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Robust Variable Structure and Adaptive Control of Single-Arm Dynamics

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Abstract

In this paper, we present a new control law which is a combination of the continuous variable structure control (VSC) law and the switching-σ modification control law for the control of a single-arm manipulator with rigid links holding a rigid object. In the analysis, we show that this new control law has a smaller error bound than a simply continuous VSC law or switching -σ modification control law. We also present some simulations, where the proposed control law has better tracking precision performance both in transient state and steady state which agree with the theoretical analysis, and it still maintains the good tracking precision even if actuator unmodelled dynamics are considered.

Key words: Robotics, Variable Structure, Robustness.

1 Introduction

The area of robotics is a very important one and has received the attention of many researchers and practitioners from several fields of engineering. A particular class of problems arising in this area are control problems, where one is interested in creating a controller which will make the robot arms move in a certain way. Owing to the nonlinearity, disturbance, and unknown parameters (e.g., loads) of the manipulator's dynamics, the design of the manipulator's controller is a difficult problem. This leads to interesting and difficult robust variable structure control (see [9]-[18]) and adaptive control (see [20]-[27]) problems.

VSC ([9]-[18]) is a high-speed switching feedback control, where the control law switches to different values according to some rule. This control law can drive the nonlinear plant's state trajectory onto a sliding surface which is designed by the designer, and maintain the plant's state trajectory on the designed sliding surface afterwards. Its control law is very robust to system's disturbance. Adaptive control (see [20]-[27]) is different from the conventional fixed parameter control. It estimates the unknown parameter then uses the estimated parameter into its control law, and thus its tracking performance is better than that of the conventional fixed parameter control. Adaptive control is a very effective tool for the control of a manipulator in the presence of the uncertainties in the manipulator's dynamic equation.

Slotine and Li ([23]-[26]) exploited the structure of manipulator dynamics, which is assumed to be disturbance free, to develop a globally convergent adaptive scheme for position control of a single-arm manipulator. This scheme does not require measurements of the manipulator's joint accelerations, nor inversion of the estimated inertia matrix. The analysis and simulations show that the unknown parameters can be precisely estimated if disturbances are not involved in the manipulator dynamics. They also mentioned the possibility of applying the VSC for the control of the single-arm dynamics with bounded disturbance. But there are several important aspects they did not consider. First, they did not apply the estimation law which can guarantee that the estimated parameter will not drift to infinity in the presence of bounded disturbance. Second, they did not find the control law in Cartesian space which can derive the relationship between the reference variable in joint space and the reference variable in Cartesian space. This relationship is necessary for the control of single-arm dynamics as the desired trajectory is in the Cartesian space. Finally, they did not simulate and discuss VSC law if disturbances and unmodeled dynamics are considered. Reed and Ioannou (see [19],[27]) developed two new robust adaptive controllers which are based on the switching-σ modification control and the computed torque method [1] for the control of a single-arm manipulator with rigid links. Their control laws can guarantee that the tracking error belong to some error set in the presence of bounded disturbance and time-varying parameters. But they did not apply the VSC law to their adaptive controllers. In this paper, we will apply VSC and switching-σ modification control to the control of the single-arm dynamics with some unknown parameters and bounded disturbance.

In Sections 2 to 5, we will analyze and compare four control laws which are related to VSC and switching-σ modification control for the single-arm dynamics with some unknown parameters and bounded disturbance. The new proposed control law which is a combination of the continuous VSC law and the switching-σ modification control law is presented and analyzed in Section 5. Although Slotine and Li ([23]-[26]) have already analyzed the single-arm manipulator's tracking error of VSC law based on the sliding surface equation, our approach, which is related to the analysis in [27] but in multi-variable sense, based on Lyapunov function is different from theirs. Besides the error bounds for these four control laws derived from our approach are of a similar type, and thus it is easy to compare them. In Section 6, we derive the control law in Cartesian space and find the relationship between the reference variable in joint space and the reference variable in Cartesian space. In Section 7, we simulate and discuss these control laws in the presence of bounded disturbance and unknown parameters with and without unmodeled dynamics involved. The simulations show that the control laws works very well even in the presence of unmodeled dynamics which is not considered in our analysis. As was to be expected from our theoretical analysis, the simulations demonstrate that the combination of the continuous VSC law and the switching-σ modification control law works better. (Proofs are not provided here and can be found in [28].)

