Robust Variable Structure and Adaptive Control of Multi-Arm Dynamics

Li-Wen Chen
Chinese Naval Academy, Tsoying, Taiwan, R.O.C
and
G. P. Papavassilopoulos
Department of Electrical Engineering-Systems
University of Southern California, Los Angeles, CA 90089-0781, U.S.A.

ABSTRACT

In this paper we will apply the combination of the continuous VSC law and the switching-σ modification control law, the optimal hold force/ torque, and the optimal moving force/torque to the control of the multi-arm dynamics with some unknown parameter and torque error. We prove stability and calculate the error bounds. We assume that we can measure or estimate the resultant force/torque $\hat{F}_t$, which may be or may be not equal to desired resultant force/torque F_d, exerted on the object by the multi-arm manipulators. The proposed control law can achieve better tracking precision than a simply continuous VSC law and a simply switching-σ modification control law and also exerts minimal force/torque on the common rigid object. In the simulation, we can see that this control law has good tracking precision performance even if we have actuator unmodeled dynamics and the estimate resultant force/torque $\hat{F}_t$ is not equal to the desired resultant force/torque F_d.

1. INTRODUCTION

A particular class of problems arising in the robotic area are control problems, where one is interested in creating a controller which will make the robot arms move in a certain way. Owing to the nonlinearity, unknown parameters, and bounded disturbance of the manipulator's dynamics , the design of the manipulator's controller is a difficult problem. This leads to interesting robust variable structure ([2]-[3]) and adaptive control ([4]-[5]) problems for the control of multi-arm dynamics. In recent years, several papers appeared regarding the control of dual-arm and multi-arm dynamics. Zheng and Luh ([7]-[9]) studied the problems of two industrial robots holding a single object. Pittelkau [10] studied the adaptive load sharing force control for dual-arm manipulator. In his paper, he did not mention how to find the optimal hold force which is important in dual-arm and multi-arm dynamics control, and he did not generalize his scheme to the control of multi-arm dynamics. Nakamura, Nagai, and Yoshikawa [11] studied the mechanics of coordinative manipulation by multiple robotic mechanisms. The scheme they present to find the minimal internal force exerted on the common object by the multiple robots is not simple and they also did not consider the optimal moving force/torque. Seraji [14] studied adaptive control strategies for cooperative dual-arm manipulator. He linearized the nonlinear dynamic equation at the operating point, and then developed his adaptive controller based on the linearized model. Cole, Hauser, and Sastry [13] studied kinematics and control of multifingered hands with rolling contacts , but they did not deal with the calculation of the optimal hold force and moving force.

This paper is organized as follows:In Section 2, the dynamic model of the multi-arm manipulator holding one common rigid object is given. In this model we assume that the contacts between the multi-arm manipulator and the common rigid object are the point contacts so that the torques of contact are zero. In Section 3, we apply the combination of the continuous VSC law and the switching-σ modification control law, the optimal hold force/torque, and the optimal moving force/torque to the control of the multi-arm dynamics with some unknown parameter and torque error. We also state the error boundsfor this control law. In Section 4, the simulations of dual-arm manipulators, each arm with two rigid links, holding a common rigid object with controllers are presented. These simulations show that the proposed control law has better tracking performance even if actuator unmodeled dynamics and torque error are considered in the simulations. The conclusion is given in Section 5.

