Optimal Hold and Moving Force/Torque for Dual-Arm and Multi-Arm Manipulators Holding Rigid Object

Li-Wen Chen and G. P. Papavassilopoulos*

Department of Electrical Engineering
Chinese Naval Academy
Tsoying, Kaohsiung, Taiwan, R.O.C.
Department of Electrical Engineering-Systems*
University of Southern California
Los Angeles, U.S.A.

Abstract

In this paper we address two problems which are important for dual-arm and multi-arm manipulators: first, the optimal hold force/torque which can hold the common object without slipping and second the optimal moving force/torque distribution supplied by each arm. We assume that we know the resultant force/torque exerted on the common object by the manipulators, then we develop new algorithms which can determine the optimal hold force/torque mainly based on the static frictional constraints and optimal moving force/torque distribution by minimizing the cost function which is the weighted magnitude of force/torque exerted on the rigid object by the manipulators.

1. Introduction

A particular class of problems in the robotic area is to find the optimal hold force/torque and optimal moving force/torque exerted on a rigid object by the manipulators. Zheng and Luh([6]-[8]) studied the optimal load distribution of two industrial robots holding a single object. They presented several schemes to find the optimal load distribution,but their computations are complicated. They also did not study the optimal hold force/torque exerted on the rigid object by the manipulators and did not study the optimal force/torque for the multi-arm manipulators. Pittelkau[9] studied the adaptive load sharing force control for dual-arm manipulators. In his paper, he did not mention how to find the optimal hold force, which is important in dual-arm dynamic control, and did not study the optimal force/torque for multi-arm manipulators. Nakamura, Nagai, and Yoshikawa[10] studied the mechanics of coordinative manipulation by a multiple robotic mechanism. In their paper, they minimized the hold force exerted on the common object subject to static frictional constraints, which involves the minimization of a quadratic function subject to linear and quadratic constraints. However, they did not consider the optimal moving force. Cole, Hauser, and Sastry[13] studied kinematics and control of multifingered hands with rolling contacts, but did not deal with the calculation of optimal hold force and moving force. Kerr and Roth[12] approximated the frictional constraints using linear constraints and determined optimal internal force based on these constraints. Their scheme is computationally complicated, and they did not deal with optimal moving force. Cheng and Orin[16] have derived different formulas for finding the force distribution for multi-arm manipulators. Nahon[17] has studied the real-time force optimization in parallel kinematic chains under inequality constraints.

In this paper we present new algorithms which can be used to find the optimal hold force/torque and optimal moving force/torque for multi-arm manipulators holding one common rigid object. The optimal hold force/torque and optimal moving force/torque are very important for multi-arm dynamic control. The structure of this paper is as follows. In Section 2, the dynamic models for multi-arm manipulators holding one common rigid object are given. In Section 3, the optimal hold force/torque for multi-arm manipulators is calculated. In Section 4, we present a scheme which can find the optimal hold force/torque and optimal moving force/torque exerted on the common object by the multi-arm manipulators even if the moving force is not measurable. In Section 5.1, we present simulations to find the optimal hold force/torque exerted on the common object and use the same examples given in [10]. In Section 5.2, we present simulations to find the optimal hold force/torque and optimal moving force/torque exerted on the common object by the dual-arm manipulators.

