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Dynamic Games among Agents with Partial Information of the Structure of the Interactions Graph: Decision Making and Complexity Issues

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***Abstract*—We consider dynamic games on large networks, motivated by structural and decision making issues pertaining in the area of Systems of Systems. The players participating in the game do not know the network structure and the characteristics of the dynamics and costs of the players involved. Instead, they know some local characteristics of the topology, as well as a statistical description of the network. An approximate equilibrium concept is introduced and a complexity notion that describes the minimum amount of structural and feedback information needed for the players in order to behave approximately in Nash equilibrium, is defined. An example of a Linear Quadratic game on a ring is finally studied and an asymptotic upper bound for the complexity of the game is derived.**

I. INTRODUCTION

Several features characterizing Systems of Systems, such as operational and managerial independence, geographical distribution, heterogeneity of systems and networks of systems [1] can be captured by Dynamic Game models. In this work we study dynamic games on large random interaction structures where the players participating in the game have only partial information about the other agents participating in the game. Particularly, the agents interact on a network and they have information about their neighborhood as well as a statistical model (ex. [2]) for the network. An example of a System of Systems, where the game theoretic analysis is quite suitable, is the smart grid where several producers / consumers are connected to the same network.

Two kinds of approaches have been mainly used to predict the behavior of the participants in large games. The first approach is based on equilibrium concepts. The dominant notion in this approach is the Nash equilibrium and a complete knowledge of a large amount of information is needed. The second kind of approaches assumes limited rationality for the participants and it is based on dynamic formulations. In particular, some deterministic or stochastic rules describing the future actions of the agents as a function of their current actions are postulated and then evaluated experimentally or theoretically. This kind of approach does not require a complete knowledge of the game. However, the dynamic rules used are not universal, in the sense that there is no reason to believe that all the players will follow some specified rule to determine their future actions. This work aims to stand between these two approaches, assuming only a partial knowledge of the game and players of a full rationality.

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A. Related Topics

The interest for the games with large number of players is not new. In [3], games with a continuum of players, called Oceanic Games, were introduced and a value for such games was defined. The Mean Field Games [4] have been recently introduced to study games with large number of players. The closely related methodology of Nash Certainty Equivalence was also developed, in order to obtain asymptotic Nash equilibrium results, as the number of players tends to infinity [5],[6]. These approaches study games, where each player interacts with the mass of the other players, which is approximated by a continuum. Large games involving a coordinator (major player) were studied in [7] and some extensions of the Mean Field game models are presented in [8].

Another related topic is Games with Local Interactions, in which each player interacts with some players important to him/her on some organized structure. In [9], equilibria for complete and incomplete information Local Interaction Games were found, based on contraction mapping ideas. The dynamic game counterpart is presented in [10]. Models with discrete choice were introduced in [11].

Games where players move on a graph were analyzed in [12] and finite games on graphs, where each node corresponds to a participant of the game, were studied in [13]. Repeated games with random matching of the opponents were introduced in [14], in the context of sustainability of cooperation and social norms. The probability of existence of a Nash equilibrium for games on random graphs is studied in [15]. A quadratic game on networks is studied in [16], using centrality notions. Games on networks with incomplete

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information are studied in [17]. Dynamic games on evolving (state dependent) graphs were studied in [18] and stochastic games in [19]. A review of network games is given in [20].

Dynamic rules for updating the actions of the agents on lattices were studied in the context of Interacting Particle Systems [21] and in [22]. Several dynamic rules for games on graphs were introduced and studied analytically and computationally [23]. Several social applications of evolutionary games on graphs were studied in [24].

The impact of the quality of information that the agents receive on their costs is studied in [25]. The notion of the price of uncertainty was introduced in [26] and [27], in order to describe the difference in the costs of the players under different information using dynamic and equilibrium formulations respectively. The price of information was introduced in [28] to describe the difference of the cost that the players have in deterministic dynamic games under different information patterns (feedback and open loop).

