An Adaptive Game for Electricity Markets

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Abstract: In the present paper we describe an adaptive game applied in a deregulated power market consisting of an Independent System Operator (ISO) and Power Generators (Players) who submit their offers to the ISO in the form of curves. The ISO purchases energy from the generators, starting from the more economical offer in order to cover the electricity demand. The power generators do not know the costs, the offers and the payoffs of their competitors and therefore they use an adaptive learning tool to compensate their lack of knowledge trying to maximize their profit. Sequential iterations of the game are being executed with different parameter settings in order to study mainly the impact of the number of participants on the market's clearing price in parallel with the way they alter their offers during the game. After repeated runs of the simulation model under specific market rules and conditions, many interesting phenomenona have been observed regarding the behavior of the players and the stability of the market.

Key words: Adaptive Learning, Game Theory, Power Markets, Deregulation

1. Introduction

The on-going restructuring of the energy markets in the western world and the opening of the Greek power market to competition combined with the recent experience of California's electricity market crisis, makes more intense the necessity for further analysis of the energy markets and the behavior of their participants in the new deregulated environment. Power generators, challenged to act in this new business environment characterized by imperfect information and absence of historical data, are interested mainly in issues such as strategy formulation and risk minimization, while regulatory authorities and organizations, designated to ensure the regular operation and supply of the market, address issues concerning the formation and optimization of the regulatory framework, competition and price mechanisms [1, 2].

There have been recorded so far many significant efforts coping with the newly emerged issues concerning the deregulated electricity markets [3, 4] and a considerable number of them that use game theoretical models [5-9] to approach various market structures with different rules and assumptions. The present paper is introducing a different approach in order to study electricity markets, using a power market model where players with adaptive learning skills act in an incomplete information environment. More specifically, we simulate a deregulated power market consisting of an Independent System Operator and independent power generators who know only their own cost, previous offers and corresponding payoffs. Generators use a stochastic learning algorithm, in order to maximize their profit. Each generator readjusts its offers by increasing, decreasing or keeping constant each one of his offer curve's parameters, for a stated number of turns. Randomly chosen values from a probabilistic profile of behavior define the readjustment of the offer for each generator. This behavior profile is being gradually and continuously formed, by appraising the impact of his last readjustment of the offer curve, to its income. In fact, we are facing a Nash game [10, 11] where players don't know each other's costs, actions or payoffs and therefore they use an adaptive learning scheme to counterbalance their lack of knowledge [12-15]. The present paper focuses on the impact of the number of participants on the market's clearing price and on the way players alter their offers during the game.

The basic conclusion is that participants in a power market where prices are defined on an offer-based procedure, tend to lead prices, through their offers, at levels significantly higher than these of their real marginal cost. However, prices gradually decrease as the number of players increase, i.e. as competition increases. It was also observed that power generators tend to offer their first quantities at prices lower than their marginal cost trying to take a greater market share, while they pass sharply to considerably higher price levels when it comes to the upper part of their capacity range. In that manner players try to lead the market to a higher clearing price and take advantage of it, as their whole dispatched generation is sold at that price. Moreover, competition tends to increase the intensity of this particular behavioral pattern.

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2. Description of game's general model

A general theoretical model of a power market, based on some simplified assumptions, has been developed in order to test the learning process and study through it the behavior of participants. The modeled power market consists of:

a) an ***Independent System Operator*** who buys electricity from the power generators according to their submitted offers, in order to cover the demand D.

b) i ***Power Generators*** (*Players)* who have a capacity range $[x_{i_{\min}}, x_{i_{\max}}]$ and their *marginal cost* is a quadratic[1] function:

$$MC_i(x) = a_i + \beta_i \cdot x + \gamma_i \cdot x^2 \qquad \text{(cost parameters: } a_i, \beta_i, \gamma_i > 0\text{)}, \tag{1}$$

The total power generators' capacity covers the expected demand:

$$\sum_{i=1}^{i} x_{\max_i} > D \tag{2}$$

They submit their offers in the same form as of their marginal cost by slightly increasing, decreasing or keeping constant the cost coefficients. Their choice is randomly made and is initially defined by an arbitrary probabilistic distribution of their potential actions.

The Independent System Operator sets an upper bound for the offered prices (*Price Cap*), which is calculated approximately as a multiple of the price where power generators would equilibrate if they submit their marginal cost. Each generator submits his offer for his whole capacity range to the ISO in the form of a monotonically increasing and continuous quadratic function such that the offered price does not exceed Price Cap at any level of production. The players do not know each other's costs, offers and payoffs.

