

# Leasing Policies of a Satellite Operator: Comparison of two Methodologies

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**Abstract**-In this paper we present a decision making tool that determines the business plan of a satellite operator. It is a mathematical model that evaluates different scenarios in terms of profitability and risk. The study was done for Hellas-Sat (the Greek satellite operator) and addresses a real situation. The international developments of the satellite market services have been taken into account since pricing data for different services and thus different capacity demand have been gathered. These data have been classified and statistically computed in the first part of the work. The outcome was a tool in the form of decision tree that determines the best leasing policies. The verification of the model was achieved by the development of another methodology based on dynamic stochastic formulation with similar results.

### III. INTRODUCTION

The potentiality of the revenue management in the area of telecommunication services has become obvious the last years. Some attempts have been made in creating models that quantify the optimal business plan of a telecommunication satellite operator. Current methodology mainly consists of evaluating the business plan of a satellite operator in terms of statistical computations. The evolutionary penetration of the satellite services and demand in the telecommunication market the last decades has emerge the need for a decision making tool for a satellite operator. We present a model that compares different scenarios of combinations of customers that want to hire of satellite capacity. There is a variety of services that a satellite can support. For each of these services, that a customer requests, there is a different bandwidth demand and a different pricing. The purpose of this work is to find the most profitable case for the incumbent operator. Each scenario leads to a decision tree. This maximization problem is described in terms of real and expected revenues, along with the corresponding probability of getting them.

The satellite operator needs to construct a business plan that will maximize the revenues. This is an optimization problem that addresses a real case. The ability of real time decisions has a significant effect to the viability and profitability of the company in the growing and competitive satellite market. The projected decision making model is an applied tool for managerial purposes which deals with real problems.

We created a mathematical model that will take into account the uncertainty and hence the risk. The model

includes the recording of possible states. Accordingly their quantification and analysis take place, considering the possible benefits but also the resulting risk. Depending on these data different alternative enterprising steps that the company might follow will result.

Apart from the determination of uncertainty the model is dynamic, incorporating the time evolution in the decision making. This characteristic involves the analysis of the risk of a decision in a competitive environment. This fact is compatible with the studied application of a satellite provider since it operates in a aggressive and growing market. The international developments of the satellite market have been taken into account since pricing data for different services and thus different capacity demand have been gathered.

We included in our work the development of another model. This model is based on dynamic programming formulation with stochastic elements [1], [3]. The evolution in time has also been included to the calculations. The stochastic nature of the problem depends on the satellite services demand and pricing at the beginning of the modeling, as long as at each intermediate time. Available data has been gathered to determine these estimates. The purpose of creating the second model is to verify the results and compare the applicability of the proposed model which turned out to be quiet satisfying.

Related theoretical works available in the open literature is not very common due to the relatively recent appearance of the satellite operator business, but nonetheless see refs. [2] and [4] for related work.

### IV. DESCRIPTION OF FRAMEWORK

This study has first involved the recording and the evaluation of the pricing data coming from the international market of leasing satellite capacity as well as the statistical treatment of those. The main features of these data are shown in Table 1.

The goal of the proposed model, implemented in the next stage, is to enable the decision maker to determine the best possible scenario for the satellite operator that is the scenario with the larger amount of income.

We start with the hypothesis that a satellite operator has different lease demands from different customers. Each customer wants to hire satellite capacity with certain bandwidth, time of lease and has a certain charge. The

outcome of the model gives guides to which combination of them is more profitable for the operator.

We begin our model by entering the characteristics of the possible customers which are: *Service* that is the type

TABLE 1  
PRICING DATA

Date (day-month-year)	
Satellite Operator	
Satellite name	
Service Application	
Purchaser type	
Frequency Band	
Power (Max e.i.r.p. dBw)	
Power (Min e.i.r.p. dBw)	
Total Bandwidth (in MHz)	
Price per MHz (per month)	
Duration (months)	
Contract Price	
Uplink Regions	
Downlink Regions	

TABLE 2  
INPUT PARAMETERS

Accession number of customer	Type of Service	t start (month)	t end (month)	Duration of lease (months)	Requested w (MHz)
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

of the requested service,  $t_{start}$ , that is the month of beginning of lease,  $t_{end}$  that is the month of end of lease and produces the requested *duration of lease* with respect to the time evolution,  $w$  (MHz), that is the requested bandwidth, and  $C$  (Euro) that is the corresponding cost of lease. In the structure of the model we have included the possibility of beginning the hire in different time periods (different months). Table 2 shows an example of the input parameters of the model.