B. Our Approach

We assume that the agents have a statistical knowledge about the game they are involved, instead of a full knowledge. Particularly, there is an ensemble (set, collection) of possible games that the players may participate in. A probability measure on the ensemble, common to all players, is assumed. The players can also measure the interaction structure locally, i.e. they know the characteristics and the interactions of some players, that are important for them. A model that divides the stochasticity into two parts is introduced. Specifically, the one type of stochasticity is due to the lack of predictability in the dynamics of the game and the other is the uncertainty due to the lack of information.

Based on such a model, a new approximate equilibrium concept is defined. Specifically, we study sets of strategies depending only on local information that constitute an ϵ - Nash equilibrium with very high probability. A new notion of complexity for an ensemble of games is defined as the minimum amount of information needed, in order to achieve an approximate equilibrium. Let us note that the current paper extends our previous work [29], where the static case was analyzed.

C. Organization

The rest of the paper is organized as follows: In Section II, the game is described. In section III, an approximate equilibrium concept is defined and compared with other equilibrium concepts. In Section IV, the complexity function for an ensemble of games is defined. Section V contains an example of a Linear Quadratic games on a ring and derives some bounds on the complexity functions. Section VI concludes.

D. Notation

A directed or undirected graph will be denoted by $G = (V, E)$, where V is the set of vertices and E the set of edges. For a $v \in V$, the neighborhood of v is defined as $N_v^1(G) = \{v\} \cup \{v' \in V : (v', v) \in E\}$. The neighborhood of order n of v is defined as $N_v^n(G) = \cup_{j \in N_v^1(G)} N_v^{n-1}(G)$. For a set of real numbers or functions u_i, $i = 1, \dots, N$ we denote by $(u_i)_i$ the ordered tuple $(u_1, \dots, u_N)$ and by u_{-i} the tuple $(u_1, \dots, u_{i-1}, u_{i+1}, \dots, u_N)$.

The asymptotic notation will be also used. For the real functions f and g, we write $f(x) \in O(g(x))$, if there exists a constant $c > 0$, such that $|f(x)| \le c|g(x)|$ for large x.

II. Game Description

The game for a specified interaction structure is first defined and the ensemble of interaction structures, as well as the information available to the players are then described.

The dynamics and the cost functions of the players participating in the game depend on an interaction structure. Each player i has a type θ_i, belonging to a set of types Θ. The interaction structure contains a set of N players, $p_1, \dots, p_N$ and is described as:

$$S = (\Pi, G), \tag{1}$$

where $\Pi = ((p_1, \theta_1), \dots, (p_N, \theta_N))$ and $G = (V, E)$ is a graph (directed or not), describing the interactions. Each vertex of the graph G represents an agent from $\{p_1, \dots, p_N\}$ and each edge represents the influence of an agent to another.

Each player i has his/her own state variable denoted by x^i. The evolution of the state variables is described by the following equation:

$$x_{k+1}^i = \sum_{j \in N_i^1(G)} f_1^{\theta_i, \theta_j}(x_k^i, x_k^j, u_k^i, u_k^j, w_k^i) + \sum_{j \in V} f_2^{\theta_i, \theta_j}(x_k^i, x_k^j, u_k^i, u_k^j, w_k^i), \tag{2}$$

where u_k^i is the control variable of player i at time step k and w_k^i are ranom variables. The random variables w_k^i and x_0^i are defined on a probability space $(\Omega, \mathcal{F}, P)$.

The cost functions of the players participating in the game have the form:

$$J_i = E_P \left\{ \sum_{k=0}^{T} \rho^k \left[\sum_{j \in N_i^1(G)} L_1^{\theta_i, \theta_j}(x_k^i, x_k^j, u_k^i, u_k^j) + \sum_{j \in V} L_2^{\theta_i, \theta_j}(x_k^i, x_k^j, u_k^i, u_k^j) \right] \right\} \tag{3}$$

where the time horizon T can be finite or infinite, $L_1, L_2 \ge 0$ and $\rho \in (0, 1)$ is the discount factor. The cost function of the player p_i depends differently on the players with whom he/she has a direct connection, than the rest of the players.

Before stating the assumptions on the information structure of the game, we shall describe the knowledge that the agents have about the game. The players instead of possessing a full knowledge of the interaction structure, they know that it belongs to an ensemble of possible interaction structures denoted by $\mathcal{E}$. The players also have a probability measure Q on $\mathcal{E}$. We assume that the ensemble $\mathcal{E}$ and the measure Q are common knowledge among the players, i.e. we study games with "known statistics". For an interaction structure $S \in \mathcal{E}$, let us denote by g^S the corresponding game.