System's Marginal Price λ is calculated on a least-cost base (the offered cost of the last and most expensive kWh dispatched) from the generators' offers in respect of the demand. Generators are paid at this price the whole quantity they sell to the System Operator.

In Fig.1 we illustrate a System with two power generators who submit offers for their range of capacity and the System Operator covers the demand D, buying x_1 and x_2 respectively ($x_1 + x_2 = D$) at the *SMP*.

Generators submit offers for n sequential rounds and remain into the game even if they don't manage to get a market share for long periods of time.

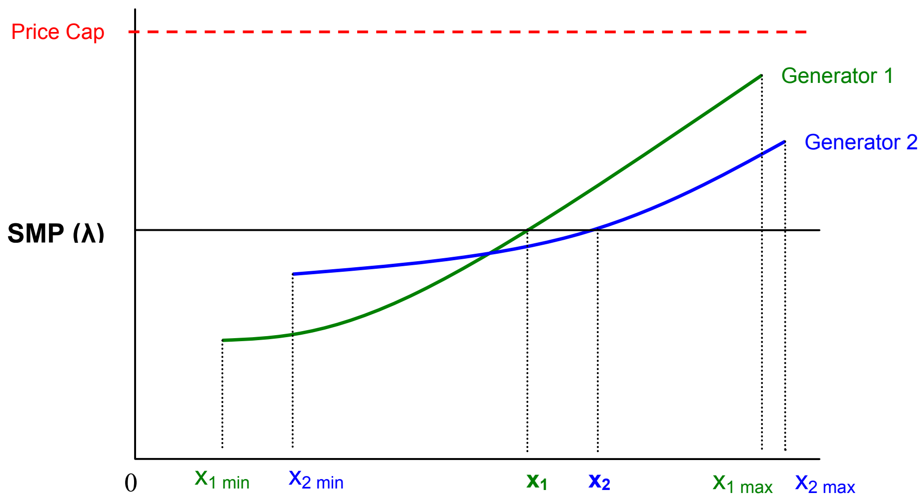

Figure 1. Calculation of SMP and corresponding market shares in a system with two power generators.

[1] Though linear functions describe sufficiently the marginal cost of a power generator, we have here selected the quadratic form in order to give players more tactical flexibility in the formation of their offers during the game.

At the end of each round, players are acquainted only with the market share they obtained and the SMP. They compare the result, in terms of profit, with that of the previous round and if it is better they reward the last randomly chosen action by increasing its probability value in the probability distribution of the potential actions. Otherwise, they decrease that probability value. New randomly chosen values define the next offer curve. The sizes of the alterations in the actions' probability values and in the values of the offer curve's coefficients are defined at the beginning of the game and they are called *steps*. Thus, during the game each player gradually forms a probabilistic profile regarding his potential moves, which is actually a behavioral tool based on his recent experience, allowing him to react proportionately to different market's trends.

Starting the Game

The first offer that players submit to the System Operator is actually their marginal cost with the cost parameters α_i, β_i, γ_i , increased by a small percentage $e_{A_i}, e_{B_i}, e_{C_i}$ respectively (step). The value of these steps designate the cost parameters' variation ability during the game and it is assigned before the game starts. We can assign different step values per player and per cost coefficient in order to portray the differentiation in players' reaction.

The adjusted coefficients:

$$A_{i_1} = a_i \cdot (1 + e_{A_i}) \tag{3.a}$$

$$B_{i_1} = \beta_i \cdot (1 + e_{B_i}) \tag{3.b}$$

$$C_{i_1} = \gamma_i \cdot (1 + e_{C_i}) \tag{3.c}$$

define the first offer curve for each power generator:

$$F_{i_1}(x) = A_{i_1} + B_{i_1} \cdot x + C_{i_1} \cdot x^2 \tag{4}$$

System's Marginal Price (λ_1) and dispatched generation (x_{i_1}) for each generator are then calculated. The corresponding net revenue for each player is[1]:

$$J_{i_1} = x_{i_1} \cdot \lambda_1(x_{i_1}) - \int_0^{x_{i_1}} MC_i(x)dx \tag{5}$$

At the end of the round, generator *i* knows only the SMP and the quantity x_{i_1} the ISO bought from him.

Repeated Rounds

At the beginning of the next round generators may modify their offer, before they submit it to the System Operator, by changing the values of the coefficients (A_i, B_i, C_i). Players can modify ***one*** of the three coefficients of their offer curve at each round and only the same coefficient for a determinant number of sequential rounds (*modification period*). The duration of these periods is different per player and per coefficient and it is defined or randomly assigned at the beginning of the game.