2. Formulation of the Dynamic Model of Multi-Arm Manipulators Holding Rigid Object

Figure 1 shows a multi-arm manipulators which has m arms holding a common rigid object together. In Figure 1, ([1]-[4]) x_b, y_b, z_b denote the base frame, x_o, y_o, z_o denote the object frame fixed at the center of mass of the common rigid object, the (3×1) vectors $f_1, f_2, \cdots, f_m$ denote the end-effector forces exerted on the common object by each arm with reference to the base frame, the (3×1) vectors $n_1, n_2, \cdots, n_m$ denote the end-effector torques exerted on the common object by each arm with reference to the base frame, the (3×1) vector p_o denotes the position vector of the mass center of the common rigid object with reference to the base frame, the (3×1) vectors $p_1, p_2, \cdots, p_m$ denote the position vectors of the contact points with reference to the base frame, and the (3×1) vectors $r_1, r_2, \cdots, r_m$ denote the position vectors of each contact point with reference to the object frame, respectively. We also denote by the (3×1) vectors f_t, n_t resultant force and torque exerted on the common object, respectively. We denote by $g = [0\ 0\ -9.8]^T \in R^3$ the gravitational accelerational vector, by M the mass of the common object , and by R_o the rotational matrix of object frame x_o, y_o, z_o with reference to the base frame x_b, y_b, z_b. Assuming the contacts between the arms and the common rigid object are the point contacts,i.e.,$n_i, i = 1, 2, \cdots, m$ are zero, then the dynamic equation of the common object is as follows:([1]-[4],[18])

$$M_c \ddot{x}_c + F_{t0} = Wf \tag{1}$$

where

$$M_c = \begin{bmatrix} MI_3 & 0 \\ 0 & I \end{bmatrix}, \ddot{x}_c = \begin{bmatrix} \ddot{p}_o - g \\ \dot{\omega} \end{bmatrix}$$

$$F_{t0} = \begin{bmatrix} 0 \\ \omega \otimes (I\omega) \end{bmatrix}, f = \begin{bmatrix} f_1 \\ \vdots \\ f_m \end{bmatrix}$$

$$W = \begin{bmatrix} I_3 & I_3 & \cdots & I_3 \\ (R_o r_1)\otimes & (R_o r_2)\otimes & \cdots & (R_o r_m)\otimes \end{bmatrix},$$

where $\ddot{p}_o \in R^3$ is the accelerational vector of the mass center of the object with reference to the base frame, $\omega, \dot{\omega} \in R^3$ are the angular position and velocity vectors of the object with reference to the base frame, $I \in R^{3\times3}$ is the inertial matrix of the object with respect to the base frame. $\otimes$ denotes the cross product of two column vector. I_3 is a (3×3) identity matrix, M_c is a (6×6) matrix, $\ddot{x}_c$ is a (6×1) vector, F_{t0} is a (6×1) vector, W is a $(6 \times 3m)$ matrix, and f is a $(3m \times 1)$ vector.
We also let

$$Wf = F_t = \sum_{i=1}^{m} F_i, F_i = \begin{bmatrix} f_i \\ (R_o r_i) \otimes f_i \end{bmatrix} \tag{2}$$

where $F_1, F_2, \cdots, F_m \in R^6$ are force/torque exerted on the object by each arm, and $F_t \in R^6$ is resultant force/torque exerted on the object by the multi-arm manipulators.

3. The Calculation of the Optimal Hold Force/Torque Exerted on the Common Rigid Object by the Multi-Arm Manipulators

In this section we deal with the calculation of the optimal hold force/torque exerted on the common rigid object by the multi-arm manipulators. We present a scheme which can find the optimal hold force/torque, which is in the direction of the chosen basis of W in (1), exerted on the common rigid object by the manipulators as the moving force/torque is measurable. The moving force/torque is a force/torque contributing to the motion of the object, but the hold force/torque is a force/torque only holding the object, not contributing to the motion of the object. The optimal hold force/torque is the minimal one which can hold the common object without slipping, and the optimal moving force/torque is the minimal one in magnitude.