Apart from the statistical model, each agent has also some local information about the interaction structure and the state variables of the participants of the game. Particularly, we assume that the player i knows the structure of the subgraph of its neighborhood of order n, denoted by $N_i^n(G)$, as well as the types of the players involved in that subgraph and their state variables. Thus, the information available for the player i at the time step k is:

$$I_k^{i,n} = (N_i^n(G), (\theta_j)_{j \in N_i^n(G)}, (x_t^j)_{j \in N_n^i(G)}^{t=0,\dots k}), \qquad (4)$$

i.e. the information available to the agent i at time step k depends on the size of the information neighborhood n.

The strategy of each agent is a function of the information available to him/her. We consider symmetric sets of strategies, where players with the same information (and hence type) behave the same. Furthermore, we focus on feedback (without memory) time invariant strategies. The strategies under consideration have thus the form:

$$u_k^i = \gamma(\bar{I}_k^{i,n}), \qquad (5)$$

where:

$$\bar{I}_k^{i,n} = (N_i^n(G), (\theta_j)_{j \in N_i^n(G)}, (x_k^j)_{j \in N_n^i(G)}). \qquad (6)$$

Remark 1: (i) The dynamics and the cost functions given by equation (2) and (3) describe two types of interactions. The first sum corresponds to local interactions and the second term to mean field interactions.

(ii) The members of the ensemble do not need to have the same number of players.

(iii) There are two types of stochasticity presented in the model. The first is due to the lack of predictability and it is described by the probability space $(\Omega, \mathcal{F}, P)$. The second is the uncertainty due to the lack of information and it is described by the probability space $(\mathcal{E}, \mathcal{A}, Q)$. The elements of $\mathcal{E}$ contain the structural features of the game.

Remark 2: There are some possible extensions of the model studied. For example:

(i) Models involving graphs with information on their edges could also be studied, as well as structures more general than the graphs relating more than two agents (ex. [30]).

(ii) The probability measure P could depend on the interaction structure.

(iii) Continuous time modes could also be considered.

In the following sections, asymptotic results for large interaction structures are derived. In order to do so, we define a sequence of ensembles describing games with increasing number of players. Let us denote by $\mathcal{E}_\nu$ the ν-th member of that sequence. The results of the next sections will refer to the final part of the sequence $\mathcal{E}_\nu$.

III. Approximate Equilibrium

Consider a large game in which the actions of the players depend only on local and statistical information. Due to the fact that the agents do not know in which game they participate in, it is reasonable to expect that a set of strategies in the form (5) could not typically constitute a Nash equilibrium. An approximate equilibrium concept is thus defined, based on the concept of ϵ - Nash equilibrium. We first recall the definition of the ϵ - Nash equilibrium for a single game.

Definition 1: Consider a game g^S with $S \in \mathcal{E}$ and the set of dynamics and cost functions given by (2) and (3). Then a set of strategies $(u_i)_i$ in the form (5) constitute an ϵ - Nash equilibrium, if for every player i it holds:

$$J_i(u_i, u_{-i}) - \min_u J_i(u, u_{-i}) < \epsilon, \qquad (7)$$

where the minimum is considered within the set of full state measurement closed loop strategies.

An approximate equilibrium concept is then defined for the ensemble $\mathcal{E}$. Particularly, we are interested on sets of strategies that constitute an ϵ - Nash equilibrium for most of the games in $\mathcal{E}$.

Definition 2: Consider an ensemble of interaction structures $\mathcal{E}$ and the set of dynamics and cost functions as before. Then a set of strategies $(u_i)_i$ in the form (5) is ϵ - fine if it holds:

$$Q(\{S \in \mathcal{E} : (u_i)_i \text{ is an } \epsilon\text{- Nash equilibrium}\}) > 1 - \epsilon, \quad (8)$$

i.e. $(u_i)_i$ constitutes an ϵ - Nash equilibrium with high probability.

