatenated player, we have a classical linear quadratic infinite time game with two players. (We took the R's and the B's equal to 1 and α>0.) We can derive the associated Ricatti equations for the Nash solution. After some transformations it turns out that we have to solve the following system of equations.

$$x_1 = 25q_f + A^2(x_1 + x_1{}^2) \qquad (63)$$

$$x_0 = q_0 + A^2(x_0 + x_0{}^2) \qquad (64)$$

$$A = \frac{a}{1 + x_0 + x_1} \qquad (65)$$

A is the closed loop matrix and in order to have real solutions $x_1$, $x_0$ and A stable it must be:

$$A < \frac{1}{\sqrt{q_0} + \sqrt{q_0 + 1}}, A < \frac{1}{\sqrt{25q_f} + \sqrt{25q_f + 1}}, 0 < A < 1 \qquad (66)$$

The cost of the PPC is proportional to $x_0$ and the cost of the concatenated short term player is proportional to 1/5 $x_1$. In our experiments the solutions turned out to be unique for each triplet of the parameters α, $q_0$, $q_f$.

A similar concatenation can be done if we have a time horizon of 7 (or 6 or any other length) for the minor players. In this case the coefficients 5 and 25 appearing in (61)-(65) will be substituted by 7 and 49 respectively.

We present some numerical results of solving (61)-(63) for the same values of α, $q_0$, $q_f$ in Table VII in the Appendix.

By comparing the values of $x_0$, $x_1$ (for the same parameter values) of section IV (Table I) and section V (Table VII) several conclusions can be drawn about the validity and usefulness of concatenating the many small players into one.

## VI. CONCLUSIONS

In our future search we intend to use the Stackelberg equilibrium model for the players. Of interest is also the case where the time duration of the short time players is a random variable taking values in a certain interval. Similarly we can consider cases where the appearance of a small duration player at each instant of time is itself a random event.

## APPENDIX

TABLE I
TIME HORIZON 3 YEARS

| N/N | qf | q0 | a | x0 | x1 |
|---|---|---|---|---|---|
| **1** | 10 | 1 | 0.1 | 0.247746958 | 10.00112664 |
| **2** | 10 | 1 | 3 | 0.814297118 | 11.02225805 |
| **3** | 10 | 1 | 10 | 0.952273151 | 20.54378178 |
| **4** | 5 | 1 | 0.1 | 0.664506239 | 5.001080428 |
| **5** | 5 | 1 | 3 | 0.982631706 | 5.999368294 |
| **6** | 5 | 1 | 10 | 8.604233333 | 11.0291 |
| **7** | 1 | 5 | 0.1 | 4.332757209 | 1.000288124 |
| **8** | 1 | 5 | 3 | 9.82253976 | 1.104559829 |
| **9** | 1 | 5 | 10 | 98.00184336 | 1.019764964 |
| **10** | 1 | 10 | 0.1 | 8.499743975 | 1.000128025 |
| **11** | 1 | 10 | 3 | 16.04242311 | 1.047403077 |
| **12** | 1 | 10 | 10 | 103.4918343 | 1.017749419 |
| **13** | 5 | 5 | 0.1 | 3.998499794 | 5.000750206 |
| **14** | 5 | 5 | 3 | 5.964327649 | 5.605595428 |
| **15** | 5 | 5 | 10 | 70.88182708 | 5.447591161 |
| **16** | 0.5 | 0.5 | 0.1 | 0.439440465 | 0.500869005 |
| **17** | 0.5 | 0.5 | 3 | 4.638513638 | 0.664816936 |
| **18** | 0.5 | 0.5 | 10 | 95.52344669 | 0.507965998 |
| **19** | 10 | 10 | 0.1 | 2.331353146 | 10.00099019 |
| **20** | 10 | 10 | 3 | 10.7222562 | 10.59588665 |
| **21** | 10 | 10 | 10 | 32.20642151 | 13.44634573 |

