# A step closer to competition: A different concept of Cournot duopoly under incomplete information

Nikolaos Chrysanthopoulos<sup>1</sup> and George P. Papavassilopoulos<sup>2</sup>

<sup>1,2</sup>School of Electrical and Computer Engineering National Technical University of Athens Greece <sup>1</sup>nikoschrys@mail.ntua.gr <sup>2</sup>yorgos@netmode.ntua.gr

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The available information plays an important role in oligopolistic market structures since it affects the market outcome. From the literature that deals with information exchange, it is known that oligopoly under conditions of linear demand and Cournot behavior promotes no information sharing when the unknown value is common, for example when a parameter of a common faced demand is at stake [6]. In contrast, when private marginal costs are unknown, quantity based competition offers incentives for information transmission among firms and perfect revelation is identified as a dominant strategy [2]. In this work, using a Cournot duopoly model, we study how an incomplete information concept, in lack of any information exchange, may affect the strategic interaction of the firms and the market outcome in a finite time horizon and under the existence of a pretending action [3]. This work is motivated by modelling decisions in a liberalized energy market where producers, distributors, consumers of energy are present and firms do not have exact knowledge of their rivals' costs.

We consider a duopoly in a multiperiod framework, where firms (i = 1, 2) choose simultaneously their output level  $(q_{i,t})$ , having no information about their rival's marginal cost. Since we assume that there is neither a mechanism for truthful information exchange nor a structure for allowing any signaling activity, each firm's expectation about its rival's marginal cost should to be replaced by an estimation that is related with the observed actions in the past. We assume that both firms adopt an estimation technique that guarantees the convergence, for any given initial conditions, to the Cournot-Nash equilibrium of the corresponding complete information game. Given that both firms produce a homogenous product at constant marginal cost and that the market demand is linear (p = a - bQ), the discrete-time system formed by the firms' best responses is described by (1). Hence, the sufficient assumption made is that each firm expects its rival's action to be the same with the last one observed  $(\hat{q}_{-i,t} = q_{-i,t-1})$  and as a result the estimated marginal cost of its rival is a function of previous observed actions  $(\hat{c}_{-i,t} = a - 2bq_{-i,t-1} - bq_{i,t-2})$ .

$$q_{i,t} = \frac{a - c_{i,t}}{2b} - \frac{1}{2}\hat{q}_{-i,t}, \qquad i = 1,2$$
(1)

The fact that the actual cost of each firm is never revealed to its competitor may offer an opportunity to firms to achieve an improved outcome at the steady state. By allowing duopolists to choose their type before they are engaged in repeated interaction, each one will be able to adopt a different cost in a behavioral manner. That is, firms may choose their output level as if they had a different cost while their actual cost is maintained as their production's expenditure. Hence, we define the "pretend" action as the action that refers to the adoption of such an optimal cost to behave with, which is called the optimal behavioral pretending cost  $(c_i^{pr*})$ . Given the profit in the steady state (3), the optimal behavioral pretending cost is derived as the solution of (2). It's worth mentioning that, when one of the firms adopts the optimal behavioral pretending cost, the output levels and the payoffs yield are the same with those of a complete information Stackelberg game, where both have their actual costs and that firm is the leader. The other action included at the action set available before the beginning of the repeated game is the "not pretend" action where the firm commits itself to behave in accordance with (1).

$$c_i^{pr*} = \arg\max_{c_i^{pr}} \{\pi_i^{ss}(c_i^{pr}, c_i, c_{-i})\} = \frac{6c_i - a - c_{-i}}{4}$$
(2)

$$\pi_i^{ss}(c_i^{pr}, c_i, c_{-i}) = \frac{(a + c_i^{pr} + c_{-i} - 3c_i)(a - 2c_i^{pr} + c_{-i})}{9b}$$
(3)

Therefore, we define a multiperiod game  $(G_T)$  where firms, after selecting their behavioral cost action, are involved in a repeated interaction for a finite time horizon T. For sufficient large time horizon, the time averaging payoffs become approximately equivalent with the payoffs occurred at the steady state. Thus, firms when selecting their behavioral cost action are concerned only about the outcomes occurred at the equilibrium. Since the "pretend" action is dominant,  $G_T$  has a unique Nash equilibrium where both firms by pretending to have lower marginal cost, actually overproduce. If  $G_T$  is considered as the stage game, then the respective repeated game that has horizon a multiple of T, describes the market in the long-run and the equilibrium of the stage game is replicated at the supergame.