The agents act without knowing in which game they play. Thus, the best they could expect is to play well in many of the possible games. For large information neighborhoods, i.e. for large n, we may expect that, under some conditions, there exists an ϵ - fine set of strategies with small ϵ and that $\epsilon \to 0$ as the information of the players approaches the full information.

Remark 3: The reason for studying ϵ - fine sets of strategies with small ϵ is that, with a very high probability, no player has non-negligible benefit from changing his/her strategy. Particularly, if a player i is allowed to measure the whole interaction structure S, as well as the state variables of all the other players, he/she could only improve his/her cost by at most ϵ, assuming that the strategies of the other players remain the same.

Remark 4: Definition 2 transfers some ideas from Statistical Physics to games. A widely used model in Statistical Physics considers an ensemble of systems with a probability measure on that ensemble (ex. [31]). The actual system under consideration is one of the systems of the ensemble. Several properties dealing with some statistic of the state variables of the particles of the system are then found to have a value very close to a deterministic constant, for all but a set of systems with a very small measure. This situation is transferred to an ensemble of games in Definition 2. Let us note that the idea to transfer concepts from Statistical Physics to Game Theory or Multiagent Systems is not new (see for example [21], [22], [4], [5]).

Remark 5: An alternative way to model the uncertainty is to use a common probability structure for both the unpredictable disturbances and the structural information, i.e. to use a single probability space to describe w_k^i, G and θ^i. In that case, the notion of Bayesian Nash equilibrium could be used. Some differences among the two approaches are then listed:

(i) It is very difficult to compute a Bayesian Nash equilibrium even for simple dynamic games. In the case of LQ stochastic Dynamic Games with imperfect state feedback information, Nash equilibria have been computed only for special information patterns [32]. In the case where the structural information is also incomplete, the optimization problems are vey difficult even for single person games (control problems) due to the fact that the dual control problem arises [33].

(ii) In contrast to Bayesian Nash equilibrium, an ϵ - fine set of strategies is insensitive to new information. Particularly, assume that $(u_i)_i$ is an ϵ - fine set of strategies and some players receive more information $I_i' \supset I_i$. Then $(u_i)_i$ remains ϵ - fine.

(iii) In contrast to an ϵ - fine set of strategies, a Bayesian Nash equilibrium could not typically constitute of feedback (memoryless) functions.

IV. COMPLEXITY

With a small amount of information it is probably not possible to have an ϵ - fine set of strategies. Thus, we are interested in the following question:

Question 1: "Given a positive constant ϵ, what is the minimum amount of information that the agents need to have in order to achieve an ϵ - fine set of strategies?"

Based on the answer to this question, a complexity notion for an ensemble of games is defined. In several cases, it is easier to answer Question 1, when the game has a large number of players. The following definition refers to a sequence of ensembles $\mathcal{E}_\nu$ and a complexity function is defined.

Definition 3: (i) Consider an ensemble $\mathcal{E}$ and the cost functions given by equation (3). Let us define the following function:

$$\bar{n}(m) = \inf\{n \in \mathbf{N} : \exists (u_i)_i, u_i = \gamma(\bar{I}_k^{i,n}) \text{ which is an } 2^{-m} \text{ - fine set of strategies}\}. \tag{9}$$

The Necessary Information Complexity (NIC) function is defined as:

$$C(m) = \max\{\# N_i^{\bar{n}(m)}\}, \tag{10}$$

where the maximum is taken over the players and over the games of the ensemble where $(u_i)_i$ is a 2^{-m} - Nash equilibrium.

(ii) Consider a sequence of ensembles $\mathcal{E}_\nu$ with cost functions J_i^ν and dynamics described by $f_1^{\cdot,\cdot,\nu}$, $f_2^{\cdot,\cdot,\nu}$. Denote by $C^\nu(\cdot)$ the NIC function of the ν-th ensemble. The Asymptotic Necessary Information Complexity (ANIC) function is given by:

$$C_a(m) = \limsup_{\nu \to \infty} C^\nu(m). \tag{11}$$

The sequence of ensembles will be called asymptotically simple if the function $C_a(m)$ is bounded and asymptotically complex if for some $m \in \mathbf{N}$ it holds $C_a(m) = \infty$.