Figure 1 shows a m-arm manipulators holding a rigid object together. In this paper, the hold force exerted on the rigid object by the i^{th} arm denoted by $f_i^h \in R^3$, the hold force exerted on the rigid object by the m-arm manipulators denoted by $f^h \in R^{3m}$, the hold force/torque exerted on the rigid object by the i^{th} arm denoted by $F_i^h \in R^6$, the moving force exerted on the rigid object by the i^{th} arm denoted by $f_i^m \in R^3$, the moving force exerted on the rigid object by the m-arm manipulators denoted by $f^m \in R^{3m}$, the moving force/torque exerted on the rigid object by the i^{th} arm denoted by $F_i^m \in R^6$, The hold force denoted by $f^h \in R^{3m}$ is in the null space of W in (1). Therefore, $f_i^h, f^h, f_i^m, f^m, F_i^h, F_i^m, i = 1, 2, \cdots, m$ satisfy the following equation:

$$Wf^h = \sum_{i=1}^{m} F_i^h = 0, Wf^m = m_c \ddot{x}_c + F_{t0} \tag{3}$$

where

$$f^m = \begin{bmatrix} f_1^m \\ f_2^m \\ \vdots \\ f_m^m \end{bmatrix}, f^h = \begin{bmatrix} f_1^h \\ f_2^h \\ \vdots \\ f_m^h \end{bmatrix}$$

$$F_i^h = \begin{bmatrix} f_i^h \\ (R_o r_i) \otimes f_i^h \end{bmatrix}, F_i^m = \begin{bmatrix} f_i^m \\ (R_o r_i) \otimes f_i^m \end{bmatrix}$$

where $W, M_c\ddot{x}_c, F_{t0}, (R_o r_i)\otimes$ defined in (1) and the other terms defined above. In order to avoid slipping, the hold force f_i^h has to satisfy the following static frictional constraints(Figure 2):

$$N_i^T(f_i^h + f_i^m) \geq \eta_i \|f_i^h + f_i^m\|, i = 1, 2, \cdots, m \tag{4}$$

where $\eta_i = 1/\sqrt{1+\mu_i^2}$, N_i is the inner unit normal vector of the surface of the common rigid object at the contact point, and μ_i is the maximum static frictional coefficient between the manipulators and common rigid object. Let the dimension of the null space of W be p, and the orthonormal basis of the null space of W be $u_1, u_2, \cdots, u_p$. Then the hold force $f^h \in R^{3m}$ can be expressed as follows:

$$f^h = \sum_{j=1}^{p} \gamma_j u_j = U\gamma, \tag{5}$$

$$U = (u_1, u_2, \cdots, u_p), \gamma = (\gamma_1, \gamma_2, \cdots, \gamma_p)^T$$

$$u_j = \begin{bmatrix} u_{j1} \\ u_{j2} \\ \vdots \\ u_{jm} \end{bmatrix}, u_{ji} = \begin{bmatrix} u_{ji1} \\ u_{ji2} \\ u_{ji3} \end{bmatrix}$$

where $j = 1, 2, \cdots, p$ and $i = 1, 2, \cdots, m$

where γ_j is a real number, $u_j \in R^{3m}$ is a unit vector, $U \in R^{3m\times p}$, $\gamma \in R^p$ and $u_{ji} \in R^3$. From (3) and (5), $f_i^h \in R^3$ can be expressed as follows:

$$f_i^h = \sum_{j=1}^{p} \gamma_j u_{ji} \tag{6}$$

In order to find the optimal hold force $f_i^{oh} \in R^3$, we have to solve the following nonlinear programming problem:

$$\text{minimize } f^{h^T} f^h = \sum_{j=1}^{p} (\gamma_j)^2 \quad (7)$$

subject to
$N_i^T(f_i^m + \sum_{j=1}^{p} \gamma_j u_{ji}) \geq \eta_i \| f_i^m + \sum_{j=1}^{p} \gamma_j u_{ji} \|$
where $j = 1, 2, \cdots, p$ and $i = 1, 2, \cdots, m$.
In order to avoid solving the nonlinear programming problem, we assume the optimal hold force is in the direction of the chosen basis of W in (1). Since all hold forces f_i^h should point to the common object, we can find one of basis of W in (1) which points to the common object as the direction of hold force f^h. Assume u_k is qualified as a hold force direction, then the hold forces f^h, f_i^h can be expressed as follows:

$$f_i^h = \beta_i u_{ki}, f^h = \beta u_k \quad (8)$$

where

$$u_{ki} = (u_{ki1}, u_{ki2}, u_{ki3})^T, u_k = (u_{k1}^T, u_{k2}^T, \cdots, u_{km}^T)^T$$