TABLE II
TIME HORIZON 4 YEARS

| N/N | qf | q0 | a | x0 | x1 |
|---|---|---|---|---|---|
| **1** | 10 | 1 | 0.1 | 0.664765713 | 10.00063368 |
| **2** | 10 | 1 | 3 | 0.794150148 | 10.58302386 |
| **3** | 10 | 1 | 10 | 0.645152668 | 17.65086319 |
| **4** | 5 | 1 | 0.1 | 1.220399452 | 5.000607635 |
| **5** | 5 | 1 | 3 | 0.973759738 | 5.590923489 |
| **6** | 5 | 1 | 10 | 20.75662544 | 7.497445366 |
| **7** | 1 | 5 | 0.1 | 4.99939988 | 1.00020006 |
| **8** | 1 | 5 | 3 | 8.924652739 | 1.103027039 |
| **9** | 1 | 5 | 10 | 95.95011748 | 1.020199407 |
| **10** | 1 | 10 | 0.1 | 9.285420257 | 1.000098014 |
| **11** | 1 | 10 | 3 | 15.27263543 | 1.046249779 |
| **12** | 1 | 10 | 10 | 101.3290077 | 1.018155849 |
| **13** | 5 | 5 | 0.1 | 3.998559831 | 5.000480084 |
| **14** | 5 | 5 | 3 | 5.076796304 | 5.420021264 |
| **15** | 5 | 5 | 10 | 57.40997258 | 5.568527783 |
| **16** | 0.5 | 0.5 | 0.1 | 0.446381488 | 0.500631813 |
| **17** | 0.5 | 0.5 | 3 | 2.440979724 | 0.816438634 |
| **18** | 0.5 | 0.5 | 10 | 95.01547727 | 0.507969417 |
| **19** | 10 | 10 | 0.1 | 8.998679926 | 10.00044004 |
| **20** | 10 | 10 | 3 | 9.558866595 | 10.3985896 |
| **21** | 10 | 10 | 10 | 16.13618209 | 14.37704012 |

TABLE III
TIME HORIZON 5 YEARS

| N/N | qf | q0 | a | x0 | x1 |
|---|---|---|---|---|---|
| 1 | 10 | 1 | 0.1 | 4,554129883 | 10,000356420 |
| 2 | 10 | 1 | 3 | 1,119064198 | 10,369730350 |
| 3 | 10 | 1 | 10 | 1,293726313 | 14,842455780 |
| 4 | 5 | 1 | 0.1 | 0,314056467 | 5,000433269 |
| 5 | 5 | 1 | 3 | 1,113122694 | 5,378089324 |
| 6 | 5 | 1 | 10 | 0,615899776 | 10,593893270 |
| 7 | 1 | 5 | 0.1 | 3,999199820 | 1,000200060 |
| 8 | 1 | 5 | 3 | 8,112666758 | 1,098785773 |
| 9 | 1 | 5 | 10 | 93,918150710 | 1,020622924 |
| 10 | 1 | 10 | 0.1 | 8,285322242 | 1,000098014 |
| 11 | 1 | 10 | 3 | 14,659253250 | 1,044265620 |
| 12 | 1 | 10 | 10 | 99,189455650 | 1,018555487 |
| 13 | 5 | 5 | 0.1 | 0,314056467 | 5,000433269 |
| 14 | 5 | 5 | 3 | 5,139082174 | 5,287513421 |
| 15 | 5 | 5 | 10 | 17,694225760 | 7,179090223 |
| 16 | 0.5 | 0.5 | 0.1 | 0,498123240 | 0,500469337 |
| 17 | 0.5 | 0.5 | 3 | 1,032002046 | 0,936738724 |
| 18 | 0.5 | 0.5 | 10 | 93,555035820 | 0,508129951 |
| 19 | 10 | 10 | 0.1 | 1,629990457 | 10,000397130 |
| 20 | 10 | 10 | 3 | 9,126166344 | 10,278327690 |
| 21 | 10 | 10 | 10 | 12,345295190 | 13,349442450 |