The results are also illustrated through a simulation example motivated from energy markets where two, at the beginning, identical firms interact for a long-run horizon in a market that links technology improvement and cost reduction with the achieved profits. A stochastic process is the mechanism for sustaining that link, by favoring the firm with the better outcome in the attendance of a cost reduction. Before the beginning of each sort-run period, the actual marginal cost of firm *i* that is determined by nature, follows (4) where the reduction factor is given in (6) as a function of the cumulative payoffs achieved in the previous sort-run period. The probability for a cost reduction to occur, increases if the cost remained constant in the previous period or returns at a default level ( $x \sim U[0,1]$ ;  $X_0$ , K,  $\varepsilon$  exogenous constants). The evolution of firms' payoffs in the long-run (n = 200, T = 100) for the cases where firms adhere to the same strategy for every stage game is depicted in Figure 1.

$$c_{i,nT} = c_{i,(n-1)T} \left( \mathbf{1}_{\{x \le X_n\}} \delta_n + \mathbf{1}_{\{x > X_n\}} \right)$$
(4)

$$X_{n} = \begin{cases} X_{n-1} + K \left( \sum_{t=(n-1)T}^{nT-1} \frac{\pi_{i,t}}{\pi_{i,t} + \pi_{-i,t}} \right), & c_{i,(n-1)T} = c_{i,(n-2)T} \\ X_{0}, & c_{i,(n-1)T} \neq c_{i,(n-2)T} \end{cases}$$
(5)

$$\delta_n = 1 - \varepsilon \frac{\sum_{t=(n-1)T}^{nT-1} \pi_{i,t}}{T \max_{q_{i,t} \in [(n-1)T, nT-1]} \pi_{i,t}}$$
(6)



Figure 1. Payoffs' evolution when cost reduction related with the achieved profits

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N. Chrysanthopoulos, G. P. Papavassilopoulos

School of Electrical & Computer Engineering National Technical University of Athens

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Cournot Duopoly			

#### The duopoly model

We consider a **Cournot duopoly** model, in an environment of repeated interaction, where firms have **private information**.

The simplest form of Cournot:

- Products are homogeneous
- Marginal costs are constant
- Demand is linear

& each firm knows nothing about its rival's cost!

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#### **Cournot Duopoly with Private Information**

#### Information Exchange

Incentives for duopolists to honestly share information change depending upon the nature of the competition (Cournot or Betrand) and the nature of the information structure.

#### **Common Parameter:**

(e.g. a demand parameter)

 Concealing is a dominant strategy

(Vives, 1984)

#### **Private Parameter:**

- (e.g. private costs)
- Promotes information sharing (Gal-Or, 1986)

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Cournet Duepely wit	h Private Inform	ation & Learning	

# Why learning?

Marginal costs are not common knowledge and since players are engaged in repeated interaction they can learn.

Learning theories use information about past events to come up with a prediction/description for the present play.

# **Common Learning Schemes/Theories:**

- Cournot Adjustment
- Fictitious Play
- Reinforcement Learning

Different ...

- information used
- way of use

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#### Cournot Duopoly with Private Information & Learning

### Different Information Used in Common Learning Schemes:

- Cournot Adjustment
  - Last round's action of other player
  - Own payoff matrix
- Fictitious Play
  - Past actions of other player
  - Own payoff matrix
- Reinforcement Learning
  - Own past actions
  - Associated payoffs

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#### Cournot Duopoly with Private Information & Learning

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 Cournot Adjustment (Myopic best responses)
 Players play their best response to their rival's last action (Cournot, 1838)

### Pros:

- 1. Simple learning model of simultaneous adjustment
- 2. Stable steady state for 2 players (Theocharis, 1960)

# Cons:

- 1. Different path for different starting values
- 2. Weak rationality (dynamic rule but "static" behavior)
- 3. Stability related to the number of players (Theocharis, 1960)

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#### Question at stake

Each firm knows nothing about its rival's cost and can only extract it from the history of the game. May firms pretend to have a different cost so as to achieve a better outcome?

#### Idea behind the concept

- Allow each firm to have a "behavioral" cost that may differ from the actual one.
- ► Allow each firm to select its "behavioral" cost optimally

Before being engaged in repeated interaction, players choose an action from the set ("Not Pretend", "Pretend").

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Cournot Duopoly with Private Information, Learning & Pretending

#### **Repeated Game**

► The multiperiod game (G<sub>T</sub>) is considered to be the stage game of a repeated game that has a multiple of T as horizon.



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Cournot Duopoly with Private Information, Learning & Pretending

#### **Repeated Game**

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#### Cournot Duopoly with Private Information, Learning & Pretending

#### Motivated by energy markets

This work is motivated by modeling decisions in a **liberalized** energy market where firms do not have exact knowledge of their rival's costs.

#### The Application

- ▶ Identical, at the beginning, firms interact for a long horizon.
- Cost reduction is linked with profits (Technology improvement).
- Market's evolution varies with the adopted strategies.

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Private Information Model: No Player Pretending





**Private Information Model: No Player Pretending** 







#### Private Information Model: No Player Pretending





#### Private Information Model: No Player Pretending





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Private Information Model: Player 1 Pretending



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Private Information Model: Player 1 Pretending



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Private Information Model: Player 1 Pretending





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#### Private Information Model: Both Players Pretending





#### **Private Information Model: Both Players Pretending**



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The Multistage Game			

- In the first stage, players choose between the "Not Pretend" and the "Pretend" actions
- ► Afterwards, they interact repeatedly over the market for finite time period *T*.
- ► For a sufficiently large *T*, undiscounted average payoffs converge to the payoffs of the respective equilibria.