The indicative cost for each service, was calculated using the data collected from the global satellite market. From the gathered data there was a classification of the possible services that a satellite operator could offer. These services correspond to different bandwidth demands and are presented in Table 3. The statistical analysis included the pricing for the equivalent capacity per 36 MHz per month and the duration of lease in months. For each service, the mean value and the standard deviation of these sizes was determined. Similarly, we did the calculations for the duration of hire as well as the cross-correlation of them.

At the third stage a dynamic programming formulation model has been developed. Apart from the heuristic approach, we included the stochastic discrete optimization approach to the methodology, developing a second model, in order to verify the preceding work. The comparison of the two methodologies showed similar results.

## V. MODEL FORMULATION

The model does the comparison of 10 at most customers with maximum asked time of lease 60 months. These limitations are considered to be a normal input for a satellite provider and easily can be expanded to more customers wanting satellite capacity for more than 2 years.

The number of all the combinations of ten customers in groups of ten is  $2^{10}$ . We take into account combinations of ten customers in groups of two or more. That leads us to 1028 combinations. All the possible combinations of customers are calculated. A description of these combinations is taking place in terms of possibility to implement or not.

A Negotiable Combination is the combination of customers that exceeds by 1 MHz at most the highest possible capacity that a transponder can serve i.e. 36 MHz, which can probably constitute an issue of negotiation between the provider and the consumer. A *Possible Combination* is the feasible combination of customers from the point of view of the maximum capacity of the transponder and a *Not Possible Combination* is a not feasible one. The next step is to decide the combinations which are the best to compare by examining the most promising ones. This is done by selecting the combinations with the highest amount of total occupied capacity, which is the sum of the requested capacity of each customer of the combination. Obviously the more bandwidth is occupied from a transponder the more profit will be for the firm. These are the '*Real Revenues*'.

The next step is to decide which of these combinations with high occupied capacity is the more profitable. The criteria that can be used to lead to the optimum choice of combination are:

- The amount of *Real Revenues*, which are the revenues that an operator will gain from the hire of capacity to the customers of each combination.
- The calculation of additional expected future profits for the satellite operator taking into consideration standard deviations of the prices and consequently the corresponding risk.

Each combination has different maximum requested time of hiring the capacity. Therefore in order to be able to properly compare the scenarios it is necessary to reduce both of them to the same period of time i.e. at the same

TABLE 3  
TYPES OF SERVICES

Services
VSAT
Telephony
IP Gateway
Corporate
Broadcast
Video Contribution
Media company
Government

maximum month of hiring, and then to calculate the additional possible income that we can acquire from this left over free capacity that is called ‘Remaining Capacity’ (*C Remaining*). It is also possible that at specific months not all the available capacity of the transponder of the satellite will be occupied with each combination. This leads to the undesirable fact of not having maximum occupancy of the transponder of the satellite in each month. So the satellite operator could probably hire this available capacity to another possible future customer that is not included to the combination and gain more revenues. We call this ‘Empty Capacity’ (*C Empty*).

#### A. Calculation of revenues from *C Remaining*

For each scenario the possible revenues from the potential leasing of the Remaining Capacity are calculated. This calculation of the theoretical potential revenues is categorized depending on the type of service. Therefore the decision tree as shows in Fig. 1 arises for each scenario.

The Expected Values of theoretical income for the Remaining Capacity depending on the type of service are being calculated as:

$$\text{TheRevCrem}_{(PC_f),s} = t_{left(PC_f)} \cdot \overline{m2}_s$$

The probable revenues for each service which are called Theoretical Revenues are calculated and categorized for each service as:

$$\text{ExpValCrem}_{(PC_f),s} = P_A \cdot P_{Bs} \cdot t_{left(PC_f)} \cdot \overline{m2}_s$$

Where  $P_{Bs}$ , is the probability of appearance of a customer in 1 month asking for service  $s$ ,

$\overline{m2}_s$ , is the mean value price per 36 MHz per month, for each service  $s$ , referring to bandwidth of 36 MHz,

$t_{left(PC_f)}$ , is the time interval for each combination

between the max requested time by the customers of the combination until the max requested time of all compared scenarios,

$P_A$ , is the probability of appearance of a customer asking for satellite services.

The Expected Values of revenues is an intermediate calculated volume that is used only for comparison reasons of the Scenarios enclosing the corresponding risk while the size Theoretical Revenues is the real amount of money in Euros that can be acquired following each brand of the decision tree.

The standard deviation for the Expected Values and the Theoretical Revenues for each service are also calculated in order to have an estimation of the risk involved.

All the calculations of probabilities as well as the mean values of the prices of lease and standard deviations of prices are based the real data the international market.