Assume N_i, η_i and f_i^m are available, we can obtain the interval of β_i from the following equations

$$\begin{aligned} N_i^T(f_i^m + \beta_i u_{ki}) &\geq \eta_i \| f_i^m + \beta_i u_{ki} \| \\ \beta_i &\geq 0, i = 1, 2, \cdots, m \end{aligned} \quad (9)$$

Since all $u_{ki}, i = 1, 2, \cdots, m$ point to the common object, we have to put the constraint $\beta_i \geq 0$ in (9). We define β^{min} as follows:

$$\beta^{min} = min(\beta_1 \cap \beta_2 \cdots \cap \beta_m) \quad (10)$$

where $\cap$ is intersection of the intervals. The optimal hold force f^{oh}, f_i^{oh} can be calculated by the following equation:

$$f^{oh} = \beta^{min} u_k, f_i^{oh} = \beta^{min} u_{ki}, i = 1, 2, \cdots, m \quad (11)$$

The optimal hold force/torque $F^{oh}, F_i^{oh}, i = 1, 2, \cdots, m$ can be calculated by the following equation:

$$F_i^{oh} = \begin{bmatrix} f_i^{oh} \\ (R_o r_i) \otimes f_i^{oh} \end{bmatrix}, F^{oh} = \begin{bmatrix} F_1^{oh} \\ F_2^{oh} \\ \vdots \\ F_m^{oh} \end{bmatrix} \quad (12)$$

Therefore, the algorithm for finding the optimal hold force/torque exerted on the common rigid object by the multi-arm manipulators can be summarized as follows:

1. Find the null space of W in (1), and find one basis of W in (1) as the direction of optimal hold force f^{oh}, f_i^{oh}.
2. Find $\beta_i, i = 1, 2, \cdots, m$ from (9).
3. Find the optimal hold force f^{oh}, f_i^{oh} from (10) and (11).
4. Find the optimal hold force/torque F^{oh}, F_i^{oh} from (12).

4. The Calculation of the Optimal Moving Force/Torque and Optimal Hold Force/Torque Exerted on the Common Rigid Object by the Multi-Arm Manipulators

Our objective in this section is to find the optimal moving force/torque and optimal hold force/torque exerted on the common object by each arm. Assume that the resultant force/torque exerted on the common rigid object by the multi-arm manipulators is available. Let F_i^s in (13) be the force/torque exerted on the common rigid object by the i^{th} arm measured by the force sensor, and F_i^m, F_i^h defined in Section 3, then

$$F_i^s = F_i^m + F_i^h, i = 1, 2, \cdots, m \quad (13)$$

Since $\sum_{i=1}^{m} F_i^h = 0$, we obtain

$$\sum_{i=1}^{m} F_i^s = \sum_{i=1}^{m} F_i^m = F_t \quad (14)$$

where F_t is the resultant force/torque exerted on the common object by the multi-arm manipulators.
Assuming all F_i^m are in the same direction, we have

$$F_i^m = \alpha_i F_t \quad (15)$$

$$\text{where } 0 \leq \alpha_i \leq 1, \text{ and } \sum_{i=1}^{m} \alpha_i = 1. \quad (16)$$

The moving force/torque $F^m \in R^{6m}$ and total force/torque $F \in R^{6m}$ can be expressed as follows:

$$F^m = \begin{bmatrix} \alpha_1 F_t \\ \alpha_2 F_t \\ \vdots \\ \alpha_m F_t \end{bmatrix}, F = \begin{bmatrix} \alpha_1 F_t + F_1^h \\ \alpha_2 F_t + F_2^h \\ \vdots \\ \alpha_m F_t + F_m^h \end{bmatrix} \quad (17)$$