2 Discontinuous VSC Law

In this section we study the control of a single-arm manipulator with bounded disturbance and unknown parameters with discontinuous VSC law. This problem was also studied in [24]-[26]. Our results are the same as those of [24]-[26], but our analysis is different since we use Lyapunov function for analysis whereas [24]-[26] use an analysis based on the sliding surface equation (2.4). The dynamic equation of single-arm manipulator with n links in joint space is as follows (see [1]-[4]).

$$\tau = D(q)\ddot{q} + H(q,\dot{q})\dot{q} + G(q) - d(t) \tag{2.1}$$

where $\tau \in R^n$ is the vector of joint torques supplied by the actuators; $D(q) \in R^{n\times n}$ is the arm mass (inertial) matrix which is symmetric and positive definite; $q, \dot{q}, \ddot{q} \in R^n$ are the vectors of joint displacement, velocity and acceleration, respectively; $H(q,\dot{q}) \in R^{n\times n}$ is the matrix from centrifugal, Coriolis and frictional forces; $G(q) \in R^n$ is the vector of gravitational force; $d(t) \in R^n$ is an upper uniformly bounded disturbance. Our objective is to find a controller which uses the control law τ as a function of the state $q, \dot{q}$ and the estimated unknown parameter $\hat{P}$ in (2.3) which will make (2.1) to have $q \to q_d$ in the presence of disturbance $d(t)$ and unknown parameter P, where q_d is the desired trajectory.

In designing a VSC law for a nonlinear system, we need to specify a sliding surface. If all the terms in VSC law are continuous with respect to the sliding surface variable, we call the VSC law continuous VSC law, otherwise discontinuous VSC law. Since the sgn(S) term in the control law (2.9) changes infinitely-fast between two different values -1 and 1 according to the sign of the sliding surface variable S, we call (2.9) discontinuous VSC law, where our sliding surface is chosen to be as the set of points where $S(t) = 0$, where $S(t)$ as in (2.4) (see [2],[24]-[26]). Since we know that the joint positions $q(t)$ of the single-arm will be equal to the desired joint positions $q_d(t)$ if $S(t)$ is equal to zero and Λ which is chosen by the designer is positive definite (from (2.4) and (2.5)), our objective in this section is to find the discontinuous VSC law of the dynamic equation (2.1) such that $S(t)$ will be equal to zero.

We choose the following function as a Lyapunov function candidate of the dynamic equation (2.1).

$$V(t,S,\Phi) = (1/2)S(t)^T D(q)S(t) + (1/2)\Phi(t)^T \Gamma \Phi(t) \tag{2.2}$$

where $D(q) \in R^{n\times n}$ is the arm inertia matrix which is symmetric and positive definite, $\Gamma \in R^{m1\times m1}$ is a diagonal positive definite constant matrix chosen by the designer, and $\Phi(t) \in R^{m1}$ is:

$$\Phi(t) = \hat{P}(t) - P \tag{2.3}$$

where m1 is the number of unknown parameter, P is an unknown constant parameter vector, $\hat{P}(t)$ is the estimate of P, and $\Phi(t)$ is the estimate error of the parameter vector. We assume that the desired trajectory q_d is twice differentiable, then we define $S(t) \in R^n$ as follows:

$$S(t) = \dot{\bar{q}}(t) + \Lambda \bar{q}(t), \dot{S}(t) = \ddot{\bar{q}}(t) + \Lambda\dot{\bar{q}}(t) \tag{2.4}$$

where

$$\bar{q}(t) = q(t) - q_d(t), \dot{\bar{q}}(t) = \dot{q}(t) - \dot{q}_d(t), \ddot{\bar{q}}(t) = \ddot{q}(t) - \ddot{q}_d(t) \tag{2.5}$$

$q_d(t), \dot{q}_d(t)$ and $\ddot{q}_d(t) \in R^n$ are the desired joint position, velocity and acceleration of the single arm, and $\Lambda \in R^{n\times n}$ is a constant diagonal positive definite matrix chosen by the designer. We also define the reference variable $q_r(t) \in R^n$ as follows:

$$q_r(t) = q_d(t) - \Lambda \int_0^t \bar{q}(t)dt \tag{2.6}$$

Therefore

$$\dot{q}_r(t) = \dot{q}_d(t) - \Lambda\bar{q}(t), \ddot{q}_r(t) = \ddot{q}_d(t) - \Lambda\dot{\bar{q}}(t) \tag{2.7}$$

From (2.4), (2.5) and (2.7), we can get

$$S(t) = \dot{q}(t) - \dot{q}_r(t) = \dot{\tilde{q}}_r(t) \tag{2.8}$$

Lemma 1 *$V(t,S,\Phi)$ is positive definite and decresent.*

Lemma 2 *Consider the following adaptive control law for equation (2.1):*

$$\tau = \hat{D}(q,\hat{P})\ddot{q}_r + \hat{H}(q,\dot{q},\hat{P})\dot{q}_r + \hat{G}(q,\hat{P}) - K_d S(t) - d_0 sgn(S) \tag{2.9}$$

$$\dot{\Phi}(t) = \dot{\hat{P}}(t) = -\Gamma^{-1}W^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)S \tag{2.10}$$

