An Adaptive Game for Pay-as-Bid and Uniform Pricing Power Pools Comparison

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ABSTRACT: In the present paper we model a deregulated wholesale power market, using an adaptive game in order to study and compare the behavior of the market under two different auction rules: uniform pricing and pay-as-bid. The market consists of an Independent System Operator (ISO) and Power Generators (Players) who submit their offers to the ISO in the form of curves. The ISO purchases energy from the generators in order to cover the electricity demand. The power generators do not know the costs, the offers and the payoffs of their competitors and therefore they use an adaptive learning tool to compensate their lack of knowledge trying to maximize their profit. Sequential iterations of the game, conducted for different number of players and for both pricing systems, have shown that electricity price under uniform pricing decreases intensely as the market tends to perfect competition, while discriminatory pricing seems to be more effective in oligopoly. Furthermore, wide variations in market shares amongst generators have been recorded under uniform pricing, while pay-as-bid seems to lead in rather uniform market shares distributions.

Keywords: Adaptive Learning, Power Markets, Deregulation, Game Theory, Pay-as-Bid, Uniform Pricing.

I. INTRODUCTION

The lately announced reforms of the operating framework in the recently deregulated Greek power market combined with the related experience from other deregulated power markets worldwide, makes more intense the necessity for further study of these markets, their operation and the behavior of their participants. Power generators, challenged to act in this new business environment characterized by imperfect information and absence of historical data, are interested mainly in issues such as strategy formulation and risk minimization, while regulatory authorities and organizations, designated to ensure security of supply and operating efficiency, address issues concerning the formation and the optimization of the regulatory framework, competition and price mechanisms [1, 2].

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There have been recorded so far many significant efforts coping with the newly emerged issues concerning the deregulated electricity markets [3] and a considerable number of them that use game theoretical models [4-6] to approach various market structures with different rules and assumptions.

The present paper focuses on the comparison of two different auction rules applied in a centralized wholesale power market: uniform pricing and pay-as-bid. Therefore, we are introducing a different approach in order to simulate and study electricity markets, using a power market model where players with adaptive learning skills act in an incomplete information environment. More specifically, we simulate a deregulated power market consisting of an Independent System Operator and independent power generators who know only their own cost, previous offers and corresponding payoffs. Generators use a stochastic learning algorithm, in order to maximize their profit. Each generator readjusts its offer by increasing, decreasing or keeping constant its offer curve parameters. Randomly chosen values from a probabilistic profile of behavior determine the readjustment of the offer for each generator. This behavior profile is being gradually and continuously formed, by appraising the impact of the last readjustment of the offer curve, to generator's income. In fact, we are facing a Nash game [7] where players don't know each other's costs, actions or payoffs and therefore they use an adaptive learning scheme to counterbalance their lack of knowledge [8-10].

The impact of the number of participants on electricity price, in a market consisting of generators with similar attributes, i.e. cost, capacity range, adaptation and reaction capability, is examined for both pricing systems. Namely, higher price levels are recorded under uniform pricing when the market functions as oligopoly, while an intense decreasing trend that leads prices below the corresponding pay-as-bid levels, is observed as the market tends to perfect competition. Moreover, the market shares that generators obtain, present wider variance under uniform pricing than pay-as-bid, reflecting, at the same time, their in-between differences in the production cost, i.e. generators with lower marginal cost obtain respectively higher market shares.

II. DESCRIPTION OF THE GAME

A general theoretical model of a power market, based on some simplified assumptions, has been developed in order to apply the learning process and study through it the behavior of the deregulated market under the two different pricing systems.

A. The Market

The modeled power market consists of:

1) i Power Generators (Players) with a capacity range $[x_{i_{\min}}, x_{i_{\max}}]$ that defines: (*i*) the technical minimum, below which the *i*-generator cannot operate and (*ii*) the maximum output that each generator can produce. The total generators capacity exceeds the expected demand.
2) an Independent System Operator (ISO), whose aim is to cover the demand D at the lowest possible cost and therefore purchases electricity from the power generators evaluating their offers,

The total cost of the *i*-generator is a quadratic function of the following form:

$$TC_i(x) = FC_i + a_i x + b_i x^2 \tag{1}$$

where FC is generation's fixed cost and a_i, b_i the cost coefficients (a_i, $b_i > 0$).