Therefore, in order to find the smallest weighted magnitude of F, our problem can be expressed as follows:

minimize

$$\begin{aligned} C(\alpha) &= (1/2)[\alpha_1 F_t + F_1^h]^T Q_1 [\alpha_1 F_t + F_1^h] \\ &+ (1/2)[\alpha_2 F_t + F_2^h]^T Q_2 [\alpha_2 F_t + F_2^h] \\ &+ \cdots + (1/2) \\ &[\alpha_m F_t + F_m^h]^T Q_m [\alpha_m F_t + F_m^h] \end{aligned} \quad (18)$$

subject to

$$\sum_{i=1}^{m} \alpha_i = 1, 0 \leq \alpha_i \leq 1, i = 1, 2, \cdots, m$$

where $Q = diag(Q_1, Q_2, \cdots, Q_m), Q_i = (1/F_{i1}^2(max), \cdots, 1/F_{i6}^2(max))$, $i = 1, 2, \cdots, m$ are all positive definite 6×6 matrices. From the Kuhn-Tucker conditions([11]), we know that if $C(\alpha^*)$ is the minimum point for this problem, then there are a scalar $\lambda \in R$ and a vector $\mu \in R^m$ with every element $\mu_i \geq 0$ such that

$$\nabla C(\alpha^*) + \lambda \nabla (\sum_{i=1}^{m} \alpha_i * -1) + \nabla(-\alpha^*)\mu = 0 \quad (19)$$

$$\mu_i \alpha_i^* = 0, i = 1, 2, \cdots, m \quad (20)$$

where

$\mu^T = [\mu_1, \mu_2, \cdots, \mu_m], {\alpha^*}^T = [\alpha_1^*, \alpha_2^*, \cdots, \alpha_m^*]$

From (18),(19), we obtain

$$\mu_i = \alpha_i^* F_t^T Q_i F_t + F_t^T Q_i F_i^h + \lambda, i = 1, 2, \cdots, m \quad (21)$$

If $\alpha_1, \alpha_2, \cdots, \alpha_n$ are positive, and $\alpha_{n+1}, \alpha_{n+2}, \cdots, \alpha_m$ are zero, we get the following equations from (20)

$$\begin{aligned} \mu_i &= 0, i = 1, 2, \cdots, n \\ \mu_i &\geq 0, i = n+1, n+2, \cdots, m \end{aligned} \quad (22)$$

From (21) and (22), we obtain

$$\alpha_i^* = \frac{-F_t^T Q_i F_i^h - \lambda}{F_t^T Q_i F_t}, i = 1, 2, \cdots, n \quad (23)$$

$$\lambda \geq -F_t^T Q_i F_i^h, i = n+1, n+2, \cdots, m \quad (24)$$

Without loss of generality, we let

$$-F_t^T Q_1 F_1^h \geq -F_t^T Q_2 F_2^h \geq \cdots \geq -F_t^T Q_m F_m^h \quad (25)$$

$$\alpha_j^* = \frac{-F_t^T Q_j F_j^h - \lambda_j}{F_t^T Q_j F_t}, j = 1, 2, \cdots, m \quad (26)$$

where

$$\lambda_j = \frac{-1 + \sum_{i=1}^{j} \frac{-F_t^T Q_i F_i^h}{F_t^T Q_i F_t}}{\sum_{i=1}^{j} \frac{1}{F_t^T Q_i F_t}} \quad (27)$$