where

$$d(t) = (d_1(t),\cdots,d_n(t))^T \in R^n \tag{2.11}$$

is the disturbance, $d_0 = diag(d_{10},\cdots,d_{n0})$ is a square matrix, where

$$|d_i(t)| \le d_{i0}, i = 1,2,\cdots,n \tag{2.12}$$

$$sgn(S) = (sgn(S_1), sgn(S_2),\cdots, sgn(S_n))^T \in R^n$$

where

$$sgn(S_i) = \begin{cases} 1 & if\ S_i > 0 \\ -1 & if\ S_i < 0 \end{cases} \tag{2.13}$$

$$W(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Phi = \tilde{H}(q,\dot{q},\hat{p})\dot{q}_r + \tilde{D}(q,\hat{P})\ddot{q}_r + \tilde{G}(q,\hat{P}) \tag{2.14}$$

where

$$\tilde{H} = \hat{H} - H, \tilde{D} = \hat{D} - D, \tilde{G} = \hat{G} - G \tag{2.15}$$

where $\hat{H},\hat{D},\hat{G}$ are the estimate of H,D,G, and $\tilde{H},\tilde{D},\tilde{G}$ are the errors. K_d and Γ are arbitrary constant diagonal positive definite matrices chosen by the designer. Then

$$q(t) \to q_d(t), as\ t \to \infty. \tag{2.16}$$

Although the discontinuous VSC law can guarantee that the tracking error goes to zero, it has two drawbacks. First, it has the infinitely-fast switching term $d_0 sgn(S)$ which may excite the high-frequency unmodeled dynamics. Second, we can not implement the infinitely-fast switching term $d_0 sgn(S)$ of this control law owing to physical limitation. In the next section, We present the following switching-σ modification control law which does not have infinitely-fast switching term $d_0 sgn(S)$.

3 Switching-σ Modification Control Law

In this section, we consider the switching-σ modification control law for a single-arm manipulator with bounded disturbance and unknown parameters. The switching-σ modification control law can guarantee that there is no estimated parameter drift in the presence of the bounded disturbance by adding a sigma term to the estimation law. This sigma term can prevent the estimated parameter drifting by changing the sign of the derivative of the estimated parameter. The results presented are the same with those of [19], the detail proof is in [19].

Lemma 3 *Consider the following adaptive control law for equation (2.1) :*

$$\tau = \hat{D}(q,\hat{P})\ddot{q}_r + \hat{H}(q,\dot{q},\hat{P})\dot{q}_r + \hat{G}(q,\hat{P}) - K_d S(t) \tag{3.1}$$

$$\dot{\Phi}(t) = \dot{\hat{P}}(t) = -\Gamma^{-1}W^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)S - \sigma\Gamma^{-1}\hat{P} \tag{3.2}$$

where

$$\sigma = \begin{cases} 0 & \|\hat{P}\| \le P_0 \\ \sigma_0(\frac{\|\hat{P}\|}{P_0} - 1) & P_0 < \|\hat{P}\| \le 2P_0 \\ \sigma_0 & 2P_0 < \|\hat{P}\| \end{cases} \tag{3.3}$$

*$\sigma_0 > 0$ is a scalar, and $P_0 > \|P\|$, where $\|*\|$ is l_2 norms for the vectors *, and the other variables are defined in Section 2. Then we can guarantee q is close to q_d in a bound related to (3.4), as $t \to \infty$*

$$\lim_{T\to\infty}(1/T)\int_{t_0}^{t_0+T} S^T K_d S dt \le \|d_{00}^T K_e^{-1}\|^2 \tag{3.4}$$

where

$$K_e K_e^T = K_d, d_{00} = (d_{10}, d_{20},\cdots,d_{n0})^T$$

(3.4) means that the mean value of $S^T K_d S$ will be bounded from above by $\|d_{00}^T K_e^{-1}\|^2$ as T goes to infinity. The variables K_d and $d_{i0}, i = 1,2,\cdots,n$ are defined in Section 2.

The switching-σ modification control law can relax the two drawbacks of discontinuous VSC law, but it can only guarantee that the tracking precision is inside some error bound. Since there is a term $K_d S(t)$ in (3.1), the switching-σ modification control law has the same robustness as the PD control law with respect to the structured uncertainties. Ideally, the switching-σ modification control law (see [19],[27]) can guarantee the tracking error to be zero if K_d goes to infinity. But in a real world we can not have a controller with an infinite gain, and very large K_d will make the system control bandwidth so large that it may excite the high frequency unmodeled dynamics.