The modification of the in turn coefficient consists in the increment or decrement of the coefficient's value by a percentage equal to the corresponding step (i.e. e_{Ai}). A third option players have is to maintain the same value of the coefficient (stabilization). The ***action*** (increase, decrease or stabilization) to be followed is randomly selected from a probability distribution of values corresponding to each action. Therefore, to each coefficient per player correspond three probability values P^{in}, P^{de}, P^{st} (increase, decrease, stabilization), such that for player *i*:

$$P_{i_A}^{in} + P_{i_A}^{de} + P_{i_A}^{st} = 1 \tag{6.a}$$

$$P_{i_B}^{in} + P_{i_B}^{de} + P_{i_B}^{st} = 1 \tag{6.b}$$

$$P_{i_C}^{in} + P_{i_C}^{de} + P_{i_C}^{st} = 1 \tag{6.c}$$

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[1] It is evident that the total cost is a cubic function *($TC = \alpha x + {}^1/_2\, \beta x^2 + {}^1/_3\, \gamma x^3 + FC$)*. For simplicity reasons we assume that the fixed cost (*FC*) is negligible and therefore is not taken into account since it does not influence the generality.

The initial, arbitrarily defined, probability distribution of the three actions for each coefficient might not necessarily be equiponderant regarding the actions.

The randomly selected action, depending on the in turn coefficient's modification period, defines the new coefficient values of the offer to be submitted to the System Operator, as follows:

If modification period of A *If modification period of B* *If modification period of C*

$$A_{i_n} = A_{i_{n-1}} \cdot (1+\varepsilon) \qquad A_{i_n} = A_{i_{n-1}} \qquad A_{i_n} = A_{i_{n-1}}$$

$$B_{i_n} = B_{i_{n-1}} \quad \text{or} \quad B_{i_n} = B_{i_{n-1}} \cdot (1+\varepsilon) \quad \text{or} \quad B_{i_n} = B_{i_{n-1}}$$

$$C_{i_n} = C_{i_{n-1}} \qquad C_{i_n} = C_{i_{n-1}} \qquad C_{i_n} = C_{i_{n-1}} \cdot (1+\varepsilon)$$

$$\text{where} \quad \varepsilon = \begin{cases} e_{A_i}\ ,\ e_{B_i} \text{ or } e_{C_i} & \text{if the selected action is } \textit{increase} \\ 0 & \text{if the selected action is } \textit{stabilization} \\ -e_{A_i}\ , -e_{B_i} \text{ or } -e_{C_i} & \text{if the selected action is } \textit{decrease} \end{cases}$$

The net revenue J_{i_n} for player i resulting after round n is compared with the net revenue $J_{i_{n-1}}$ of the previous round and the probability distribution of player's available actions is then adjusted. If, for player i, the difference ($J_{i_n} - J_{i_{n-1}}$) corresponding to two sequential rounds is positive then the probability value of the selected action in round n is increased (*reward*) by a predefined step θ, expressed as a percentage, and the probability values of the other two actions are equally decreased. In case that the net revenue is inferior to the one of the previous round the probability value of the selected action is decreased (*punishment*) by the same step θ and the probability values of the other two actions are equally increased. Step's size can be different per player, signifying diversification in players' learning capabilities.

3. An application of the game

Based on the power market model and the game described above we applied a limited version of the game with specific features in order to extract some general conclusions regarding mainly the relation between the SMP and the number of players participating in the game, and also the way players evolve their offer curves in their effort to maximize their profit. Therefore, some parameters of the game were defined as of static nature while differentiation among players was minimized.

More specifically, we assume that electricity demand D remains constant throughout the game, players participating in the game have the same production capacity range and the sum of the production capacity of all players is three times the electricity demand. Values of players' cost function coefficients are randomly spread within an interval ±30% from the corresponding values of the first player and values of steps e and θ are equal for all players and all coefficients:

$$e_{A_i} = e_{B_i} = e_{C_i} = 2\% \qquad \text{and} \qquad \theta_i = 5\%\ , \qquad \forall \text{ player } i$$

The initial values of probability P^{in}, P^{de}, P^{st} that correspond to the three *actions* are taken also equal per player and per coefficient while they vary per action:

$$P_i^{in} = P_i^{de} = 35\% \qquad \text{and} \qquad P_i^{st} = 30\%\ , \qquad \forall \text{ player } i \text{ and coefficient } A_i, B_i, C_i$$

The number of consecutive iterations defining each coefficient's modification period for each player is randomly selected from a common interval of values between 30 and 80. The random values applied in the first game remain the same for all the repeated games.