TABLE IV
TIME HORIZON 6 YEARS

| N/N | qf | q0 | a | x0 | x1 |
|---|---|---|---|---|---|
| 1 | 10 | 1 | 0.1 | 5.66542912 | 10.00024751 |
| 2 | 10 | 1 | 3 | 0.181249765 | 10.26649779 |
| 3 | 10 | 1 | 10 | 0.696100067 | 13.25743429 |
| 4 | 5 | 1 | 0.1 | 2.331983226 | 5.000270027 |
| 5 | 5 | 1 | 3 | 0.625254685 | 5.274009629 |
| 6 | 5 | 1 | 10 | 1.801413987 | 8.583985187 |
| 7 | 1 | 5 | 0.1 | 4.110300954 | 1.000162039 |
| 8 | 1 | 5 | 3 | 7.556488691 | 1.091284933 |
| 9 | 1 | 5 | 10 | 91.90531682 | 1.021050735 |
| 10 | 1 | 10 | 0.1 | 9.666306636 | 1.000072008 |
| 11 | 1 | 10 | 3 | 14.22279057 | 1.041685111 |
| 12 | 1 | 10 | 10 | 97.07241184 | 1.018959505 |
| 13 | 5 | 5 | 0.1 | 2.331983226 | 5.000270027 |
| 14 | 5 | 5 | 3 | 5.481827577 | 5.206834557 |
| 15 | 5 | 5 | 10 | 7.058578692 | 7.709085443 |
| 16 | 0.5 | 0.5 | 0.1 | 0.345840656 | 0.50039717 |
| 17 | 0.5 | 0.5 | 3 | 0.889118343 | 0.844682801 |
| 18 | 0.5 | 0.5 | 10 | 93.04690587 | 0.508129953 |
| 19 | 10 | 10 | 0.1 | 1.498591939 | 10.00028162 |
| 20 | 10 | 10 | 3 | 9.428511247 | 10.20157895 |
| 21 | 10 | 10 | 10 | 11.64497359 | 12.37384924 |

TABLE V
TIME HORIZON 7 YEARS

| N/N | qf | q0 | a | x0 | x1 |
|---|---|---|---|---|---|
| 1 | 10 | 1 | 0.1 | 0.427277784 | 10.00021561 |
| 2 | 10 | 1 | 3 | 1.027374155 | 10.19174043 |
| 3 | 10 | 1 | 10 | 1.275667714 | 12.26158508 |
| 4 | 5 | 1 | 0.1 | 1.035724749 | 5.000218718 |
| 5 | 5 | 1 | 3 | 0.782017776 | 5.201302496 |
| 6 | 5 | 1 | 10 | 0.216308957 | 7.688161834 |
| 7 | 1 | 5 | 0.1 | 4.499231877 | 1.000128025 |
| 8 | 1 | 5 | 3 | 7.136605693 | 1.083182175 |
| 9 | 1 | 5 | 10 | 89.91101306 | 1.02148334 |
| 10 | 1 | 10 | 0.1 | 8.666234628 | 1.000072008 |
| 11 | 1 | 10 | 3 | 13.66136502 | 1.039809285 |
| 12 | 1 | 10 | 10 | 96.05345233 | 1.018959505 |
| 13 | 5 | 5 | 0.1 | 3.998874936 | 5.000187513 |
| 14 | 5 | 5 | 3 | 4.686370025 | 5.165074462 |
| 15 | 5 | 5 | 10 | 7.233408323 | 6.92368935 |
| 16 | 0.5 | 0.5 | 0.1 | 0.043274787 | 0.500363352 |
| 17 | 0.5 | 0.5 | 3 | 0.139765609 | 0.841513921 |
| 18 | 0.5 | 0.5 | 10 | 92.53877591 | 0.508129953 |
| 19 | 10 | 10 | 0.1 | 0.427277784 | 10.00021561 |
| 20 | 10 | 10 | 3 | 9.155219226 | 10.15508136 |
| 21 | 10 | 10 | 10 | 11.75882068 | 11.74551947 |