Remark 6: Several static and dynamic games, that have only mean field interactions, have been studied in the literature [4], [6]. In these cases, under some conditions, each player interacts with the mass of the other players which behaves asymptotically deterministically, as the number of players increases. Thus, each player needs to know only his/her type in order to behave nearly optimally. Thus, the Mean Field Games is a first example of simple games.

V. A LQ GAME ON A RING

This section studies an example of a game with a known number of players $N = \nu$ lying on a ring having interactions only with their nearest neighbors. Specifically, there is a set of players $p_1, \ldots, p_N$ and each player p_i has a connection with p_{i-1} and p_{i+1}, where the convention $N + l \equiv l$ is used. Each agent p_i has a type $\theta_i \in [0, 1]$. The random variables θ_i are independent and distributed uniformly on $[0, 1]$.

The dynamics of the state vector of the player i is described by the following equation:

$$x_{k+1}^i = a x_k^i + u_k^i + w_k^i, \tag{12}$$

where x^i is the scalar state variable of the agent i, u_k^i the control variable of the agent i and w_k^i are zero mean i.i.d. random variables with finite second moments. The initial conditions are given by $x_0^i = w_{-1}^i$.

The cost function for the player i is given by:

$$J^i = E\left\{ \left(z_T^i\right)^2 + \sum_{k=0}^{T-1} \left[\left(z_k^i\right)^2 + r_i \left(u_k^i\right)^2 \right] \right\}, \tag{13}$$

where $r_i = (1 + \theta_i)/2$ and

$$z_k^i = x_k^i - \lambda \left(x_k^{i-1} + x_k^{i+1} \right). \tag{14}$$

Thus, a Linear Quadratic game with coupling only through the cost functions is considered.

Remark 7: The random variables θ_i are defined on $(\mathcal{E}, \mathcal{A}, Q)$ and the disturbances w_k^i on $(\Omega, \mathcal{F}, P)$.

It will be shown that the ANIC is at most linear under some specified conditions. To do so, we shall use simultaneous dynamic programming from the time step $k = T - 1$, backwards to $k = 0$. Consider the last step of the simultaneous dynamic programming. The cost functions are given by:

$$J_{T-1}^i = E\Big[(z_{T-1}^i)^2 + \left(x_T^i - \lambda \left(x_T^{i-1} + x_T^{i+1} \right) \right)^2 + r_i \left(u_{T-1}^i \right)^2 | x_{T-1} \Big]. \tag{15}$$

It is not difficult to see that the equilibrium condition is given by:

$$u_{T-1}^i = W_{T-1}^i((u_{T-1}^j)_{j=1}^N), \tag{16}$$

where the mapping $W^i_{T-1}((u^j_{T-1})^N_{j=1})$ is given by:

$$W^i_{T-1}((u^j_{T-1})^N_{j=1}) = \frac{\lambda}{1+r_i}\left[u^{i-1}_{T-1} + u^{i-1}_{T-1}\right] + + \frac{a}{1+r_i}\left[-x^i_{T-1} + \lambda(x^{i-1}_{T-1} + x^{i+1}_{T-1})\right]. \quad (17)$$

The proof of the following Proposition 1 is based on the contractivity of the following mapping:

$$W_{T-1}((u^j_{T-1})^N_{j=1}) = \left(W^i_{T-1}((u^j_{T-1})^N_{j=1})\right)^N_{i=1}. \quad (18)$$

Analogous mappings are then defined for the other time steps $T-2,\dots,0$.

Proposition 1: For small coupling constant λ, the ANIC of the ensemble of games described by (12) and (13) is at most linear, i.e. $C_a(m) \in O(m)$.