The generators submit their offers in the same form as of their marginal cost[1], i.e. an increasing linear function. Each generator submits an offer for its whole capacity range in a way that the offered price, at any output level, does not exceed the upper bound that has been set by the Independent System Operator (Price Cap). Price Cap is determined approximately as a multiple of the price where power generators would equilibrate if they submitted their actual marginal cost. At each round, which corresponds to a short period of time, generators can modify only one of their offer curve coefficients, by increasing, decreasing or keeping constant its value. Their choice is randomly made using a probabilistic distribution of the potential actions, which is gradually formed through their experience.

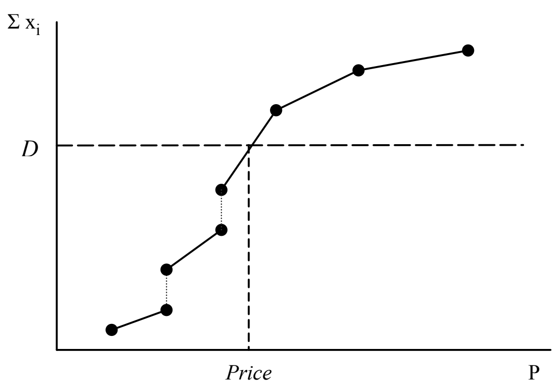

Figure 1. Aggregate Supply Curve and Market Clearing Price Assessment.

Assuming, for simplicity reasons, an inelastic demand with constant value throughout the game, the ISO receives the offer curves of all generators and constructs the aggregate supply curve. The point where supply curve crosses demand defines the market-clearing price (Fig.1) and, thus, the ISO allocates the production to generators in the most efficient way. However, discontinuities that may appear in the aggregate supply curve due to constraints set by generators capacity range and the fact that there is no demand side bidding, may oblige the dispatching of a generator at its technical minimum even though the production then, exceeds the demand. In Fig.2 we illustrate a System with three power generators who submit offers for their range of capacity and the ISO covers the demand D, purchasing, only from two of them (x_2 and x_3 respectively, such that: $x_2 + x_3 = D$).

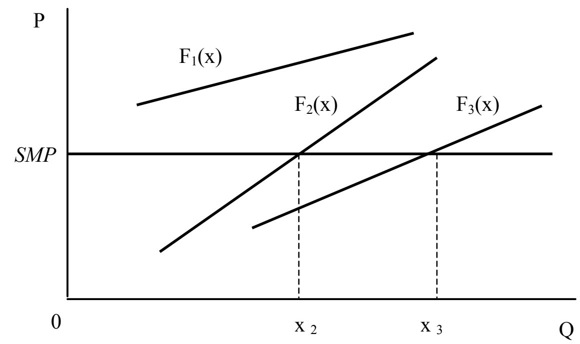

Figure 2. System Marginal Price and Power Production Allocation.

There are two different pricing regimes that are examined in the present paper: uniform pricing and pay-as-bid. In the uniform pricing all generators are paid the whole quantity they sell to the ISO at the System Marginal Price (SMP), which is the price of the most expensive power unit allocated. In the pay-as-bid generators are paid each power unit at the offer price they declared.

B. The Learning Process

During the game, generators submit offers for n sequential rounds and remain into the game even if they don't manage to get a market share for a long time. They do not know each other's costs, offers and payoffs and at the end of each round are acquainted only with the market-clearing price and with their own market share and revenue. They compare their outcome, in terms of profit, with the one they obtained in the previous round and if it is better they reward the last randomly chosen action by increasing its probability in the probability distribution of potential actions. Otherwise, they decrease that probability value. New randomly chosen values from the adjusted probability distribution determine the next offer.