#### B. Calculation of revenues from *C Empty*

We proceed analogously calculating all possible revenues (Expected Values and Theoretical Revenues) that can result from the leasing of the ‘Empty Capacity’ up to the 36 MHz.

This calculation is categorized depending on the bandwidth that is not used during each time period of hire

for the specific Scenario and could potentially being hired.

The selected ranges of capacity in MHz, as shown in Fig. 1, along with the corresponding probability of appearance of a customer in one month asking for each BW range has been statistically computed by the available gathered data. A decision tree arises for each scenario.

Where the Expected Values of theoretical income for the ‘Empty Capacity’ depending on the not-leased bandwidth each time period are being calculated as:

$$\text{TheRevCempty}_{(PC_f)} = \sum_{j=1}^{60} \text{Cempty}_{(PC_f),j} \cdot \overline{m1}_n$$

While the Theoretical Revenues depending on the number of MHz that are not used results as:

$$\text{ExpValCempty}_{(PC_f)} = P_A \cdot \sum_{j=1}^{60} P_{Cn} \cdot \text{Cempty}_{(PC_f),j} \cdot \overline{m1}_n$$

Where  $\text{Cempty}_{(PC_f),j}$ , is the Empty Capacity for each

combination of customers ( $PC_f$ ) for the  $j$ -th month,

$\overline{m1}_n$ , is the mean value price per MHz per month asking for the  $n$ -th range of bandwidth,

$P_{Cn}$ , is the pprobability of appearance of a customer asking for the  $n$ -th range of bandwidth

The standard deviation for the Expected Values and the Theoretical Revenues for each bandwidth are also calculated.

#### C. Dynamic Programming Formulation

A discrete time stochastic dynamic programming formulation has also been created. We consider all the possible combinations of customers and we determine the most profitable ones. This model takes into account the state of the system  $x_k$  and summarizes past information that is relevant for future optimization, the control  $u_k$  which is the decision variable to be selected at each time  $k$ , and the disturbance  $w_k$  which is the unknown noise parameter.

As the state of the system  $x_k$  we take the sum of the capacity of all of the customers of each combination, from the time  $k$  and forward. We consider as  $u_k$  the control of selling the Empty Capacity or not. So  $u_k$  could take the values 1 or 0. The noise factor  $w_k$  is less or equal to the Empty Capacity, and it is the amount of the Empty Capacity that the operator decides to lease at each time period.

The system has the form

$$x_{k+1} = f_k(x_k, u_k, w_k)$$

with  $N$  the time horizon

The closed loop optimization model has the form of minimization of the expected cost function including future costs:

$$\min E \left\{ g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k) \right\}$$

with  $-g_N(x_N)$  the Total Expected Revenues for the satellite operator.

The dynamic programming formulation takes the form of

$$J_N(x_N) = g_N(x_N)$$

$$J_k(x_k) = \min_{u_k, w_k} E \{ g_k(x_k, u_k, w_k) + J_{k+1}(f_k(x_k, u_k, w_k)) \}$$

### VI. CONCLUSIONS

All of these evaluated data are presented in the form of a unified decision tree (Fig. 1), which is the output of the model.

The amount of Total Expected Revenues that will estimate the optimum policy for the firm is the sum of all potential revenues. This sum consists of the Real Revenues, plus the Expected Value resulting from the leasing of the Empty Capacity plus the Expected Value resulting from the leasing of the Remaining Capacity

including their standard deviations.

The Total Expected Revenues gives a range of values, defining the best and worst case scenario for the revenues of the satellite operator.

$$\begin{aligned} \text{Total Expected Revenues} = & \text{Real Revenues} + \\ & (\text{Expected Value from } C \text{ Empty} \pm \\ & \text{Standard Deviation Expected Value } C \text{ Empty}) + \\ & (\text{Expected Value Revenues from } C \text{ Remaining} \pm \\ & \text{Standard Deviation Expected Value } C \text{ Remaining}) \end{aligned}$$

The real amount of money that will possibly result to the satellite operator is:

$$\begin{aligned} \text{Total Revenues} = & \text{Real Revenues} + \\ & (\text{Theoretical Revenues from } C \text{ Empty} \pm \\ & \text{Standard Deviation Theoretical Revenues } C \text{ Empty}) + \\ & (\text{Theoretical Revenues from } C \text{ Remaining} \pm \\ & \text{Standard Deviation Theoretical Revenues } C \\ & \text{Remaining}) \end{aligned}$$