We can easily show that if $-F_t^T Q_j F_j^h \geq \lambda_j \geq -F_t^T Q_{j+1} F_{j+1}^h$, then we can calculate α_j^* from (26) and $\alpha_{j+1}^* = \alpha_{j+2}^* = \cdots = \alpha_m^* = 0$.
The following recursive formula for calculating λ_{j+1} can be easily verified

$$\lambda_{j+1} = \lambda_j + \frac{-F_t^T Q_{j+1} F_{j+1}^h - \lambda_j}{F_t^T Q_{j+1} F_t (\sum_{i=1}^{j+1} \frac{1}{F_t^T Q_i F_t})} \quad (28)$$

From (28), we can easily verify that if $\lambda_j \geq -F_t^T Q_n F_n^h$ and $\lambda_j \leq -F_t^T Q_{n+1} F_{n+1}^h$, then λ_j is a nondecreasing function from $j = 1$ to $j = n$ and starts to decrease at $j = n+1$. Therefore, the following algorithm can be used for finding α^*:

1. We calculate λ_1 from (27). Let j=2.
2. We calculate λ_j from (28). If $\lambda_j > \lambda_{j-1}$, then we go to step 3. Otherwise we calculate α_{j-1}^* from (26) and $\alpha_j^* = \alpha_{j+1}^* = \cdots = \alpha_m^* = 0$. We then go to step 4.
3. We let j=j+1 and go back to step 2.
4. stop.

The optimal moving force/torque $F_i^{om}, i = 1, 2, \cdots, m$ exerted on the common object by the i^{th} arm is as follows:

$$F_i^{om} = \alpha_i^* F_t, F_i^{om} = \begin{bmatrix} f_i^{om} \\ (R_o r_i) \otimes f_i^{om} \end{bmatrix} \quad (29)$$

The algorithm for finding the optimal moving/hold force/torque can be summarized as follows:

1. Inserting the first three equations of F_i^{om} in (29) into (9),(10) and (11), we obtain the optimal hold forces f_i^{oh} exerted on the common rigid object by the i^{th} arm as we did in Section 3. We can also obtain the optimal moving forces f_i^{om} exerted on the common rigid object by the i^{th} arm from the above algorithms in Section 4.
2. If we can get the rotational matrix R_0 by using (1), (3), f_i^{om}, and the resultant force/torque F_t, we then go to step 3, otherwise we go to step 4.
3. The optimal hold force/torque and optimal moving force/torque exerted on the common rigid object by the manipulators can be calculated from (12) and (29).
4. This force/torque cannot be applied using the configuration.

5. Simulation Results

In this section, we simulate the schemes presented in Sections 3 and 4. Figure 3 shows four different objects held by the dual-arm manipulators in the x-y plane.The origin of the object frame x_o, y_o is fixed at the center of mass of the common rigid object, and the base frame is x_b, y_b.

5.1 Simulations of the Optimal Hold Force/Torque

In this section, we assume that the orientation of the base frame and the object frame is the same, i.e., the rotational matrix of the object frame with reference to the base frame R_o is identity matrix. We also assume that $mg = (0, -1, 0)^T$ and that the coefficients related to the maximum static frictional coefficients for the left and the right contact points are $\eta_1 = \eta_2 = 0.5$.

- Object A
From Object A of Figure 3, we know that the left and right contact points with reference to the object frame are $r_1 = (-1, 0, 0)^T$ and $r_2 = (2, 0, 0)^T$. From (1), we know the matrix W is as follows:

$$W = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 & 2 & 0 \end{bmatrix}. \quad (30)$$

From the matrix W, we can find $u_k = [1, 0, 0, -1, 0, 0]^T$, which is qualified as the direction of the optimal hold force f^{oh}, and $u_{k1} = [1, 0, 0]^T, u_{k2} = [-1, 0, 0]^T$ which are qualified as the direction of the optimal hold forces f_1^{oh}, f_2^{oh}, respectively. The inner unit normal vectors of the surface of the object A at the left and right contact points are $N_1 = (\sqrt{2}/2, -\sqrt{2}/2, 0)^T$ and $N_2 = (-\sqrt{3}/2, -1/2, 0)^T$. From (1), (3), $M_c \ddot{x}_c = [0, 1, 0]^T$, and $F_{to} = [0, 0, 0]^T$, we know $W f^m = (0, 1, 0)^T$. Subsequently, we can determine that the moving forces exerted on the common object by the left and right arms are $f_1^m = (0, 2/3, 0)^T$ and $f_2^m = (0, 1/3, 0)^T$.