ISSN# 0-7803-0233-8/91 $1.00©1991 IEEE

2. FORMULATION

Figure 1 shows a m-arm manipulator which has m arms, each one of which has n rigid links, holding one common rigid object together. In Figure 1, X_b, Y_b, Z_b denote the base frame, X_o, Y_o, Z_o denote the object frame fixed at the mass center of the common rigid object, the (3×1) vectors $f_1, f_2, \cdots, f_m$denote the end-effector forces exerted on the common object by each arm with reference to the base frame, the (3×1) vectors $n_1, n_2, \cdots, n_m$ denote the end-effector torques exerted on the common object by each arm with reference to the base frame, the (3×1) vector p_o denotes the position vector of the mass center of the common rigid object with reference to the base frame, the (3×1) vectors $p_1, p_2, \ldots, p_m$ denote the position vectors of the contact points with reference to the base frame, and the (3×1) vectors $r_1, r_2, dots, r_m$ denote the vectors from the mass center of the common object to the each contact point with reference to the object frame, respectively. We also denote by the (3×1) vectors f_t, n_t the resultant force and torque exerted on the common object, respectively. We denote by $g = [0\ 0\ -9.8]^T \in R^3$ the gravitational accelerational vector, by M the mass of the common object , and by R_o the rotational matrix of object frame X_o, Y_o, Z_o with reference to base frame X_b, Y_b, Z_b. Assuming the contacts between the arms and the common rigid object are the point contacts,i.e., the torques exerted on the object by each arm $n_i, i = 1, 2, \cdots, m$ are zero, then the following equations hold:([11]-[13])

$$f_t = \sum_{i=1}^{m} f_i + Mg \tag{2.1}$$

$$n_t = \sum_{i=1}^{m} (R_o r_i) \otimes f_i \tag{2.2}$$

where $\otimes$ in (2.2) denotes the cross product of two column vector. Then the dynamic equations of the common object can be represented by the following Newton and Euler equations.

$$M\ddot{p_o} = f_t \tag{2.3}$$

$$I\dot{\omega} + \omega \otimes (I\omega) = n_t \tag{2.4}$$

where $\ddot{p_o} \in R^3$ is the linear accelerational vector of the mass center of the object with reference to the base frame, $\omega, \dot{\omega} \in R^3$ are the angular position and velocity vectors of the object with reference to the base frame, and $I \in R^{3\times3}$ is the inertial matrix of the object with reference to the base frame. Note that the inertial matrix varies with the orientation of the common rigid object([11]-[13]). We can write the dynamic equation of the common object from (2.1) to (2.4) as follows.

$$M_c \ddot{x_c} + F_{t0} = Wf \tag{2.5}$$

where

$$M_c = \begin{pmatrix} MI_3 & 0 \\ 0 & I \end{pmatrix}, \ddot{x_c} = \begin{pmatrix} \ddot{p_o} - g \\ \dot{\omega} \end{pmatrix}$$

$$W = \begin{pmatrix} I_3 & I_3 & \cdots & I_3 \\ (R_o r_1)\otimes & (R_o r_2)\otimes & \cdots & (R_o r_m)\otimes \end{pmatrix}$$

$$f = (f_1, \ldots, f_m)^T, F_{t0} = (0, \omega \otimes (I\omega))^T$$

where W is a $(6 \times 3m)$ matrix, and f is a $(3m \times 1)$ vector.

We also let

$$Wf = F_t = \sum_{i=1}^{m} F_i, F_i = \begin{pmatrix} f_i \\ (R_0 r_i) \otimes f_i \end{pmatrix} \tag{2.6}$$

$$i = 1, 2, \ldots, m.$$

where $F_1, F_2, \cdots, F_m \in R^6$ are the force/ torque exerted on the object by each arm, and $F_t \in R^6$ is the resultant force/torque exerted on the object by the m-arm manipulators. The dynamic equation of each arm with n links in joint space is as follows[1].

$$\tau_i = D(q_i)\ddot{q_i} + H(q_i, \dot{q_i})\dot{q_i} + G(q_i) + (J(q_i))^T F_i \tag{2.7}$$

where $\tau_i \in R^n$ is the vector of joint torques supplied by the i^{th} robot arm actuators; $D(q_i) \in R^{n\times n}$ is the i^{th} robot arm mass (inertial) matrix which is symmetric and positive definite; $q_i, \dot{q_i}, \ddot{q_i} \in R^n$ are i^{th} robot arm vectors of joint displacement ,velocity and acceleration, respectively; $H(q_i, \dot{q_i}) \in R^{n\times n}$ is the matrix from centrifugal, Coriolis and frictional forces for i^{th} robot arm; $G(q_i) \in R^n$ is the i^{th} robot arm vector of gravitational torque ; $J(q_i) \in R^{6\times n}$ is the Jacobian for i^{th} robot arm; $F_i \in R^6$ is the Cartesian force/torque vector acting on the mass center of the object by i^{th} robot arm. Finally, we can write the dynamic equation of m-arm manipulators in joint space as follows.