TABLE VI
TIME HORIZON 8 YEARS

| N/N | qf | q0 | a | x0 | x1 |
|---|---|---|---|---|---|
| 1 | 10 | 1 | 0.1 | 2.332224505 | 10.0001584 |
| 2 | 10 | 1 | 3 | 1.311424795 | 10.14657709 |
| 3 | 10 | 1 | 10 | 1.1565471 | 11.70146966 |
| 4 | 5 | 1 | 0.1 | 0.665457001 | 5.000172811 |
| 5 | 5 | 1 | 3 | 0.777641647 | 5.155499929 |
| 6 | 5 | 1 | 10 | 2.277887489 | 6.781397716 |
| 7 | 1 | 5 | 0.1 | 3.499103853 | 1.000128025 |
| 8 | 1 | 5 | 3 | 6.784080553 | 1.075582122 |
| 9 | 1 | 5 | 10 | 87.93465947 | 1.02192077 |
| 10 | 1 | 10 | 0.1 | 7.66616262 | 1.00007201 |
| 11 | 1 | 10 | 3 | 13.29680808 | 1.037386726 |
| 12 | 1 | 10 | 10 | 92.90293532 | 1.01978171 |
| 13 | 5 | 5 | 0.1 | 0.665457001 | 5.000172811 |
| 14 | 5 | 5 | 3 | 4.97991073 | 5.12874880 |
| 15 | 5 | 5 | 10 | 5.226993432 | 6.57576716 |
| 16 | 0.5 | 0.5 | 0.1 | 0.26126147 | 0.50027095 |
| 17 | 0.5 | 0.5 | 3 | 0.415548348 | 0.741670961 |
| 18 | 0.5 | 0.5 | 10 | 91.09598546 | 0.50829212 |
| 19 | 10 | 10 | 0.1 | 2.332224505 | 10.0001584 |
| 20 | 10 | 10 | 3 | 9.05370561 | 10.12261035 |
| 21 | 10 | 10 | 10 | 10.73639171 | 11.38100576 |

TABLE VII
CONCATENATED PLAYER (5 YEARS)

| N/N | qf | q0 | α | x0 | x1 | A |
|---|---|---|---|---|---|---|
| 1 | 5 | 1 | 0.1 | 1,000001240 | 125,00976500 | 0,000787341 |
| 2 | 5 | 1 | 3 | 1,000977440 | 133,80203050 | 0,023620231 |
| 3 | 5 | 1 | 10 | 1,003950318 | 223,67110450 | 0,044311499 |
| 4 | 10 | 1 | 0.1 | 1,000000315 | 250,00988130 | 0,00039681 |
| 5 | 10 | 1 | 3 | 1,000264549 | 258,89675720 | 0,01149879 |
| 6 | 10 | 1 | 10 | 1,001625954 | 349,14635920 | 0,028478022 |
| 7 | 1 | 5 | 0.1 | 5,000312068 | 25,00676429 | 0,00322507 |
| 8 | 1 | 5 | 3 | 5,204657329 | 31,47982683 | 0,079608361 |
| 9 | 1 | 5 | 10 | 5,217728109 | 115,84957110 | 0,081922022 |
| 10 | 1 | 10 | 0.1 | 10,000848630 | 25,00501577 | 0,002777325 |
| 11 | 1 | 10 | 3 | 10,649625950 | 29,80879337 | 0,072361659 |
| 12 | 1 | 10 | 10 | 10,926800960 | 106,65424150 | 0,084330512 |
| 13 | 5 | 5 | 0.1 | 5,000017479 | 125,00917780 | 0,000763305 |
| 14 | 5 | 5 | 3 | 5,013982614 | 133,30155900 | 0,02153385 |
| 15 | 5 | 5 | 10 | 5,059925384 | 220,14407480 | 0,02153385 |
| 16 | 0.5 | 0.5 | 0.1 | 0,500038222 | 12,50861048 | 0,007138447 |
| 17 | 0.5 | 0.5 | 3 | 0,514210740 | 20,69211184 | 0,135096659 |
| 18 | 0.5 | 0.5 | 10 | 0,506052038 | 110,71303800 | 0,089111398 |
| 19 | 10 | 10 | 0.1 | 10,000016150 | 250,00921160 | 0,000 383128 |
| 20 | 10 | 10 | 3 | 10,013684110 | 258,31101440 | 0,011138971 |
| 21 | 10 | 10 | 10 | 10,088660210 | 344,12683690 | 0,028151925 |