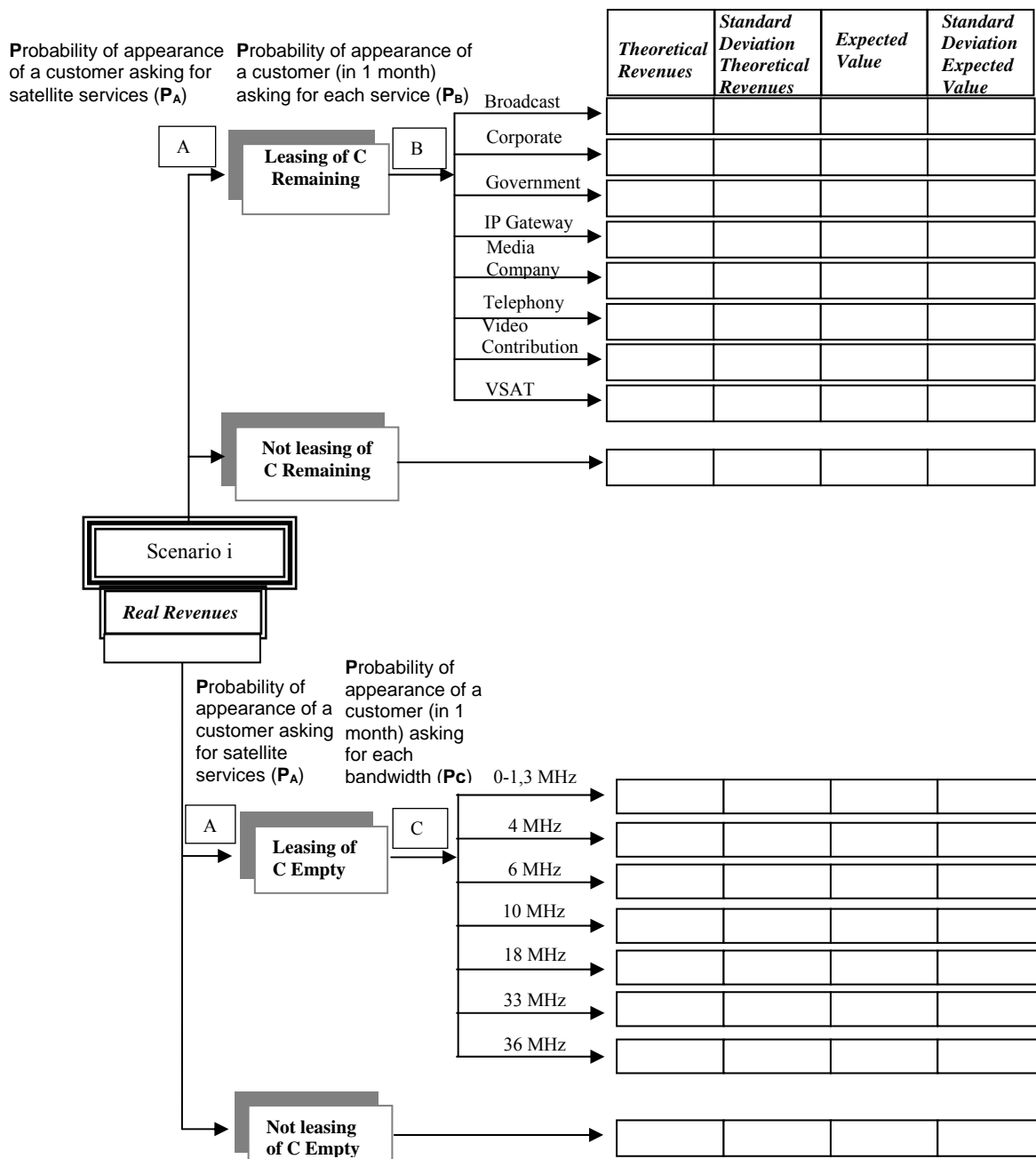


Fig. 1 Output of the model Decision Tree

This gives the opportunity to the decision maker of the satellite provider to determine the most profitable combination now and in the future. This decision depends each time period on the extent of risk the firm is willing to take and the certain policy that wants to apply.

The output of the proposed models, both the heuristic and the dynamic programming, has shown similar results. For different input parameters, i.e. different customers and specification demands, the most profitable combinations of customers have been proved similar. The simulation process of both models verified the methodology and gave a strong confidence of the work. More numerical results of the application of the two methodologies will be presented in the Conference.

The usefulness of the model and analysis presented here for any satellite operator is clear. They could also be useful to related type of activities where leasing for

specific volumes to customers is the essence of the business enterprise.

Finally let us note that a more extended version of the dynamic programming formulation and solution will be available in the forthcoming PhD Thesis of the first author, together with the related code.

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