4 Continuous VSC Law

Here we study the continuous VSC law of single arm problem with bounded disturbance and unknown parameters. This problem was also studied in [24]-[26], but our analysis is different and we provide a different error bound than that of [24]-[26]. Our error bound is of a similar type as that of the switching-σ modification control of the previous section and they are easily comparable. In our continuous VSC law, the $sat(S/\phi)$ term in the control law (4.2) changes between two different values -1 and 1 according to the sign of the variable S as S is outside the boundary layer, i.e. $|S| > \phi$;while this term is a straight line from $(-\phi, -1)$ to $(\phi, 1)$ as S is inside the boundary layer, i.e.$|S| < \phi$ (Figure 1), where ϕ (called boundary layer) is a real vector chosen by the designer.

Lemma 4 *Consider the following adaptive control law for equation (2.1):*

$$\tau = \hat{D}(q,\hat{P})\ddot{q}_r + \hat{H}(q,\dot{q},\hat{P})\dot{q}_r + \hat{G}(q,\hat{P}) - K_d S(t) - d_0 sat(S/\phi) \quad (4.1)$$

$$\dot{\Phi}(t) = \dot{\hat{P}}(t) = -\Gamma^{-1}W^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)S \quad (4.2)$$

where

$$sat(S/\phi) = (sat(S_1/\phi_1), sat(S_2/\phi_2), \cdots, sat(S_n/\phi_n))^T \quad (4.3)$$

$$sat(S_i/\phi_i) = \begin{cases} S_i/\phi_i, & |S_i/\phi_i| \le 1 \\ sgn(S_i/\phi_i), & otherwise \end{cases} \quad (4.4)$$

$$\phi = (\phi_1, \phi_2, \cdots, \phi_1)^T \quad (4.5)$$

where ϕ_i is called the boundary layer for the corresponding variable S_i, and the other variables are defined in Section 2. Then we can guarantee q is close to q_d in a bound as in (4.15) of [28].

The error bounds calculated by Slotine are as follows (see [24]-[26]):

$$|\tilde{q}| \le \Lambda^{-1}\phi, |\dot{\tilde{q}}| \le 2\phi \quad (4.6)$$

where

$$\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_n) \in R^{n \times n}$$
$$\phi^T = (\phi_1, \phi_2, \cdots, \phi_n) \in R^n$$

Using (2.4) and (4.6), we can find:

$$\lim_{T \to \infty} (1/T) \int_{t_0}^{t_0+T} S^T(K_d + d_0/\phi)S dt \le 9\phi^T(k_d + d_0/\phi)\phi \quad (4.7)$$

where

$$9\phi^T(k_d + d_0/\phi)\phi = \sum_{i=1}^{n} \frac{9\phi_i^3 k_{di}^2 + 18\phi_i^2 k_{di} d_{i0} + 9\phi_i d_{i0}^2}{k_{di}\phi_i + d_{i0}} \quad (4.8)$$

Also for the error bound that we calculated in (4.15), it holds that:

$$\|d_{00}^T K_h^{-1}\|^2 = \sum_{i=1}^{n} \frac{\phi_i d_{i0}^2}{k_{di}\phi_i + d_{i0}} \quad (4.9)$$

Comparing (4.15) of [28], (4.7)-(4.9), we can see that the error bound for $\lim_{T \to \infty}(1/T)\int_{t_0}^{t_0+T} S^T(K_d + d_0/\phi)S dt$ that we calculated in (4.15) of [28] is much smaller than the error bound in (4.7) and (4.8) obtained from Slotine's error bound (4.6). Although the continuous VSC law can relax the two drawbacks of the discontinuous VSC law, it can only guarantee that the tracking precision is inside some error bound. From (4.6) and (3.20), we know the error bound for the continuous VSC law is smaller than that of the switching-σ modification control law if the boundary layer is not very large. If we choose a large boundary layer, the control law may not excite the high frequency unmodeled dynamics and can be implemented in the real world, but we will have a large error for the tracking precision. If we choose a small boundary layer, we will have a small error for the tracking precision, but the control law may excite the high frequency unmodeled dynamics and may not be realizable. Therefore, there is a trade-off between tracking precision and robustness with respect to the unmodeled dynamics. Another drawback of this control law is that the estimated parameter may drift to infinity. In the next section, we combine the switching -σ modification control and continuous VSC laws to get the better performance and prevent the estimated parameter from drifting to infinity.

5 Combination of the Continuous VSC Law and the Switching -σ Modification Control Law

The combination of the continuous VSC law and the switching-σ modification control law for the control of a single-arm manipulator with bounded disturbance and unknown parameters is presented here for the first time. Our analysis is a combination of the techniques used earlier and the error bound is better than those of Sections 3 and 4. The block diagram of combination of the switching-σ modification control law and the continuous VSC law in joint space for the single-arm manipulator is shown in Figure 2.