Plots F(A) – A for Section IV

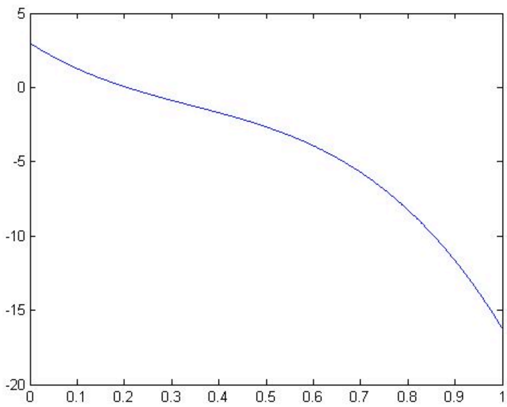



Fig. 1. Plot 24 (time horizon 5 years).

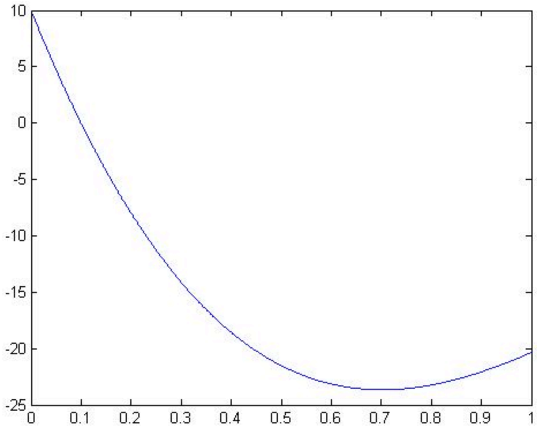



Fig. 2. Plot 25 (time horizon 5 years).

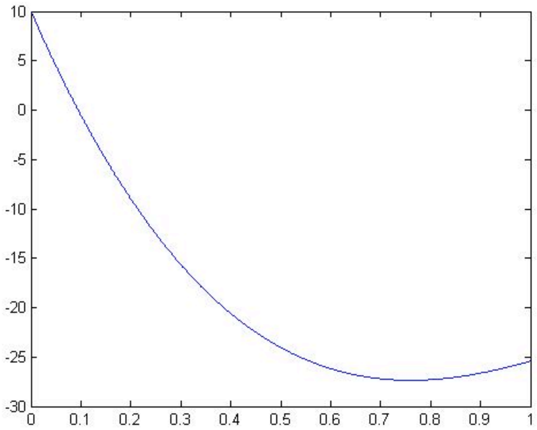



Fig. 3. Plot 28 (time horizon 5 years).

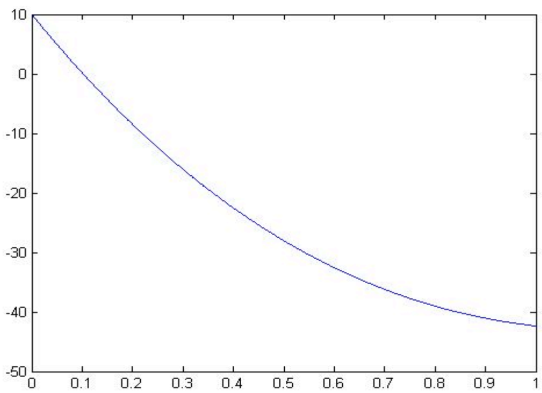



Fig. 4. Plot 34 (time horizon 5 years).

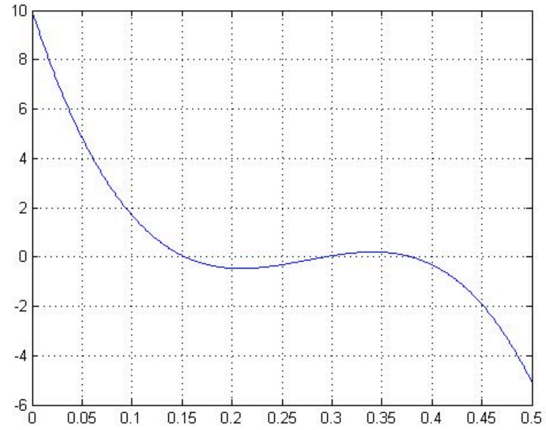



Fig. 5. Plot 6 (time horizon 3 years).