The sizes of the alterations in the actions' probability values and in the values of the offer curve coefficients are defined at the beginning of the game and they are called *steps*. Thus, during the game each player gradually forms a probabilistic profile regarding its potential moves, which is actually a behavioral tool based on its recent experience, guiding him to react proportionately to different market's trends.

The first offer that the players submit to the System Operator is their actual marginal cost and has the following form:

$$F_{i_1}(x) = A_{i_1} + B_{i_1} x \tag{2}$$

$$\text{where } A_{i_1} = a_i \quad \text{and} \quad B_{i_1} = 2b_i \tag{3}$$

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[1] $MC_i(x) = a_i + 2b_i x$

The market-clearing price and the dispatched generation (x_{i_1}) for each generator are then calculated. The corresponding net revenue for each player in uniform pricing is:

$$J_{i_1} = x_{i_1} \cdot \mathrm{SMP}_1 - TC_i\,(x_{i_1}) \qquad (4.a)$$

while in pay-as-bid is:

$$J_{i_1} = (x_{i_{\min}} \cdot F_{i_1}(x_{i_{\min}})) + \int_{x_{i_{\min}}}^{x_{i_1}} F_{i_1}(x)\, dx \; - TC_i\,(x_{i_1}) \qquad (4.b)$$

At the end of the round, generator *i* knows only the market-clearing price and the quantity x_{i_1} the ISO purchased from it.

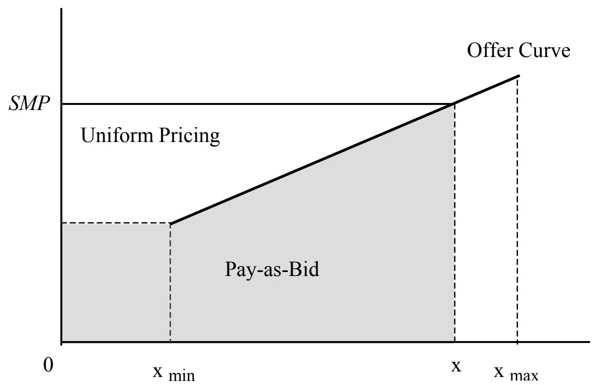

Figure 3. Mode of payment in Uniform Pricing and Pay-As-Bid.

At the beginning of the next round, generators may modify their offer, before they submit it to the ISO by changing the values of the coefficients of their offer (A_i, B_i). Players can modify just one of the coefficients, at each round, and only the same coefficient for a predefined number of sequential rounds (modification period). The duration of these periods may vary per player and per coefficient and it is assigned at the beginning of the game. The modification of the in turn coefficient consists in the increment or decrement of the coefficient's value by a percentage equal to the corresponding step (e_{Ai} or e_{Bi}). A third option players have is to maintain the same value of the coefficient (stabilization). We can assign different step values per player and per cost coefficient in order to portray the differentiation in players' reaction capability. The action (increase, decrease or stabilization) to be followed is randomly selected from a probability distribution of values corresponding to each action. Therefore, to each coefficient per player correspond three probability values P^{in}, P^{de}, P^{st} respectively, such that for player *i*:

$$P_{i_A}^{\;in} + P_{i_A}^{\;de} + P_{i_A}^{\;st} = 1 \qquad (5.a)$$

$$P_{i_B}^{\;in} + P_{i_B}^{\;de} + P_{i_B}^{\;st} = 1 \qquad (5.b)$$

The initial, arbitrarily defined, probability distribution of the three actions for each coefficient might not necessarily be equiponderant regarding the actions. During round *n,* the randomly selected action, depending on the in turn coefficient's modification period, defines the new coefficient values of the offer to be submitted to the System Operator, as follows:

modification period for A		*modification period for B*	
$A_{i_n} = A_{i_{n-1}} \cdot (1+\varepsilon)$	or	$A_{i_n} = A_{i_{n-1}}$	(6.a)
$B_{i_n} = B_{i_{n-1}}$		$B_{i_n} = B_{i_{n-1}} \cdot (1+\varepsilon)$	(6.b)

where ε is either:

- e_{A_i} or e_{B_i}, if the selected action is increase
- $-e_{A_i}$ or $-e_{B_i}$, if the selected action is decrease
- 0, if the selected action is stabilization.