Lemma 5 *Consider the following adaptive control law for equation (2.1):*

$$\tau = \hat{D}(q,\hat{P})\ddot{q}_r + \hat{H}(q,\dot{q},\hat{P})\dot{q}_r + \hat{G}(q,\hat{P}) - K_d S(t) - d_0 sat(S/\phi) \quad (5.1)$$

$$\dot{\Phi}(t) = \dot{\hat{P}}(t) = -\Gamma^{-1}W^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)S - \sigma\Gamma^{-1}\hat{P} \quad (5.2)$$

where the variables are defined in Sections 2-4. Then we can guarantee q is close to q_d in a bound as in (5.6).

6 Single-Arm Simulation

In this section we use a single-arm manipulator with two rigid links as our simulation example (Figure 3). The link lengths are both l, the first and second link's mass are m_1 and m_2, the first and second joint angles are q_1 and q_2, the first and second joint torques are τ_1 and τ_2, and d_1, d_2 are disturbances. The Lagrange-Euler equation of motion for this single-arm manipulator with two links is as follows (see [1]-[4]):

$$\begin{aligned}
\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} &= \\
& \begin{pmatrix} \frac{1}{3}m_1l^2 + \frac{4}{3}m_2l^2 + m_2cos(q_2)l^2 & \frac{1}{3}m_2l^2 + \frac{1}{2}m_2l^2cos(q_2) \\ \frac{1}{3}m_2l^2 + \frac{1}{2}m_2l^2cos(q_2) & \frac{1}{3}m_2l^2 \end{pmatrix} \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} \\
+ & \begin{pmatrix} -m_2sin(q_2)l^2\dot{q}_2 & \frac{-1}{2}m_2sin(q_2)l^2\dot{q}_2 \\ \frac{1}{2}m_2sin(q_2)l^2\dot{q}_1 & 0 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} \\
+ & \begin{pmatrix} \frac{1}{2}m_1glcos(q_1) + \frac{1}{2}m_2glcos(q_1+q_2) + m_2glcos(q_1) \\ \frac{1}{2}m_2glcos(q_1+q_2) \end{pmatrix} \\
- & \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \qquad (6.1)
\end{aligned}$$

In the simulation we let both m_1 and m_2 be 1 kilogram, l be 1 meter for simplicity, and the disturbances d_1, d_2 be as follows:

$$d_1 = d_2 = 2/(1+t) \quad (6.2)$$

where t represents time in seconds.
We also let the desired trajectory for joint 1 and 2 be as follows:

$$\begin{aligned} q_{1d} &= sin(t) + 0.1sin(3t) \\ q_{2d} &= 0.1sin(2t) + 0.1sin(4t) \end{aligned} \quad (6.3)$$

We simulate the four robust position control laws mentioned in the previous sections. We use Adams variable step-size predictor-corrector algorithm to solve these equations. We simulate the control laws without unmodeled dynamics and with actuator unmodeled dynamics involved. We model the actuator unmodeled dynamics as a first-order low pass filter, where its cut off frequency is 100 radians per second and DC gain is 1. More details about the simulations for each control law are given next.

6.1 Discontinuous VSC Law

From (2.9)-(2.15) and (6.1) we can express the discontinuous VSC law τ_1, τ_2 as follows:

$$\begin{aligned} \tau_1 = \; & \hat{m}_1(\frac{1}{3}\ddot{q}_{1r} + \frac{1}{2}gcos(q_1)) + \hat{m}_2((\frac{4}{3} + cos(q_2))\ddot{q}_{1r} + \\ & (\frac{1}{3} + \frac{1}{2}cos(q_2))\ddot{q}_{2r} - sin(q_2)\dot{q}_2\dot{q}_{1r} + \frac{-1}{2}sin(q_2)\dot{q}_2\dot{q}_{2r} + \end{aligned}$$

$$
\begin{aligned}
& \frac{1}{2}gcos(q_1+q_2)+gcos(q_1)) - k_{d1}S_1 - d_{10}sgn(S_1) \\
\tau_2 &= \hat{m}_2(\frac{1}{3}\ddot{q}_{1r}+\frac{1}{2}cos(q_2)\ddot{q}_{1r}+\frac{1}{3}\ddot{q}_{2r}+\frac{1}{2}sin(q_2)\dot{q}_1\dot{q}_{1r}+ \\
& \frac{1}{2}gcos(q_1+q_2) - k_{d2}S_2 - d_{20}sgn(S_2) \qquad (6.4)
\end{aligned}
$$

where the estimation law can be expressed as follows:

$$
\begin{aligned}
\dot{\hat{m}}_1 &= -\gamma_1^{-1}(\frac{1}{3}\ddot{q}_{1r}+\frac{1}{2}gcos(q_1))S_1 \\
\dot{\hat{m}}_2 &= -\gamma_2^{-1}((\frac{4}{3}+cos(q_2))\ddot{q}_{1r}+(\frac{1}{3}+\frac{1}{2}cos(q_2))\ddot{q}_{2r} - sin(q_2)\dot{q}_2\dot{q}_{1r}+ \\
& \frac{-1}{2}sin(q_2)\dot{q}_2\dot{q}_{2r}+\frac{1}{2}gcos(q_1+q_2)+gcos(q_1)S_1+ \\
& (\frac{1}{3}\ddot{q}_{1r}+\frac{1}{2}cos(q_2)\ddot{q}_{1r}+\frac{1}{3}\ddot{q}_{2r}+\frac{1}{2}sin(q_2)\dot{q}_1\dot{q}_{1r}+ \\
& \frac{1}{2}gcos(q_1+q_2))S_2) \qquad (6.5)
\end{aligned}
$$

where

$$
\begin{aligned}
\dot{q}_{1r} &= \dot{q}_{1d} - \lambda_1(q_1 - q_{1d}) \\
\dot{q}_{2r} &= \dot{q}_{2d} - \lambda_2(q_2 - q_{2d}) \\
\ddot{q}_{1r} &= \ddot{q}_{1d} - \lambda_1(\dot{q}_1 - \dot{q}_{1d}) \\
\ddot{q}_{2r} &= \ddot{q}_{2d} - \lambda_2(\dot{q}_2 - \dot{q}_{2d}) \\
S_1 &= (\dot{q}_1 - \dot{q}_{1d}) + \lambda_1(q_1 - q_{1d}) \\
S_2 &= (\dot{q}_2 - \dot{q}_{2d}) + \lambda_2(q_2 - q_{2d}) \qquad (6.6)
\end{aligned}
$$

We let initial values of the estimated parameter $\hat{m}_1 = 0.8, \hat{m}_2 = 1.2$, controller gains $k_{d1} = k_{d2} = 2$, and chosen scalars $\lambda_1 = 20, \lambda_2 = 15$, where $\hat{m}_1$ and $\hat{m}_2$ are the estimates of m_1 and m_2, respectively. We let d_{10} and d_{20} be upper bound of disturbance d_1 and d_2, thus $d_{10} = d_{20} = 2$. We did not simulate the discontinuous VSC law since the chattering of the control laws makes the system of differential equations very stiff. But in Section 6.3 we simulate the continuous VSC law with a very small boundary layer which is very similar to the discontinuous VSC law.

6.2 The Switching-σ Modification Control Law

From (3.1)-(3.3) and (6.1) we can express the switching-σ modification control law τ_1, τ_2 as follows:

$$
\begin{aligned}
\tau_1 &= \hat{m}_1(\frac{1}{3}\ddot{q}_{1r}+\frac{1}{2}gcos(q_1)) + \hat{m}_2((\frac{4}{3}+cos(q_2))\ddot{q}_{1r}+ \\
& (\frac{1}{3}+\frac{1}{2}cos(q_2))\ddot{q}_{2r} - sin(q_2)\dot{q}_2\dot{q}_{1r}+\frac{-1}{2}sin(q_2)\dot{q}_2\dot{q}_{2r}+ \\
& \frac{1}{2}gcos(q_1+q_2)+gcos(q_1)) - k_{d1}S_1 \qquad (6.7) \\
\tau_2 &= \hat{m}_2(\frac{1}{3}\ddot{q}_{1r}+\frac{1}{2}cos(q_2)\ddot{q}_{1r}+\frac{1}{3}\ddot{q}_{2r}+\frac{1}{2}sin(q_2)\dot{q}_1\dot{q}_{1r}+ \\
& \frac{1}{2}gcos(q_1+q_2) - k_{d2}S_2 \qquad (6.8)
\end{aligned}
$$

where the estimation law can be expressed as follows:

$$
\begin{aligned}
\dot{\hat{m}}_1 &= -\gamma_1^{-1}(\frac{1}{3}\ddot{q}_{1r}+\frac{1}{2}gcos(q_1))S_1 - \sigma_1\gamma_1^{-1}\hat{m}_1 \\
\dot{\hat{m}}_2 &= -\gamma_2^{-1}((\frac{4}{3}+cos(q_2))\ddot{q}_{1r}+(\frac{1}{3}+\frac{1}{2}cos(q_2))\ddot{q}_{2r} - sin(q_2)\dot{q}_2\dot{q}_{1r}+ \\
& \frac{-1}{2}sin(q_2)\dot{q}_2\dot{q}_{2r}+\frac{1}{2}gcos(q_1+q_2)+gcos(q_1)S_1+ \\
& (\frac{1}{3}\ddot{q}_{1r}+\frac{1}{2}cos(q_2)\ddot{q}_{1r}+\frac{1}{3}\ddot{q}_{2r}+\frac{1}{2}sin(q_2)\dot{q}_1\dot{q}_{1r}+ \\
& \frac{1}{2}gcos(q_1+q_2))S_2) - \sigma_2\gamma_2^{-1}\hat{m}_2 \qquad (6.9)
\end{aligned}
$$

where

$$
\sigma_i = \begin{cases} 0 & \|\hat{m}_i\| \le m_0 \\ \sigma_0(\frac{\|\hat{m}_i\|}{m_0} - 1) & m_0 < \|\hat{m}_i\| \le 2m_0 \\ \sigma_0 & 2m_0 < \|\hat{m}_i\| \end{cases} \quad i = 1,2 \qquad (6.10)
$$