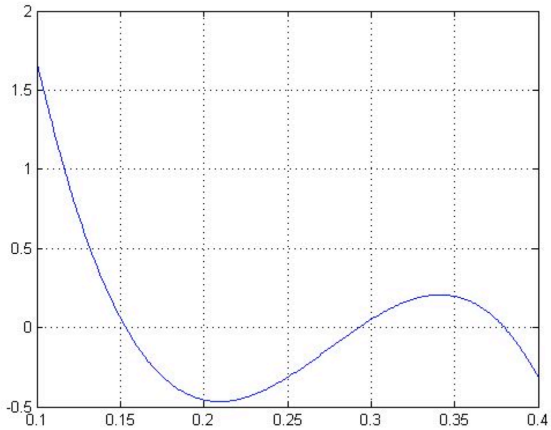



Fig. 6. Zoom of Fig. 5.

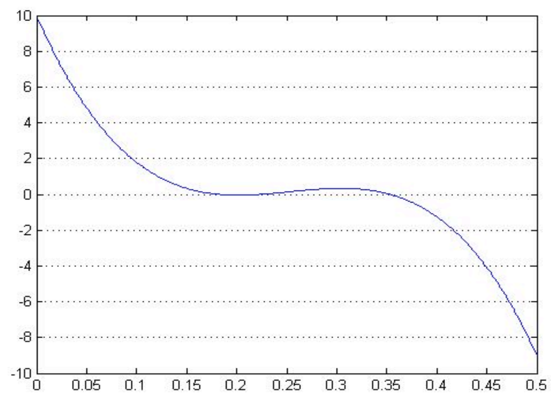



Fig. 7. Plot 6 (time horizon 4 years).

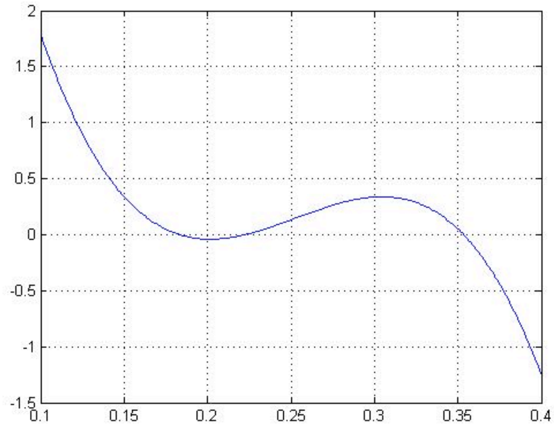



Fig. 8. Zoom of Fig. 7.

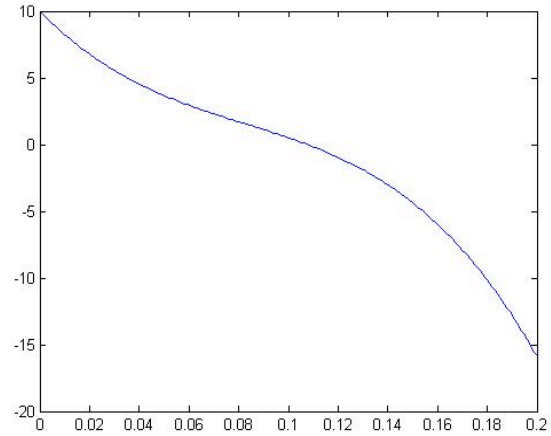



Fig. 9. Plot 4 (time horizon 7 years).

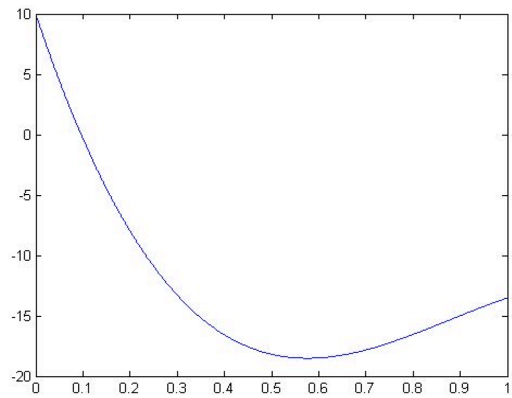



Fig. 10. Plot 14 (time horizon 7 years).