- left arm
  From (9), we obtain the interval of β_1 as $\beta_1 \geq 2.49$.
- right arm
  From (9), we obtain the interval of β_2 as $\beta_2 \geq 0.57$.

From (10), we obtain $\beta^{min} = 2.49$. Therefore, the optimal hold forces for the left and right arms $f_1^{oh} = (2.49, 0, 0)^T$ and $f_2^{oh} = (-2.49, 0, 0)^T$ can be obtained from (11). From (12),we obtain optimal hold forces/torques $F_1^{oh} = (2.49, 0, 0, 0, 0, 0)^T$ and $F_2^{oh} = (-2.49, 0, 0, 0, 0, 0)^T$.

- Object B
  Since the intersection of β_1 and β_2 is empty, we can not apply any force which can hold the object without slipping.

- Object C
  The optimal hold forces are $f_1^{oh} = (0, 0, 0)^T$ and $f_2^{oh} = (0, 0, 0)^T$ can be obtained from (11). From (12), we obtain the optimal hold forces/torques $F_1^{oh} = (0, 0, 0, 0, 0, 0)^T$ and $F_2^{oh} = (0, 0, 0, 0, 0, 0)^T$.

- Object D
  The optimal hold forces are $f_1^{oh} = (1.86, 0, 0)^T$ and $f_2^{oh} = (-1.86, 0, 0)^T$ can be obtained from (11). From (12), we obtain the optimal hold forces/torques $F_1^{oh} = (1.86, 0, 0, 0, 0, 0)^T$ and $F_2^{oh} = (-1.86, 0, 0, 0, 0, 0)^T$

5.2 Simulations of the Optimal Hold Force/Torque and Optimal Moving Force/Torque

In this section, we assume that the orientation of the base frame and the object frame are the same, i.e., the rotational matrix of the object frame with reference to the base frame R_o is identity matrix. We also assume that $mg = (0, -1, 0)^T$ and that the coefficients related to the maximum static frictional coefficients for the left and the right contact points are $\eta_1 = \eta_2 = 0.5$.

- Object A
  From Object A of Figure 3, we know that the left and right contact points with reference to the object frame are $r_1 = (-1, 0, 0)^T$ and $r_2 = (2, 0, 0)^T$. The matrix W is as (30). From the matrix W, we can find $u_k = [1, 0, 0, -1, 0, 0]^T$, which is qualified as the direction of the optimal hold force f^{oh}, and $u_{k1} = [1, 0, 0]^T, u_{k2} = [-1, 0, 0]^T$ which are qualified as the direction of the optimal hold forces f_1^{oh}, f_2^{oh}, respectively. The inner unit normal vectors of the surface of the object A at the left and the right contact points are $N_1 = (\sqrt{2}/2, -\sqrt{2}/2, 0)^T$ and $N_2 = (-\sqrt{3}/2, -1/2, 0)^T$. We also assume Q_1 and Q_2 are (6×6) identity matrix. Since the direction of the optimal hold force/torque is orthogonal to the resultant force F_t, the optimal moving forces exerted on the common object by the manipulators $f_1^{om} = f_2^{om} = (0, 0.5, 0)^T$ can be obtained from (26)-(29).
  - left arm
    From (9), we obtain the interval of β_1 as $\beta_1 \geq 1.86$.
  - right arm
    From (9), we obtain the interval of β_2 as $\beta_2 \geq 0.62$.