We choose $\sigma_0 = 5, m_0 = 1.1, \gamma_1 = \gamma_2 = 1/2.3$. In Figures 5 and 6, we can see that the estimated parameter converges to the value which is very close to 1, but the joint position error is larger than any other figures plotted by another control laws. Since Figure 5 is a plot with actuator unmodeled dynamics, the joint position error of Figure 5 is larger than those of Figure 4.

6.3 Continuous VSC Law

From (4.1) to (4.5) and (6.1) we can express the continuous VSC law τ_1, τ_2 as follows:

$$
\begin{aligned}
\tau_1 &= \hat{m}_1(\frac{1}{3}\ddot{q}_{1r}+\frac{1}{2}gcos(q_1)) + \hat{m}_2((\frac{4}{3}+cos(q_2))\ddot{q}_{1r}+ \\
& (\frac{1}{3}+\frac{1}{2}cos(q_2))\ddot{q}_{2r} - sin(q_2)\dot{q}_2\dot{q}_{1r}+\frac{-1}{2}sin(q_2)\dot{q}_2\dot{q}_{2r}+ \\
& \frac{1}{2}gcos(q_1+q_2)+gcos(q_1)) - k_{d1}S_1 - d_{10}sat(S_1/\phi_1) \\
\tau_2 &= \hat{m}_2(\frac{1}{3}\ddot{q}_{1r}+\frac{1}{2}cos(q_2)\ddot{q}_{1r}+\frac{1}{3}\ddot{q}_{2r}+\frac{1}{2}sin(q_2)\dot{q}_1\dot{q}_{1r}+ \\
& \frac{1}{2}gcos(q_1+q_2) - k_{d2}S_2 - d_{20}sat(S_2/\phi_2) \qquad (6.11)
\end{aligned}
$$

where

$$
sat(S_i/\phi_i) = \begin{cases} S_i/\phi_i, & S_i/\phi_i \le 1 \\ sgn(S_i/\phi_i), & otherwise \end{cases} \quad i = 1,2 \qquad (6.12)
$$

The estimation law is the same as (6.5), and the variables of (6.11) and (6.12) are defined in (6.6). In the simulation the different boundary layers 0.1 and 10 are used. Their plots are in the Figures 6 and 7 for the case without unmodeled dynamics, and in Figures 8 and 9 for the case with actuator unmodeled dynamics, respectively. From these figures, we can see that the bigger the boundary layer is, the larger the joint position error is. From Section 2, we know that a very small boundary layer is limited by physical limitation, and thus we have to choose proper boundary layer for realization. From the theoretical analysis, we know that the small boundary layer may excite the high frequency unmodeled dynamics which will cause instabilities, but the simulations show that the system with small boundary layer (Figure 8) still has good tracking precision even if actuator unmodeled dynamics are considered. However, comparing Figures 6,7 with Figures 8,9 we see that the joint position error without unmodeled dynamics is smaller than those with unmodeled dynamics.

6.4 Combination of the Continuous VSC and the Switching-σ Modification Control Laws

The control laws τ_1, τ_2 are the same as in (6.11) and (6.12), while the estimation law is the same as in (6.9) and (6.10). In the simulation the different boundary layers 0.1 and 10 are used. Their plots are in Figures 10,11 for the case without unmodeled dynamics, and in Figures 12,13 for the case with actuator unmodeled dynamics. Comparing Figures 10,11 with Figures 6,7 we can see that the joint position errors of Figures 10,11 are smaller than those corresponding Figures 6,7. We also can see that the joint position errors of Figures 10,11 are smaller than those of Figure 4. Thus the tracking precision performance of combining the continuous VSC and the switching-σ modification control laws are better than the switching-σ modification control law and continuous VSC law. Comparing Figures 12,13 with Figures 10,11, we see that the joint position error of combining the continuous VSC law and the switching-σ modification control laws for the case without unmodeled dynamics are smaller than those of the case with unmodeled dynamics as was to be expected.