$$\tau = D(q)\ddot{q} + H(q, \dot{q})\dot{q} + G(q) + J^T(q)F \tag{2.8}$$

where

$$D(q) = diag(D(q_1), \ldots, D(q_m))$$

$$\ddot{q} = [\ddot{q_1} \ldots \ddot{q_m}]^T$$

$$\tau = [\tau_1 \ldots \tau_m]^T$$

$$H(q,\dot{q}) = diag(H(q_1,\dot{q_1}),\ldots,H(q_m,\dot{q_m}))$$

$$G(q) = [G(q_1)\ldots G(q_m)]^T$$

$$J = diag((J(q_1))^T,\ldots,(J(q_m))^T)$$

$$F = [F_1\ldots F_m]^T$$

where $\tau \in R^{mn}, D(q) \in R^{mn\times mn}, q \in R^{mn}, H(q,\dot{q}) \in R^{mn\times mn}, J(q) \in R^{6m\times mn}$, and $F \in R^{6m}$. Therefore, our problem is to design a controller which uses the control law τ as a function of the state $q, \dot{q}$, the estimated unknown parameter $\hat{P}$ in (3.2), the estimated optimal moving force/torque F^{om} in (3.8) and the estimated optimal hold force/torque F^{oh} in (3.8) which will make (2.8) to have $q \rightarrow q_d$ in the presence of torque error and unknown parameters, where q_d is the desired trajectory.

3. ROBUST VARIABLE STRUCTURE AND ADAPTIVE CONTROL OF MULTI-ARM

For the control of the multi-arm manipulators holding one common rigid object, there are several objectives we want to achieve. First, we want the mass center of the common rigid object to move along the desired trajectory in the Cartesian space. Second, we want to achieve no slipping of the common rigid object from the manipulators, i.e., multi-arm manipulators can firmly hold the common rigid object without slipping. Third, we want to achieve that the force/torque exerted on the common rigid object by the multi-arm manipulators is minimum(optimal).

In [15], we study and simulate the combination of the continuous VSC law and switching-sigma modification control law and show that it has better tracking performance than a simply continuous VSC law, and switching-sigma modification control law for the single-arm dynamics. In [16], we present several schemes which can find the optimal hold force/torque $\hat{F}^{oh}$ and optimal moving force/torque $\hat{F}^{om}$ exerted on the common rigid object by the multi-arm manipulators. In this section, we apply the optimal hold force/torque, the optimal moving force/torque, and the combination of the continuous VSC and the switching-sigma modification control laws to the control of multi-arm manipulators holding a rigid object. Our control scheme can achieve not only the good tracking performance but also the smallest-magnitude force/torque exerted on the common rigid object by the multi-arm manipulators.

Assume the estimated resultant force/torque $\hat{F}_t$ exerted on the common rigid object by the multi-arm manipulators is available, which may be or may be not equal to the desired resultant force/torque F_d, where the desired resultant force/torque F_d is a force/torque which can move the object along the desired trajectory. We also let the multi-arm manipulators move in a coordinative way.([7]-[9]) First, we calculate the optimal hold force/torque $\hat{F}^{oh}$, and the optimal moving force/torque $\hat{F}^{om}$ based on the estimated resultant force/torque $\hat{F}_t$ and the scheme in [16], and then add $J^T(q)\hat{F}^{oh}$ and $J^T(q)\hat{F}^{om}$ into our control law, where $J^T(q)$ is Jacobian which is a mapping from velocities in joint space to velocities in Cartesian space, and its transpose maps Cartesian force/torque acting on the end-effector into equivalent joint torques. Applying the control law in (3.8) to the control of multi-arm manipulators holding one common rigid object, it can surpress the torque error and achieve better tracking performance.