	Not Pretend	Pretend
Not Pretend	$\left(\frac{(A-2c_1+c_2)^2}{9B},\frac{(A-2c_2+c_1)^2}{9B}\right)$	$\left(\frac{(A-3c_1+2c_2)^2}{16B},\frac{(A-2c_2+c_1)^2}{8B}\right)$
Pretend	$\left(\frac{(A-2c_1+c_2)^2}{8B},\frac{(A-3c_2+2c_1)^2}{16B}\right)$	$\left(\frac{(A-3c_1+2c_2)^2}{12.5B},\frac{(A-3c_2+2c_1)^2}{12.5B}\right)$

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The Repeated Game			

- ► The multiperiod game (G<sub>T</sub>) is considered to be the stage game, with the repeated game having horizon kT.
- ▶ The stage game's equilibrium is replicated at the supergame.



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Example Market			

Stochastic Process:  $c_{i,(n)T} = c_{i,(n-1)T} \left( \mathbb{1}_{\{x \le X_n\}} \delta_n + \mathbb{1}_{\{x > X_n\}} \right)$ where  $x \sim U[0, 1]$  and  $X_0, K, \varepsilon$  exogenous constants.

$$X_{n} = \begin{cases} X_{n-1} + K \left( \sum_{t=(n-1)T}^{nT-1} \frac{\pi_{i,t}}{\pi_{i,t} + \pi_{-i,t}} \right), & c_{i,(n-1)T} = c_{i,(n-2)T} \\ X_{0}, & c_{i,(n-1)T} \neq c_{i,(n-2)T} \end{cases}$$

$$\delta_n = 1 - \varepsilon \frac{\sum_{t=(n-1)T}^{nT-1} \pi_{i,t}}{T \max_{q_i,t \in [(n-1)T, nT-1]} \pi_{i,t}}$$

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Example Market			

Favors the firm with the better outcome

Stochastic Process:  $c_{i,(n)T} = c_{i,(n-1)T} \left( \mathbb{1}_{\{x \le X_n\}} \delta_n + \mathbb{1}_{\{x > X_n\}} \right)$ where  $x \sim U[0, 1]$  and  $X_0, K, \varepsilon$  exogenous constants.

$$X_{n} = \begin{cases} X_{n-1} + K \left( \sum_{t=(n-1)T}^{nT-1} \frac{\pi_{i,t}}{\pi_{i,t} + \pi_{-i,t}} \right), & c_{i,(n-1)T} = c_{i,(n-2)T} \\ X_{0}, & c_{i,(n-1)T} \neq c_{i,(n-2)T} \end{cases}$$

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Stochastic Process: 
$$c_{i,(n)T} = c_{i,(n-1)T} \left( \mathbb{1}_{\{x \le X_n\}} \delta_n + \mathbb{1}_{\{x > X_n\}} \right)$$
  
where  $x \sim U[0, 1]$  and  $X_0, K, \varepsilon$  exogenous constants.

Probability increases proportionally with the

ratio of own profits to total profits

$$X_{n} = \begin{cases} X_{n-1} + K \underbrace{\left(\sum_{t=(n-1)T}^{nT-1} \frac{\pi_{i,t}}{\pi_{i,t} + \pi_{-i,t}}\right)}_{t=(n-1)T}, & c_{i,(n-1)T} = c_{i,(n-2)T} \\ X_{0}, & c_{i,(n-1)T} \neq c_{i,(n-2)T} \end{cases}$$

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$$\delta_n = 1 - \varepsilon \frac{\sum_{t=(n-1)T}^{nT-1} \pi_{i,t}}{T \max_{q_i,t \in [(n-1)T, nT-1]} \pi_{i,t}}$$

Reduction proportional to the ratio of profits made to best possible profits

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Profits' evolution in the long-run (n = 200, T = 100).

(both firms adhere to the same strategy for every stage game)



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Simulation Results			

Profits' evolution in the long-run (n = 200, T = 100).

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#### What we have done:

- We created a framework where players have the **ability to** pretend they have a different production cost.
- We defined the optimal "Behavioral Pretending Cost" for the case of Cournot Adjustment.
- ▶ We constructed a Multiperiod Game where "Both Players Pretending" is the N.E. in pure strategies.

# Interesting points:

- This equilibrium is a more competitive one, creates less profits for producers since they both pretend to have a lower cost and thus they end up overproducing.
- Respectively, the market price is lower than the Cournot level and gets closer to the perfect competition.

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# Different adaptation & Different extensions

Study the concept's implementation:

- to different adopting/learning techniques
- to greater number of players

Incorporate also:

- trigger strategies
- discounting factor
- infinite horizon

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#### Thank you for your attention!!

### Any questions?

### Chrysanthopoulos Nikos

nikoschrys@mail.ntua.gr

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