7 Conclusions

In this paper, we have analyzed and simulated four robust position control laws for the single-arm dynamics with bounded disturbance, unknown parameter, and actuator unmodeled dynamics. We can see that the combination of the continuous VSC law and the switching-σ modification control law has better tracking precision performance than the VSC control and switching-σ modification control laws for both cases without unmodeled dynamics and with actuator unmodeled dynamics. From the theoretical analysis, we know that the small boundary layer may excite the high frequency unmodeled dynamics which will cause instabilities, but the simulations show that the system with a small boundary layer (Figure 8) still has good tracking precision even if actuator unmodeled dynamics are considered. The reason is that the high-frequency amplitude of the control torque is small. However the joint position error without unmodeled dynamics is smaller than those of with actuator unmodeled dynamics. Although the simulation shows that the small boundary layer can achieve better tracking even if actuator unmodeled dynamics are considered, it is limited by physical limitation. The combination of the continuous VSC law and the switching-σ modification control law to the control of dual-arm dynamics is under investigation.

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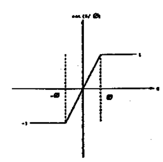

Figure 1: Continuous VSC Law

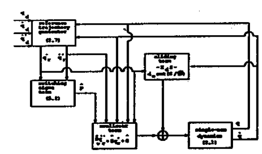

Figure 2: Block Diagram of Combination of the Continuous VSC and the Switching-σ Modification Control Laws

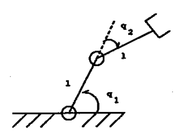

Figure 3: Single-Arm Manipulator with Two Links

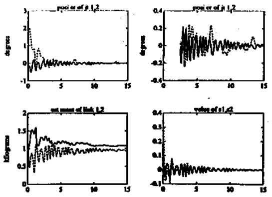

Figure 4: Switching-σ Modification Control Without Unmodeled Dynamics
solid line:joint 1,dot line:joint 2

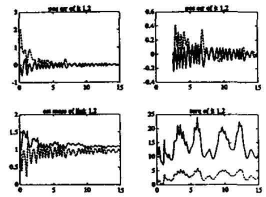

Figure 5: Switching-σ Modification Control With Actuator Unmodeled Dynamics
solid line:joint 1,dot line:joint 2

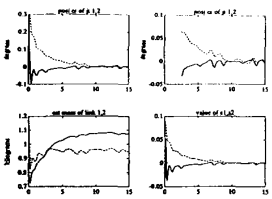

Figure 6: Continuous Variable Structure Control Without Unmodeled Dynamics
boundary layer 0.1,solid line:joint 1,dot line:joint 2

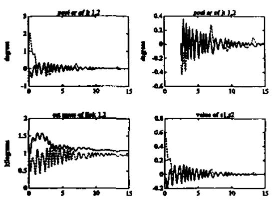

Figure 7: Continuous Variable Structure Control Without Unmodeled Dynamics
boundary layer 10,solid line:joint 1,dot line:joint 2

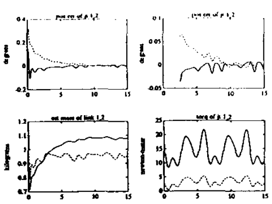

Figure 8: Continuous Variable Structure Control With Actuator Unmodeled Dynamics
boundary layer 0.1,solid line:joint 1,dot line:joint 2

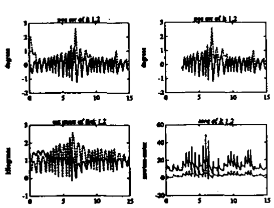

Figure 9: Continuous Variable Structure Control With Actuator Unmodeled Dynamics
boundary layer 10,solid line:joint 1,dot line:joint 2

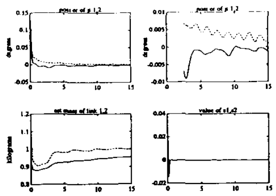

Figure 10: Combination of the Continuous Variable Structure and the Switching-σ Modification Control Laws Without Unmodeled Dynamics
boundary layer 0.1,solid line:joint 1,dot line:joint 2

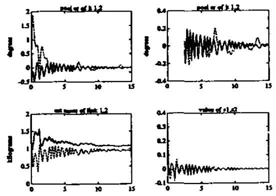

Figure 11: Combination of the Continuous Variable Structure and the Switching-σ Modification Control Laws Without Unmodeled Dynamics
boundary layer 10,solid line:joint 1,dot line:joint 2

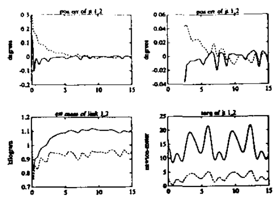

Figure 12: Combination of the Continuous Variable Structure and the Switching-σ Modification Control Laws With Actuator Unmodeled Dynamics
boundary layer 0.1,solid line:joint 1,dot line:joint 2

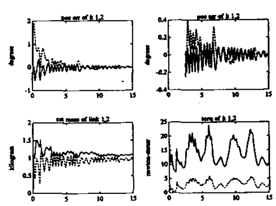

Figure 13: Combination of the Continuous Variable Structure and the Switching-σ Modification Control Laws With Actuator Unmodeled Dynamics
boundary layer 10,solid line:joint 1,dot line:joint 2