We choose the following function as a Lyapunov function candidate of the dynamic equation (2.8).

$$V(t,S,\Phi) = (1/2)S^T D(q)S + (1/2)\Phi^T\Gamma\Phi \quad (3.1)$$

where $D(q) \in R^{mn\times mn}$ is an inertia matrix which is a symmetric and positive definite, $\Gamma \in R^{mz\times mz}$ is a diagonal positive definite constant matrix chosen by the designer, and $\Phi(t) \in R^{mz}$ is defined as follows.

$$\Phi(t) = \hat{P}(t) - P \quad (3.2)$$

where mz is the number of unknown parameter, P is an unknown constant parameter vector, $\hat{P}(t)$ is the estimate of P, and $\Phi(t)$ is the estimated error of the parameter vector. We define $S(t) \in R^{mn}$ as follows.

$$S(t) = \dot{\bar{q}}(t) + \Lambda\bar{q}(t), \dot{S}(t) = \ddot{\bar{q}}(t) + \Lambda\dot{\bar{q}}(t) \quad (3.3)$$

where

$$\bar{q}(t) = q(t) - q_d(t), \dot{\bar{q}}(t) = \dot{q}(t) - \dot{q}_d(t) \quad (3.4a)$$

$$\ddot{\bar{q}}(t) = \ddot{q}(t) - \ddot{q}_d(t) \quad (3.4b)$$

$q_d(t), \dot{q}_d(t)$ and $\ddot{q}_d(t) \in R^{mn}$ are the desired joint position, velocity and acceleration of multi-arm manipulators, and $\Lambda \in R^{mn\times mn}$ is a constant diagonal positive definite matrix chosen by the designer. We also define the reference signal $q_r(t) \in R^{mn}$ as follows:

$$q_r(t) = q_d(t) - \Lambda\int_0^t \bar{q}(t)dt \quad (3.5)$$

Therefore

$$\dot{q}_r(t) = \dot{q}_d(t) - \Lambda\bar{q}(t), \ddot{q}_r(t) = \ddot{q}_d(t) - \Lambda\dot{\bar{q}}(t) \quad (3.6)$$

From (3.3), (3.4a), (3.4b) and (3.6), we get

$$S(t) = \dot{q}(t) - \dot{q}_r(t) = \dot{\tilde{q}}_r(t) \quad (3.7)$$

From Lemma 1 in [15], we know $V(t,S,\Phi)$ is positive definite and decresent. In the following we only state the control law, the detail proof is given in[17].

Lemma 1 Consider the following adaptive control law for (2.8):

$$\tau = \hat{D}(q,\hat{P})\ddot{q_r} + \hat{H}(q,\dot{q},\hat{P})\dot{q_r} + \hat{G}(q,\hat{P}) +$$

$$J^T(q)\hat{F}^{oh} + J^T(q)\hat{F}^{om} - K_d S(t) - dsat(S) \quad (3.8)$$

$$\dot{\Phi}(t) = \dot{\hat{P}}(t) = -\Gamma^{-1}W^T(q,\dot{q},\dot{q_r},\ddot{q_r})S - \sigma\Gamma^{-1}\hat{P} \quad (3.9)$$

where

$$\sigma = \begin{cases} 0, & ||\hat{P}|| \le P_0 \\ \sigma_0(\frac{||\hat{P}||}{P_0} - 1), & P_0 < ||\hat{P}|| \le 2P_0 \\ \sigma_0, & 2P_0 < ||\hat{P}|| \end{cases} \quad (3.10)$$

$\sigma_0 > 0$ is a scalar, $P_0 > \|P\|$, and $\| * \|$ is a l_2 norm. Here

$$J^T(q)(\hat{F}^{om} + \hat{F}^{oh} - F) = d$$

$$= (d_1(t), \cdots, d_{mn}(t))^T \in R^{mn} \quad (3.11)$$

$$d_0 = diag(d_{10}, \cdots, d_{mn0}) and |d_i(t)|$$

$$\le d_{i0}, i = 1, 2, \cdots, mn \quad (3.12)$$

$$sat(S) = (sat(S_1), sat(S_2), \cdots, sat(S_{mn}))^T$$

$$sat(S_i) = \begin{cases} S_i/\phi_i, & \text{if } |S_i/\phi_i| < 1 \\ 1, & \text{if } S_i/\phi_i > 1 \\ -1, & \text{if } S_i/\phi_i < 1 \end{cases} \quad (3.13)$$