Price Cap is set up to 150, which is approximately ten times more than the initial SMP.

Eleven (11) different game types, with 3, 4, 5, 6, 7, 8, 9, 10, 12, 15 and 20 players respectively, were simulated and each one of them has been repeated 20 times. Every game consists of 32,000 consecutive offers (iterations) and experience gained from these offers is used only during the current game, while repetitions of the same game start from zero point regarding players' experience. In the 3-players game type participate the three first players (1,2,3) and each time we proceed to the next game type (i.e. 4-players) we add the next player/s in row. Thus, the first three players take part in all the executed games while players from 16 to 20 only in the 20-players game type. The following table illustrates the values of all players' parameters.

Player	Cost Function Parameters			Production Capacity		Step θ	Step e (A, B, C)	Coefficients' Modification Period		
	α	β	γ	Min	Max			A	B	C
1	10.00	0.0050	0.00055	0	300	5%	2%	44	71	48
2	11.54	0.0040	0.00050	0	300	5%	2%	73	69	30
3	8.97	0.0060	0.00067	0	300	5%	2%	36	69	69
4	10.23	0.0055	0.00067	0	300	5%	2%	58	70	69
5	11.94	0.0042	0.00052	0	300	5%	2%	67	69	72
6	7.92	0.0036	0.00058	0	300	5%	2%	79	76	62
7	11.76	0.0062	0.00045	0	300	5%	2%	70	59	66
8	9.57	0.0050	0.00070	0	300	5%	2%	58	48	66
9	10.26	0.0055	0.00047	0	300	5%	2%	69	52	73
10	7.43	0.0037	0.00039	0	300	5%	2%	52	78	48
11	7.34	0.0035	0.00049	0	300	5%	2%	79	59	53
12	12.01	0.0064	0.00058	0	300	5%	2%	34	43	61
13	12.91	0.0041	0.00051	0	300	5%	2%	39	61	57
14	7.12	0.0062	0.00060	0	300	5%	2%	32	41	56
15	10.94	0.0048	0.00063	0	300	5%	2%	58	44	74
16	12.73	0.0040	0.00062	0	300	5%	2%	50	64	77
17	7.72	0.0044	0.00045	0	300	5%	2%	39	68	43
18	9.31	0.0058	0.00040	0	300	5%	2%	46	65	42
19	9.96	0.0061	0.00047	0	300	5%	2%	60	33	63
20	12.40	0.0043	0.00045	0	300	5%	2%	69	64	73

Table 1. Values of parameters for all players participated in the games

For every game and iteration the following are recorded:

1. System's Marginal Price (*SMP*)
2. Players' market shares x_i
3. Offer curve's coefficients A_i , B_i , C_i ,
4. Moving averages of all the precedents.

Great importance for the players' behavior and the necessary comparisons have the initial value of *SMP*, when offer curves slightly differ from the marginal cost curves, the market shares players obtain at the first round of each game and the cost function coefficients (a_i, β_i, γ_i) of each player.

4. Results – Observations

System's Marginal Price

In spite of the observed variations of the SMP during the games, moving average always converges and it converges at higher value levels of the initial's SMP of the corresponding game. Figure 2 illustrates the evolution of the *SMP* and its moving average in a 10-players game.

The number of players participating in the game seemed to affect the value where SMP's moving average converges and also the time it takes to converge. Namely, *SMP* converges at a higher value and at a slower rate, the fewer the number of players participate in the game.