The net revenue $J_{i\,n}$ for player *i* resulting after round *n* is compared with the net revenue $J_{i\,n-1}$ of the previous round and the probability distribution of player's available actions is then adjusted. If, for player *i,* the difference ($J_{i_n} - J_{i_{n-1}}$) corresponding to two sequential rounds is positive then the probability value of the selected action in round *n* is increased (reward) by a predefined step θ, expressed as a percentage, and the probability values of the other two actions are equally decreased. In case that the net revenue is inferior to the one of the previous round the probability value of the selected action is decreased (punishment) by the same step θ and the probability values of the other two actions are equally increased. Step size can be different per player, signifying diversification in players learning capability.

III. AN APPLICATION OF THE GAME

Based on the power market model and the game described above we applied a limited version of the game with specific features in order to test the model and extract some general conclusions. Therefore, some parameters of the game were defined as of static nature while differentiation among players was minimized.

More specifically, we assumed that electricity demand *D* remains constant throughout the game and each time is equal to the half of the summation of maximum outputs of all the players participating in the game. Generators have the same generation capacity range (5 MW - 15 MW), they all use the same fuel and the same generation technology (e.g. small oil-fired steam plants). Their fixed cost FC_i and cost coefficients (a_i, b_i) are randomly spread within an interval ±25% from the corresponding values of the first player. For player #1 we consider:

$FC_1 = 7{,}000 \qquad a_1 = 8.40 \qquad b_1 = 0.00020$

All players are considered as equivalent regarding their adaptation and learning capability, and therefore the values of steps *e* and θ are equal for all players and all coefficients in each game. The initial values of probability P^{in}, P^{de}, P^{st} that correspond to the three actions are taken also equal per player and per coefficient while they vary per action:

$P_i^{\,in} = P_i^{\,de} = 35\%$ and $P_i^{\,st} = 30\%$

The number of consecutive iterations defining each coefficient modification period for each player is randomly selected from a common interval of values between 30 and 80 offers. The random values applied in the first game remain the same for all the repeated games. Price Cap is set up to 100, which is approximately ten times more than the initial market-clearing price.

Every game consists of 32,000 consecutive offers (iterations) and experience gained from these offers is used only during the current game, while repetitions of the same game start from zero point regarding players' experience. Ten (10) different game types, with 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 players respectively were simulated for both pricing systems, assigning the value of 5% to steps e and θ, for all players and in all game types. Each game type has been repeated 100 times, raising thus the total number of conducted games to 2,000 and the total number of rounds to 64,000,000.

In the 2-players game type participate only the first two players (#1 and #2) and each time we proceed to the next game type (e.g. 4-players) we add the next two players in row (e.g. #3 and #4). Thus, the first two players take part in all the executed games while the last two players (#19 and #20) only in the 20-players game type.

For every game and iteration the following are recorded:

i) Electricity Price
ii) Market shares x_i and net revenues for each generator
iii) Offer curve coefficients A_i, B_i
iv) Surplus Capacity purchased by the ISO and corresponding cost
v) Moving averages of all the precedents

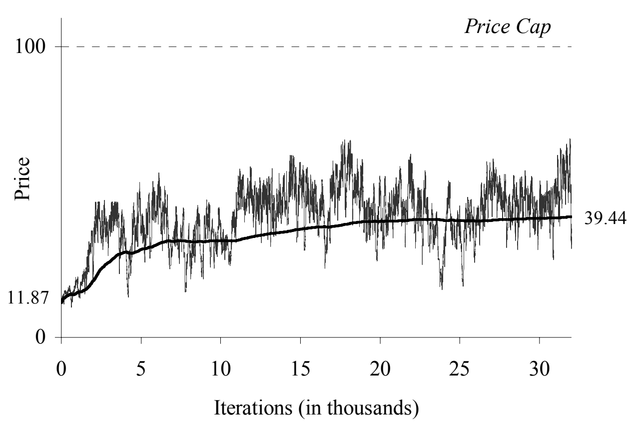

Figure 4. Electricity price and its moving average for a 6-players game under uniform pricing.