$$W(q,\dot{q},\dot{q_r},\ddot{q_r})\Phi = \bar{H}(q,\dot{q},\hat{p})\dot{q_r} + \bar{D}(q,\hat{P})\ddot{q_r} + \bar{G}(q,\hat{P}) \quad (3.14)$$

where

$$\bar{H} = \hat{H} - H, \bar{D} = \hat{D} - D, \bar{G} = \hat{G} - G \quad (3.15)$$

where $\hat{H},\hat{D},\hat{G}$ are the estimate of H, D, G, and $\bar{H},\bar{D},\bar{G}$ are the errors. K_d and Γ are arbitrary constant diagonal positive definite matrices designed by the designer. Then we can guarantee that q is closed to q_d in a bound. The block diagram of multi-arm dynamics and its controller are shown in Figure 2.

4. SIMULATION

We present one example in our simulation. Figure 3 shows a dual-arm manipulator which moves the object along a curve. We let the base frame x_b, y_b of the two arms be the same as the world frame, and the object frame x_o, y_o, its origin is located at the mass center of the object, be parallel to the base frame. We assume that all the link's length is 1 meter, link's mass is 1 kilogram, the object's length from the contact point with left arm to the contact point with right arm be 0.2 meter, the origin of the object frame x_o, y_o is initially located at (1.1, -1) meters with reference to the base frame, the left and right contact points of the object with the dual-arm manipulators are initially located at (1, -1) meters and (1.2, -1) meters with reference to the base frame, the gravitational acceleration vector is $(0, -9.8)^T$ meter squared per second, and the maximum static frictional coefficients at the contact points is $\sqrt{3}$. We assume that the mass of the object M is 2 kilogram, but the estimated mass of the object $\hat{M}$ is within the bound,i.e.,$1. \le \hat{M} \le 3$, where the unit is kilogram. Our objective is to apply the optimal hold force/torque and optimal moving force/torque calculated based on the estimated mass and desired trajectory of the object, maintain no slipping of the object from the manipulators, and the mass center of the object track the desired trajectory $(0.6 + 0.5/(1+t), -e^{-t})$ with reference to the base frame. Therefore the desired resultant force/torque required to move the mass center of the object along the desired trajectory is $F_d = (2(1+t)^{-3}, 19.6 - 2e^{-t})^T$, and the estimated resultant force/torque is within the bound $0.5F_d \le \hat{F}_t \le 1.5F_d$. According to the scheme in [16], we can get the following optimal force/torque: The desired optimal moving force/torque is: $F^{om} = (F_r^{om}, F_l^{om})^T = ((1+t)^{-3}, 9.8 - e^{-t}, (1+t)^{-3}, 9.8 - e^{-t})^T$ The desired optimal hold force/torque is: $F_l^{oh} = (1+t)^{-3} + \sqrt{1/3}(9.8 - e^{-t})(1,0)^T, F_r^{oh} = -F_l^{oh}$ The estimated optimal moving force/torque is: $0.5F^{om} \le \hat{F}_{om} \le 1.5F^{om}$ The estimated optimal hold force/ torque is: $0.5F_l^{oh} \le \hat{F}_l^{oh} \le 1.5F_l^{oh}, \hat{F}_r^{oh} = -\hat{F}_l^{oh}$ In order to maintain no slipping of the object from the manipulators, we apply the upper bound of optimal hold force/torque to the object. We also consider actuator unmodeled dynamics, its cut off frequency is 100 radians per second and DC gain is 1, in the simulation. The control law for the right and left arms can be obtained from (3.8).

In Figure 4, the Cartesian position error of the object mass center for different boundary layer 0.1 and 1 are plotted as the desired force/torque is known. In Figures 5 and 6, the Cartesian position error of the object mass center for different boundary layer 0.1,1,10, and 100 are plotted as only the estimated resultant force/torque which is within some bound stated above is known. In Figure 4, the Cartesian position error for different boundary layers 0.1,1 are very small, the Cartesian position error of the boundary layer 0.1 is smaller than those of the boundary layer 1, and the Cartesian position error goes to zero finally since the desired force/torque is known. In Figures 5 and 6, we see the smaller the boundary layer is, the smaller the position error is, and the Cartesian position error will not go to zero since the estimated force/torque is not equal to the desired force/torque. We also can see that the smaller the torque error is, the smaller the Cartesian position error is.