Proof: For simplicity reasons the proof is given for $T=2$. The proof starts at $k=1$ and then moves backwards to $k=0$. If $|\lambda| < 1/2$, then the mapping given by (18) is contractive for the infinity norm. Consider the feedback strategies obtained after m iterations of (18) with zero initial strategies. These strategies have the form:

$$u^{i,m}_1 = \sum_{l=-m}^{m} k^{i,l,m}_1 x^{i+l}_1. \quad (19)$$

Then the equation (18) is also a contraction in the space of the vectors of feedback gains with the infinity norm, i.e. the mapping $(k^{i,l,m}_1)^N_{i=1} \mapsto (k^{i,l,m+1}_1)^N_{i=1}$ is a contraction. Therefore:

$$\left\|(k^{i,l,m+1}_1)^N_{i=1} - (k^{i,l,m}_1)^N_{i=1}\right\|_\infty \le \frac{(2\lambda+1)a}{1.5}\left(\frac{2\lambda}{1.5}\right)^m \quad (20)$$

Before going back to the step $k=0$, let us compute the form of the cost functions J^i_1 when in the last step the strategies with feedback gains $(k^{i,l,m}_1)^N_{i=1}$ are applied. Equation (15) implies:

$$J^i_1 = \left(\sum_{l=-m}^{m} \xi^{i,l}_1 x^{i+l}_1\right)^2 + \left(\sum_{l=-m}^{m} k^{i,l,m}_1 x^{i+l}_1\right)^2 + (z^i_1)^2 + C_i,$$

where:

$$\xi^{i,l}_1 = a\delta_{l,0} - \lambda a(\delta_{l,1}+\delta_{l,-1}) + k^{i,l,m}_1 - - \lambda(k^{i+1,l-1,m}_1 + k^{i-1,l+1,m}_1). \quad (21)$$

It is not difficult to show that it holds:

$$J^i_1 = \sum_{l_1,l_2} q^{i,l_1,l_2}_1 x^{i+l_1}_1 x^{i+l_2}_1, \quad (22)$$

where $q^{i,l_1,l_2}_1 = q^{i,l_2,l_1}_1$, $|q^{i,l_1,l_2}_1| < M\beta^{|l_1|+|l_2|}$ and $\beta > 0$. Furthermore, $\beta \to 0$ as $\lambda \to 0$. Let us now go back one step to $k=0$ and assume that the players at time step $k=1$ will follow the strategies with feedback gains given by $k^{i,l,m}_1$. The cost functions have the form:

$$J^i_0 = (z^i_0)^2 + C_i + r_i(u^i_0)^2 + \sum_{l_1,l_2} q^{i,l_1,l_2}_1 x^{i+l_1}_1 x^{i+l_2}_1 \quad (23)$$

The equilibrium condition is, thus, given by:

$$u^i_0 = -\frac{1}{(q^{i,0,0}_1 + r_i)}\left[\sum_{l_2} a q^{i,0,l_2}_1 x^{i+l_2} + \sum_{l_2\neq 0} q^{i,0,l_2}_1 u^{i+l_2}_0\right],$$

and the mapping in the feedback gains by:

$$k^{i,l,m'+1}_0 = -\frac{1}{(q^{i,0,0}_1 + r_i)}\left[a q^{i,0,l}_1 + \sum_{l_2\neq 0} q^{i,0,l_2}_1 k^{i+l_2,l-l_2,m'}_1\right] \quad (24)$$

For small β, the mapping (24) is contractive with Lipschitz constant $2M\beta/(1-\beta)$. Therefore:

$$\left\|(k^{i,l,m'+1}_0)^N_{i=1} - (k^{i,l,m'}_0)^N_{i=1}\right\|_\infty \le aM\left(\frac{2M\beta}{1-\beta}\right)^{m'}. \quad (25)$$

To complete the proof, let us introduce some quantities. Let $\bar{J}^{i,m}_1(x_1)$ be the minimum cost to go for player i at time step $k=1$, assuming that the other players use the strategies given by (19), $J^{i,m}_1(x_1)$ be the cost to go if all the players use the strategies given by (19) and:

$$J^{i,m}_0(x_0,u^i_0) = (z^i_0)^2 + (u^i_0)^2 + E[J^{i,m}_1(x_1)|x(0)],$$
$$\bar{J}^{i,m}_0(x_0,u^i_0) = (z^i_0)^2 + (u^i_0)^2 + E[\bar{J}^{i,m}_1(x_1)|x(0)],$$

where we assume that the other players use the strategies given by (19).