## References


[1] D. P. Bertsekas, “Dynamic Programming and Optimal Control” 2nd ed., Athena Scientific-Belmont-Massachusetts , 2000

[2] T. Basar and G. J. Olsder, “Dynamic Noncooperative Game Theory” 2nd ed., SIAM Classics in Applied Mathematics, 1999

[3] A.W. Starr and Y.C. Ho, “Nonzero-Sum Differential Games”, JOTA: Vol. 3, No. 3, 1969, p.p. 184-206

[4] A.W. Starr and Y.C. Ho, “Further Properties of Nonzero-Sum Differential Games”, JOTA: Vol. 3, No. 4, 1969, p.p 207-219

[5] G.P. Papavassilopoulos, “On the Linear Quadratic Gaussian Nash Game with One-Step Delay Observation Sharing Pattern”, IEEE Trans. on Automatic Control, Vol. AC-27, No. 5, October 1982, pp. 1065-1071

[6] G.P. Papavassilopoulos, N.Kakogiannis, “Games among Long and Short Term Electricity Producers and Users”, 14th Intern. Symposium on Dynamic Games and Applications ,June 19th to 24th, 2010 The Banff Centre, Banff, Alberta, Canada.

[7] D.Fudenberg and J.Tirole “Game Theory” , MIT press 1991


## Biographies


**Nikolaos Chr. Kakogiannis** was born in Athens, Greece, and received his Diploma (2006) from the Dept. of Mechanical and Electrical Engineering at the National Technical University of Athens. He has an MsC in Economics from National Technological University of Athens and now is a PhD candidate at NTUA. His research interests are in Control and Decision Theory, Game Theory and Applications in Energy Policy in liberalised markets. He is also Investment and Development Consultant of the Vice President of the Greek Government (nikos_kakogiannis@yahoo.com)

**Panagiotis Kontogiorgos** was born in Athens, Greece, and is an undergraduate student of Dept. of Electrical and Computer Engineering at National Technological University of Athens. He is interested in Applications of Control and Game Theory in Energy Policy (panko09@hotmail.com)

**Elena Sari** was born in Athens, Greece. She holds a BSc in Physics (4 year program), a 2 year MPhil degree in Electronics and Radioelectrology, from the Depts. of Physics and Informatics all at the National & Kapodistrian University of Athens and a PhD in Electrical and Computer Engineering from the National Technical University of Athens (NTUA). Since then she works as a Research Scientist at the Control and Decision Theory Laboratory (CDL). She is also a Laboratory Instructor at the Technological Education Institute of Piraeus, Dept. of Electronic Engineering. Before starting her PhD, she worked as a Telecommunication Engineer at the Department of Design and Development of Networks and New Technologies at a Local Mobile Provider. Her current research interests are in the area of Optimization and Decision Making under uncertainty, Game Theory as well as Techno-Economic Analysis and Applications in Telecommunications and Energy. Her e-mail address is: elena@netmode.ntua.gr.

**George P. Papavassilopoulos** received his Diploma (1975) from the Dept. of Mechanical and Electrical Engineering at the National Technical University of Athens, the Master’s (1977) and PhD (1979) degrees both from the Dept. of Electrical Engineering, University of Illinois at Urbana-Champaign. He was a Professor in the Dept. of EE-Systems, University of Southern California, Los Angeles, from 1979 until 2000 when he joined the Dept. of Electrical and Computer Engineering of NTUA as Professor. He has conducted research in Dynamic and Stochastic Game Theory, Optimization and Control Theory, Parallel Algorithms, Markovian Learning, BMIs for Robust Control. He is also interested in applications in Biomedical Engineering, Energy and Telecommunication Policies. For more information on current activities see: http://www.control.ece.ntua.gr