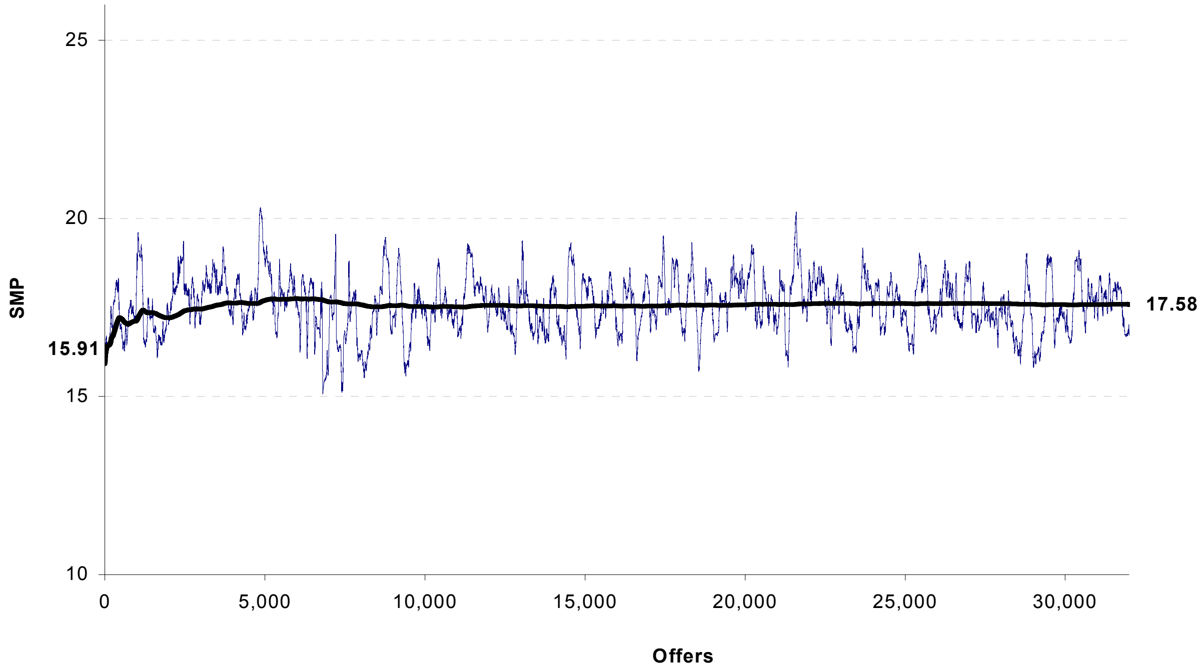


Figure 2 SMP and its moving average in a 10-players game.

Repetition	**Game Types** *(number of players in the game)*										
	3	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**12**	**15**	**20**
1	28.48	23.02	21.87	20.43	19.61	20.96	19.26	17.58	17.45	16.94	16.29
2	26.21	24.54	21.15	21.75	19.60	19.40	18.78	17.72	17.09	17.05	16.48
3	26.83	22.90	21.10	20.74	19.62	19.08	18.88	17.81	18.65	17.24	16.76
4	27.56	25.05	22.09	20.38	20.34	19.08	18.87	17.40	17.12	17.23	16.74
5	23.71	25.24	22.32	21.16	19.39	18.89	18.92	17.99	18.32	16.93	17.44
6	25.07	22.80	22.17	20.78	20.08	19.54	18.52	18.20	17.14	16.68	16.42
7	24.63	23.18	21.82	20.29	21.08	19.39	18.75	17.95	17.42	16.93	16.65
8	31.13	23.13	21.69	20.57	19.45	19.08	18.69	18.08	16.96	16.74	16.05
9	28.23	24.65	21.07	20.72	19.65	19.34	19.02	17.65	17.27	16.70	16.63
10	28.80	22.45	21.13	20.44	19.95	18.71	18.93	18.24	18.22	16.88	16.59
11	26.89	22.88	22.56	20.23	20.02	19.93	18.86	17.74	17.10	17.02	16.56
12	27.32	22.39	21.48	20.90	19.68	19.07	19.09	17.68	17.34	17.79	16.53
13	26.45	23.78	21.91	21.06	19.84	19.30	19.05	17.53	17.52	17.24	16.72
14	29.27	22.09	21.57	20.38	19.80	19.42	18.76	17.68	17.48	16.88	16.45
15	24.91	23.66	22.45	20.52	20.34	19.21	19.08	18.58	17.53	17.05	16.46
16	26.96	22.99	21.64	20.21	19.55	19.41	19.43	17.78	17.25	17.04	16.43
17	26.38	24.10	22.46	20.86	19.63	19.51	18.65	17.92	17.46	16.68	16.54
18	28.52	21.97	22.72	20.88	19.16	19.10	19.00	18.03	17.52	16.88	16.38
19	25.84	23.47	22.50	20.68	19.61	19.17	18.93	18.03	17.07	17.42	16.54
20	27.32	24.68	22.24	20.26	19.37	19.22	18.69	18.04	17.46	17.03	16.70
Average	**27.03**	**23.45**	**21.90**	**20.66**	**19.79**	**19.34**	**18.91**	**17.88**	**17.47**	**17.02**	**16.57**
Initial SMP	16.44	16.67	16.88	16.45	16.52	16.58	16.45	15.91	15.86	16.06	15.97
Ave. / Initial	**1.64**	**1.41**	**1.30**	**1.26**	**1.20**	**1.17**	**1.15**	**1.12**	**1.10**	**1.06**	**1.04**

Table 2 Convergence values of *SMP*'s averages as they resulted from the simulated games. Average *SMP* convergence value, initial *SMP* value and their in-between ratio per game type.