5. CONCLUSIONS

In this paper, we have analyzed and simulated the combination of the continuous VSC and the switching-sigma modification control laws for the multi-arm manipulators with rigid links holding a rigid object. Although the simulations show that the Cartesian position error is small if the boundary layer is small and(or) the estimated resultant force/torque is close to the desired resultant force/torque even if the actuator unmodeled dynamics are considered, the small boundary layer is limited by the physical limitation and the closeness between the estimated resultant force/torque and the desired resultant force/torque is also limited by the measurement or estimation.

REFERENCES

[1.] H. Asada and J.J.E. Slotine, Robot Analysis and Control, *John Wiley and Sons,* 1986.

[2.] V.I. Utkin, Sliding Modes and Their Application in Variable Structure System, *MIR Publishers, Moscow, Soviet Union,* 1978.

[3.] J.J.E. Slotine, "On modeling and adaptation in robot control," *Proc. IEEE International Conference on Robotics and Automation,* pp. 1387-1392, 1986.

[4.] J.S. Reed and P.A. Ioannou, "Instability analysis and robust adaptive control of robotic manipulators," *IEEE Transactions on Robotics and Automation,* June, 1989.

[5.] T.C. Hsia, "Adaptive control of robot manipulators-a review," *Proc. IEEE Int. Conf. on Robotics and Automation,* pp. 183-189, 1986.

[6.] J.J.E. Slotine and W. Li, "On the adaptive control of robot manipulators," *ASME Winter Annual Meeting,* Anaheim, CA, pp. 51-56, 1986.

[7.] Y.F. Zheng and J.Y.S. Luh, "Optimal load distribution for two industrial robots handling a single object," *IEEE International Conference on Robotics and Automation,* pp. 344-349, 1988.

[8.] Y.F. Zheng and J.Y.S. Luh, "Control of two coordinated robots in motion," *Proc. IEEE Conf. on Decision and Control,* pp. 1761-1766, 1985.

[9.] Y.F. Zheng and J.Y.S. Luh, "Joint torques for control of two coordinated moving robots," *Proc. IEEE International Conf. on Robotics and Automation,* pp. 1375-1380, 1986.

[10.] Mark E. Pittelkau, "Adaptive load-sharing force control for two-arm manipulators," *Proc. IEEE International Conf. on Robotics and Automation,* pp. 498-503, 1988.

[11.] Y. Nakamura, K. Nagai, and T. Yoshikawa, "Mechanics of coordinative manipulation by multiple robotic mechanisms," *Proc. IEEE International Conf. on Robotics and Automation,* pp. 981-998, 1987.

[12.] J. Kerr and B. Roth, " Analysis of multifingered hands," *The Int. J. of Robotics Research,* vol. 4-4, pp. 3-17, 1986.

[13.] A.B.A. Cole, J.E. Hauser, and S.S. Sastry, "Kinematics and control of multifingered hands with rolling contact," *IEEE Transactions on Automatic and Control,* vol.34, No.4, pp. 398-404, Apr, 1989.

[14.] H. Seraji, "Adaptive force and position control of manipulators," *J. of Robotic Systems,* 4(4), pp. 551-578, 1987.

[15.] L. W. Chen and G. Papavassilopoulos, "Robust variable structure and adaptive control of single-arm dynamics," *IEEE Int. Conf. on Decision and Control,* Brighton, United Kingdom, Dec. 1991.

[16.] L. W. Chen and G. Papavassilopoulos, "Optimal hold and moving force/ torque for the dual-arm and multi-arm manipulators holding one rigid object," *The European Robotics and Intelligent Sys. Conf.,* Greece, June 1991.

[17.] L. W. Chen, "Robust optimal force variable structure and adaptive control of single-arm and multi-arm dynamics," Ph. D. Dissertation, USC, L. A., Cal., 1990.