As electricity price is defined the average cost of the purchased energy for the ISO, which is identical to market-clearing price only in uniform pricing and not in pay-as-bid. The output values that result in the first round of each game, when offers are actually equal to the marginal cost, are the reference values for all the necessary comparisons.

IV. RESULTS AND OBSERVATIONS

The most interesting and important issue arising from our analysis is the behavior of electricity price. The common element in the results of our simulation is that for both pricing systems price converges in much higher value levels than the one that would result if the offers were equal to generators marginal cost (marginal price). However, in both systems, price decreases as the number of generators increases, i.e. as market tends to perfect competition, though demand and total available capacity are always in the same proportion. Nevertheless, the way price decreases with the increase of competition significantly varies in the two pricing systems. Uniform pricing presented higher sensitivity to competition as, in oligopoly, electricity price converged at higher level under uniform pricing than in pay-as-bid, while a more competitive market resulted in the inverse situation.

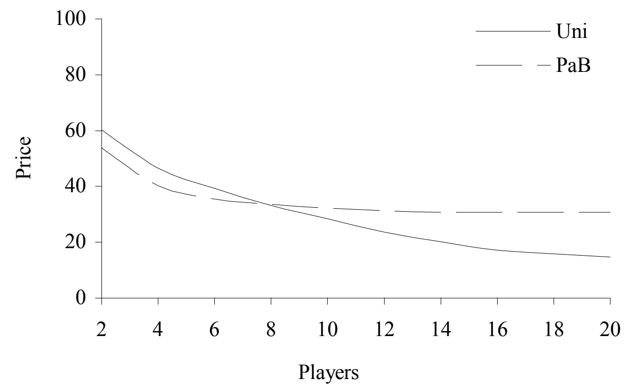

Figure 5. Average electricity price for all game types in both pricing systems.

Namely, under uniform pricing, the electricity price from 6 times the marginal price decreased to 1.5 times, as the market moved from an oligopolistic (2 players) to a more competitive state (20 players). Under the pay-as-bid auction rule, electricity price converges to level of 5 times the marginal price in oligopoly (2 players), while it remains practically stable to about 3 times the marginal price, after a certain level of competition (8-10 players). The same level of competition is also the turning point in the behavior of price mentioned above, i.e. where uniform pricing starts to seem more effective than pay-as-bid[2].

Similar patterns have been recorded for the average values of the offer coefficients for the different game types and both pricing systems. The offer coefficients values are always lower under uniform pricing than in pay-as-bid in the corresponding game type. The essential difference, in the curve form, between the two regimes is that the curve's slope is greater in uniform pricing while the constant coefficient smaller. In that way, generators offer their first quantities at low prices, even lower than of their marginal cost, in order to get into the market and when it comes to the upper part of their capacity range, they increase sharply their offer trying to lead electricity price in higher levels for to take advantage from a high clearing price since they are paid at that price their whole dispatched generation.

[2] These results have mainly qualitative value and we refer to the figures only for better visualisation.