In Table 2 it is evident the relation between SMP and the number of players in the game, which is explicitly illustrated in the next figure.

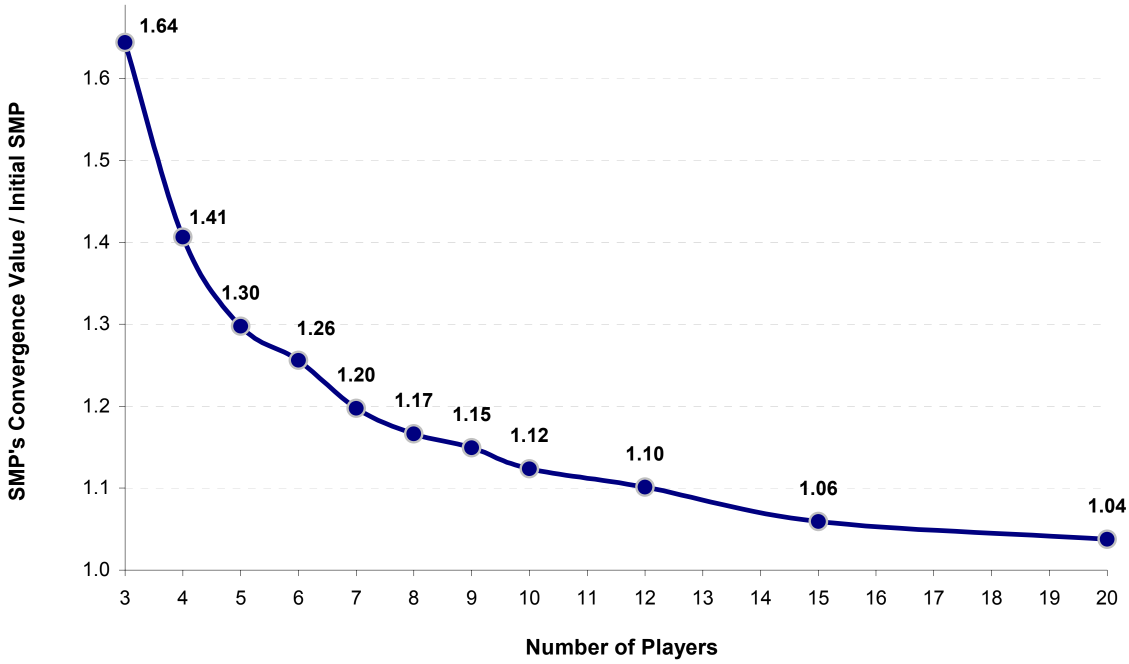


Figure 3 Relation between SMP and number of players in the corresponding game types.

Offer Curves

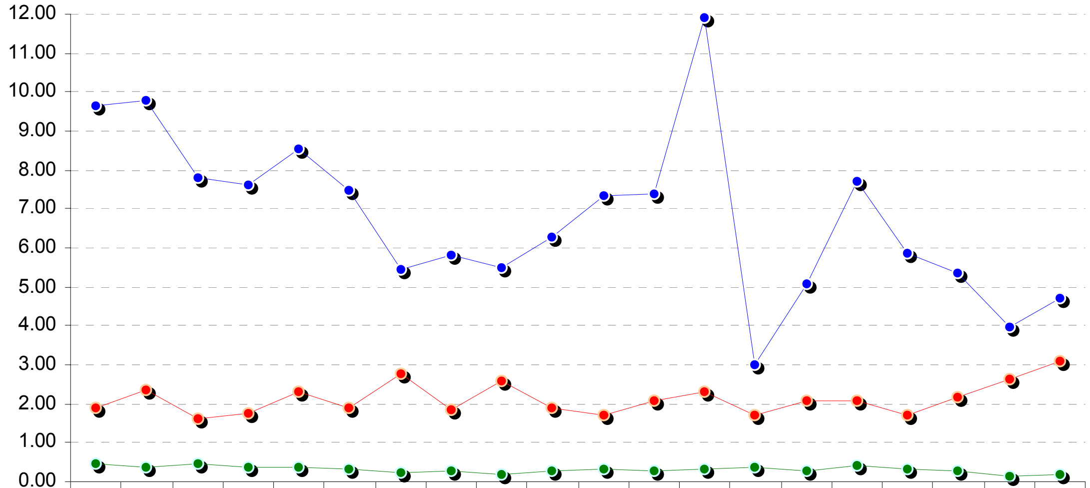
The players' behavior during the games, as it is portrayed in the evolution of the offer curves they submit to the System Operator, though it presents significant variations, it seems finally to follow a certain pattern. More specifically, it was observed that players tend to decrease the value of coefficient A, while they do the opposite for the coefficients B and C. Also the number of players in the game seems to affect the degree players alter each coefficient in relation with the corresponding cost function coefficient.