  From (10), we obtain $\beta^{min} = 1.86$, Therefore, the optimal hold forces are $f_1^{oh} = (1.86, 0, 0)^T$ and $f_2^{oh} = (1.86, 0, 0)^T$ can be obtained from (11). But from (1) and (3), we can not find a rotational matrix R_0; thus these forces/torques cannot be applied. In order to get the optimal moving force/torque and optimal hold force/torque exerted on the common rigid object by the manipulators, we need to rotate the object so that the moving forces exerted on the common rigid object by the manipulators are $f_1^{om} = f_2^{om} = (0, 0.5, 0)^T$. We also cannot find the force/torque to hold and move the Object B and Object C at the configuration.

- Object D
  The optimal hold forces $f_1^{oh} = (1.86, 0, 0)^T$, $f_2^{oh} = (1.86, 0, 0)^T$ can be obtained from (11). From (1) and (3), we can find that the rotational matrix R_0 is the identity matrix. From (12) and (29), we can determine that the optimal hold forces/torques are $F_1^{oh} = (1.86, 0, 0, 0, 0, 0)^T$, $F_2^{oh} = (-1.86, 0, 0, 0, 0, 0)^T$, and the optimal moving forces/torques are $F_1^{om} = F_2^{om} = (0, 1/2, 0, 0, 0, 0)^T$.

6. Conclusions

In this paper, we have presented schemes which can find the optimal hold force/torque and optimal moving force/torque exerted on a common rigid object by multi-arm manipulators. The theoretical analyses and simulations have showed that our schemes require less computation and are more general than current applications.

7. References

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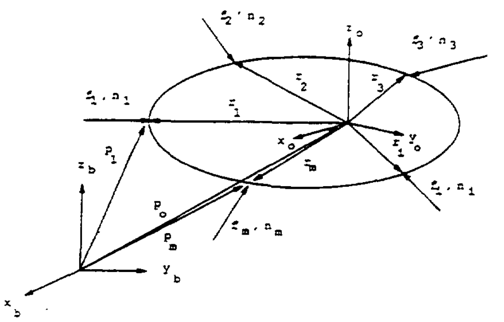

Figure 1: multi-arm manipulators holding a rigid object.

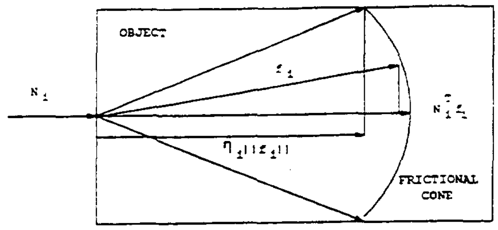

Figure 2: static frictional constraints.

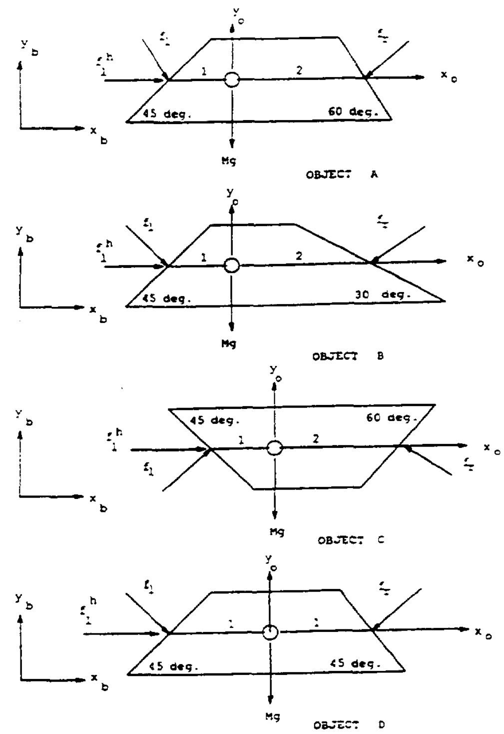

Figure 3: rigid object held by the dual-arm manipulators.