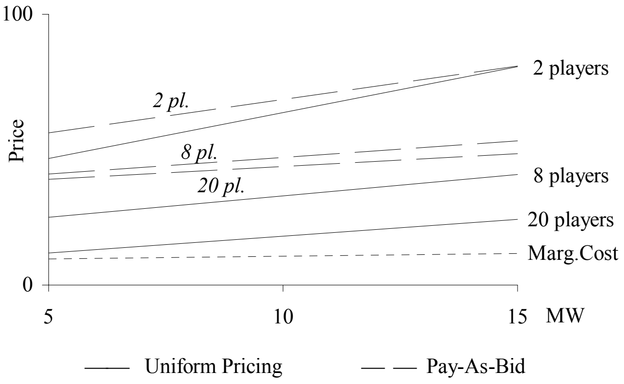

Figure 6. Average offer curve in 3 game types for both pricing systems

The wider range of price values that has been recorded under uniform pricing, in the different game types, was also observed in the values of the offer coefficients. Namely, coefficient's *A* average value in uniform pricing varied from 3.6 to 0.6 times the average value of all players a_i coefficients, while in pay-as-bid the corresponding range is from 5.2 to 4.1 times. Regarding coefficient *B,* under uniform pricing, its average value varies from 17 to 6 times the average value of all players' b_i coefficients, while in pay-as-bid the corresponding range is from 12 to 5 times. In Figure 6 are illustrated the average offer curves submitted by the generators in the 2-players, the 8-players and the 20-players game types for both pricing systems and the average marginal cost of all players, as well.

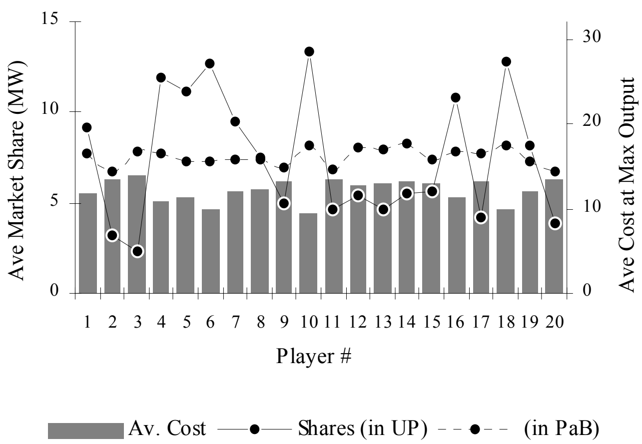

Figure 7. Average market shares of each player in the 20-players game type under different pricing systems, compared to the corresponding average production cost.

Another interesting observation concerns the distribution of market shares amongst the generators. Under the pay-as-bid regime the generators seems to share almost equally the market. There is no significant variance in the market shares and it does not seem to exist a relation between the market share of the players and their cost of production, which is rather prospective since there are no great differences in the production costs and all of them use the same generation technology. On the other hand, uniform pricing seems to create intense unequalities in the market sharing as a result of the competition. Moreover, the differentiation pattern in generators production cost is reproduced, inverted and magnified, in the market shares distribution, as it is explicitly illustrated in Figure 7.

Finally, limited number of additional runs, conducted without the Price Cap constraint, shown that electricity price does not converge in that case, as players, through their offers, lead the price continually at higher levels, for their own benefit. The relation of Price Cap level and price convergence, quantitatively defined, introduces thus an interesting issue for further research. Another interesting field, currently studied, is the behavior of a market consisting of generators with wider differentiation of their technical attributes, i.e. generation technology, costs and capacity range or other attributes such as adaptation capability, intentions and attitude to profit.

VI. CONCLUSIONS

The basic conclusion is that in a deregulated wholesale market, where electricity price is determined by an offer-based procedure, the increase of competition leads prices downwards in both pricing rules, though in a different manner. In uniform pricing, the effect is more intense since price falls to one third as market shifts from oligopoly to perfect competition. In oligopoly, pay-as-bid is more efficient than uniform pricing but price does not decrease at the same rate as competition increases, and further, it remains stable after a certain point, regardless of the number of generators participating in the market. The form of the offer curves in the two pricing rules seems to underlie this differentiated market behavior. Under uniform pricing generators end in more competitive and aggressive offer curves, characterized by low initial values and steep slope, while in pay-as-bid we observe more flat offer curves which after a certain market size do not seem to serve competition and market efficiency. Additionally, as a result of the different attitude of generators in the two regimes, the market shares distribution amongst them varies as well. The almost uniform allocation of power production to all generators in pay-as-bid contrasts with the intensely unequal market shares, that results under uniform pricing, reflecting and accentuating, thus, the differences between generators production cost.

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VII. BIOGRAPHIES

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