Figure 4 illustrates for each player the average ratios (in the games he participated) of the value that converges the moving average of each coefficient *A, B, C* to the value of the corresponding cost parameter α, β, γ. Similarities in the behavior of the coefficients *A, B, C* for all players has been observed:

- Coefficients A_i converge at lower value levels (~ 70% lower, averagely for all players) of those of the corresponding cost parameters α_i.

- Coefficients B_i converge at significantly higher value levels (~ 580% higher, averagely for all players) of those of the corresponding cost parameters β_i.

- Coefficients C_i converge at higher value levels (~ 113% higher, averagely for all players) of those of the corresponding cost parameters γ_i.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	0.44	0.38	0.46	0.39	0.37	0.31	0.24	0.26	0.19	0.29	0.31	0.27	0.31	0.38	0.26	0.41	0.32	0.26	0.13	0.17
B	9.64	9.78	7.81	7.63	8.53	7.49	5.46	5.83	5.49	6.30	7.33	7.36	11.90	2.99	5.07	7.71	5.87	5.37	3.97	4.72
C	1.90	2.37	1.61	1.76	2.33	1.90	2.77	1.85	2.57	1.91	1.71	2.08	2.29	1.70	2.09	2.09	1.71	2.18	2.65	3.07

Figure 4 Average ratios of the average convergence values of *A, B, C* coefficients to the corresponding values of the cost parameters *α, β, γ* for each player, in all games.

Another interesting observation concerns the "intensity" of this behavioral trend of the coefficients for all players, related to the number of players participating in the game. We have noticed that coefficients *A* and *C* diverge even more from the corresponding cost function's parameters (*α, γ*) values as the number of players increases, while coefficient *B* converges closer to the value of coefficient *β*. In other words, as the number of players increases, coefficients' *A* and *C* values, decrease and increase respectively even more, while coefficient's *B* increase rates start to decline.
The following figures indicate the above statement, illustrating *A, B, C* coefficients' average variation from the corresponding cost function parameters for Players 1, 2 and 3 who participated in all game types.

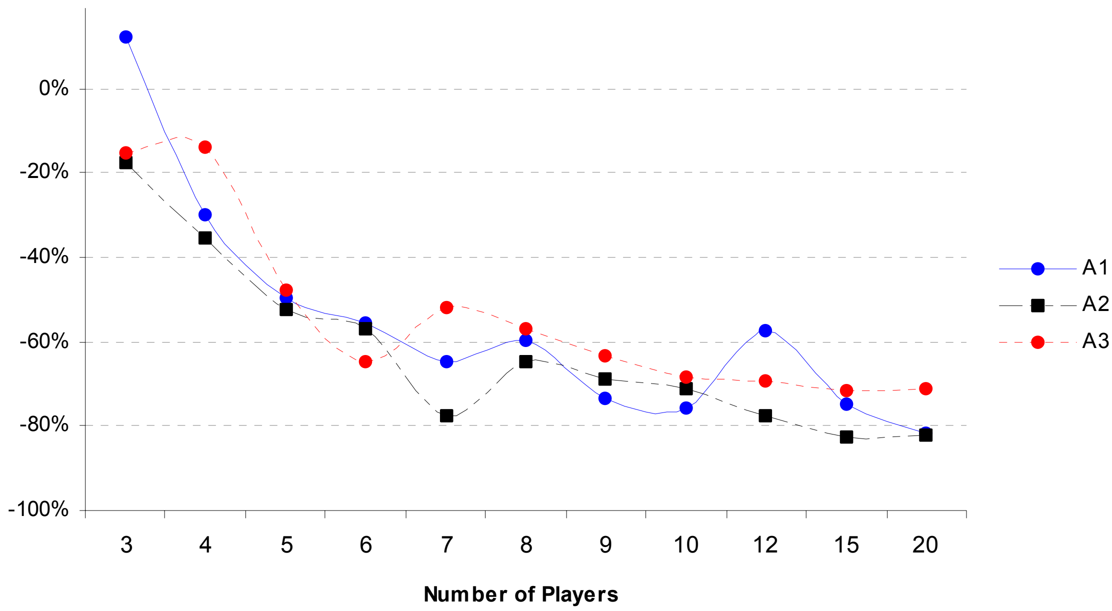


Figure 5: Coefficient's *A* average variation from the cost function parameter *α,* per game type for Players 1, 2 and 3.