The proof is based on the following facts:

Fact 1: Let $\gamma > (2\lambda/1.5)^2$. For large m it holds:

$$J^{i,m}(x_1) \le \bar{J}^{i,m}(x_1) + \gamma^m(1 + \|x_1\|^2_{\infty,i,m}),$$

where $\|x_1\|_{\infty,i,m} = max\{|x^j_1| : j = i-m,\dots,i+m\}$.

The proof of Fact 1 is immediate from equations (15) and (20).

Fact 2: Let $\gamma > (2\lambda/1.5)^2$. For large m it holds:

$$\min_{u^i_0} J^{i,m}(x_0,u^i_0) \le \min_{u^i_0} \bar{J}^{i,m}(x_0,u^i_0) + \gamma^m(1 + \|x_0\|^2_{\infty,i,m}).$$

To prove Fact 2, let us observe that:

$$J^{i,m}_0(x_0,u^i_0) = (z^i_0)^2 + (u^i_0)^2 + J^{i,m}_1(f(x_0,u^i_0)) + c_0,$$

where $f(x_0,u^i_0))$ is the expected value of x_1 given x_0 if the player i uses u^i_0 and the other players use the strategies given by (19). Furthermore, it holds:

$$\bar{J}^{i,m}_0(x_0,u^i_0) = (z^i_0)^2 + (u^i_0)^2 + \bar{J}^{i,m}_1(f(x_0,u^i_0)) + \bar{c}_0.$$

Fact 1 implies that $|c_0 - \bar{c}_0| < \gamma^m$, for large m. Denoting by $v(x_0)$ the value of u^i_0 that minimizes $\bar{J}^{i,m}_0(x_0,u^i_0)$ we have:

$$\min_{u^i_0} J^{i,m}_0(x_0,u^i_0) \le J^{i,m}_0(x_0,v) \le \min_{u^i_0} \bar{J}^{i,m}_0(x_0,u^i_0) + 2\gamma^m(1 + \|x_0\|^2_{\infty,i,m}).$$

And using a small abuse of notation (choosing a smaller value for γ) we conclude to Fact 2.

Fact 3: Let $m > (2M\beta/(1-\beta))^2$. For large m it holds:

$$J_0^{i,m}(x_0, \sum_{l=-m}^{m} k_0^{i,l,m} x_0^{i+l}) \le$$
$$\le \min_{u_0^i} J_0^{i,m}(x_0, u_0^i) + \gamma^m (1 + \|x_0\|^2_{\infty,i,m}).$$

The proof of Fact 3 is immediate from (25).

For a small value of λ, there exist a constant $\tilde{\gamma} > max\{(2M\beta/(1-\beta))^2, (2\lambda/1.5)^2\}$ and $\tilde{\gamma} < 1$ such that Facts 2 and 3 apply with $\gamma = \tilde{\gamma}$. Thus, if $\bar{\gamma} > \tilde{\gamma}$, for large m it holds:

$$E\left[J_0^{i,m}(x_0, \sum_{l=-m}^{m} k_0^{i,l,m} x_0^{i+l})\right] \le E\left[\min_{u_0^i} J_0^{i,m}(x_0, u_0^i)\right] + \bar{\gamma}^m$$

Thus, the strategies given by the feedback gains $k_0^{\cdot,\cdot,\cdot}$ and $k_1^{\cdot,\cdot,\cdot}$ are $\bar{\gamma}^m$ - fine for large m. Thus, the ANIC function is at most linear. ■

Remark 8: The proof of Proposition 1 shows that in some cases the assumption of a common knowledge of the probability measure Q can be weakened.

VI. Conclusion

A model of Dynamic Games on random interaction structures was introduced and information and complexity issues were examined. Results dealing with ensembles of games instead of single games were derived. The stochasticity was divided into two parts using different probability structures. The first part describes the lack of predictability due to random disturbances and the other the lack of information of the participants of the game about their opponents. The ϵ - fine concept was defined to describe sets of strategies that constitute an ϵ - Nash equilibrium with probability higher than $1-\epsilon$. The complexity functions NIC and ANIC were introduced to describe the minimum amount of information needed in order for the players to have an ϵ - fine set of strategies. Finally, an example of a LQ game on a ring was studied and the ANIC function was shown to be asymptotically at most linear.

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