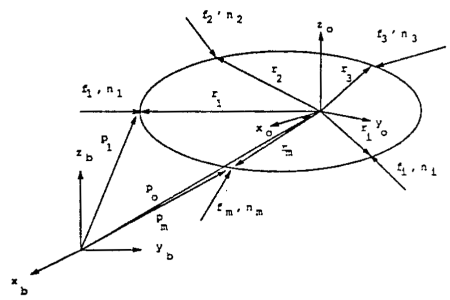

Figure 1: m-arm manipulators holding rigid object

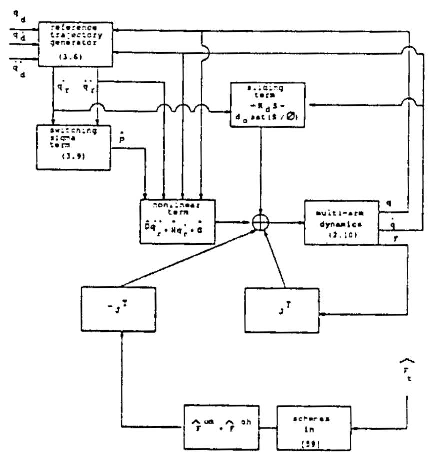

Figure 2: block diagram of multi-arm dynamics and its controllers

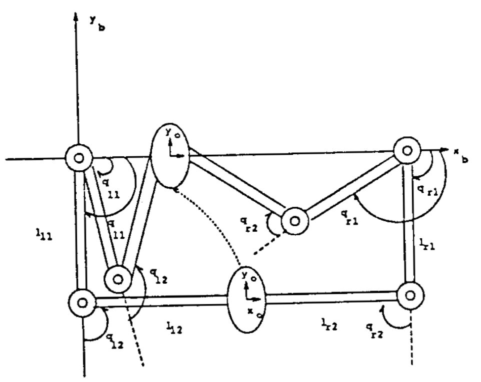

Figure 3 dual-arm manipulators move the object along a curve in the x-y plane

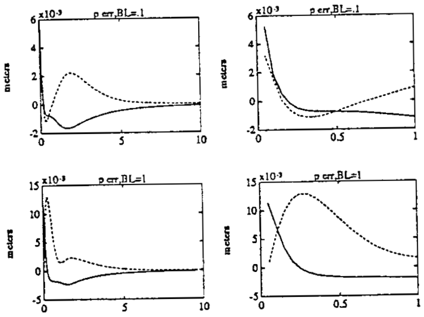

Figure 4 the Cartesian position error of the object mass center for the object moves along a curve, boundary layers 0.1 and 1,the estimated resultant force/torque is equal to the desired resultant force/torque,dot line:y position error,solid line:x position error

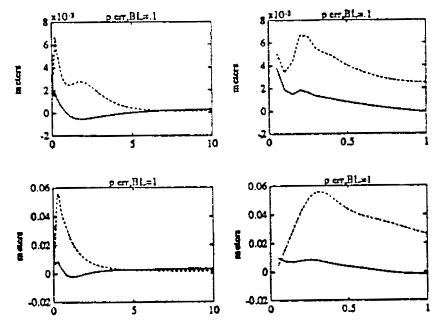

Figure 5 the Cartesian position error of the object mass center for the object moves along a curve, boundary layers 0.1 and 1,the estimated resultant force/torque is not equal to the desired resultant force/torque,dot line:y position error,solid line:x position error

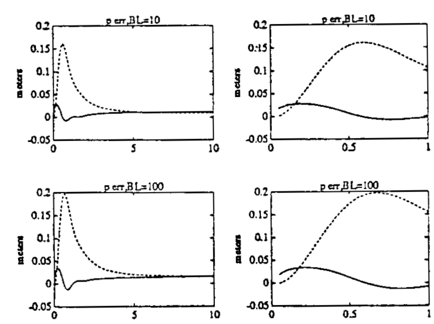

Figure 6 the Cartesian position error of the object mass center for the object moves along a curve, boundary layers 10 and 100,the estimated resultant force/torque is not equal to the desired resultant force/torque,dot line:y position error,solid line:x position error