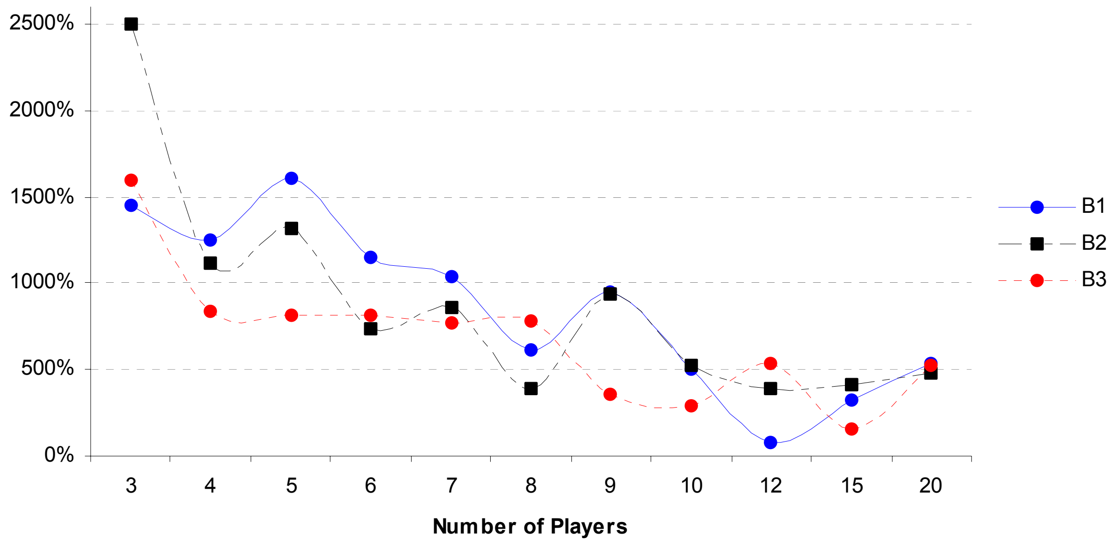


Figure 6: Coefficient's *B* average variation from the cost function parameter *β,* per game type for Players 1, 2 and 3.

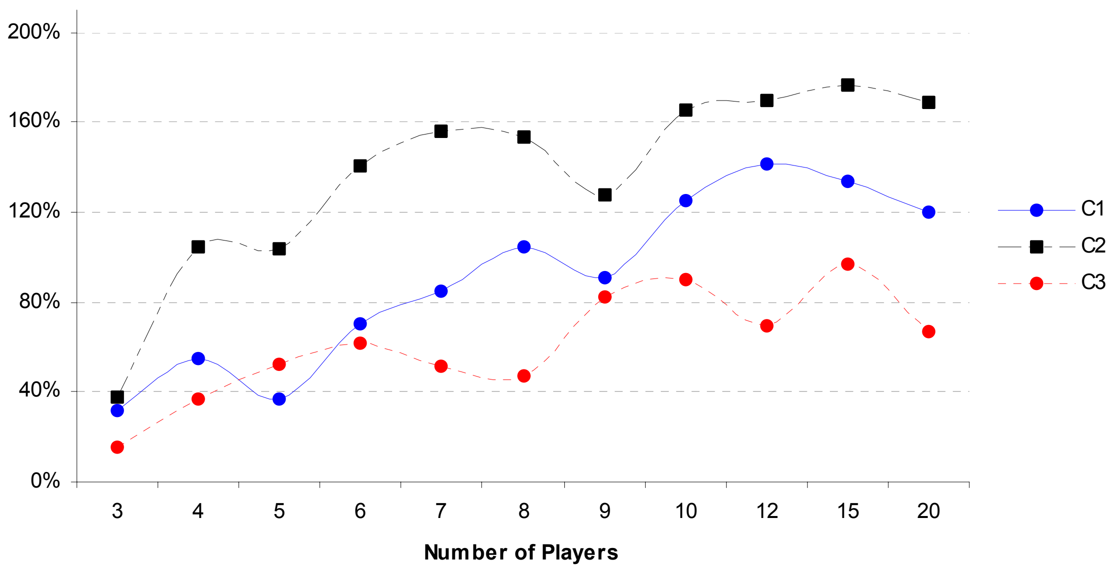


Figure 7: Coefficient's *C* average variation from the cost function parameter *γ, per* game type for Players 1, 2 and 3.

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