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Decentralized Adaptive Control in a Game Situation for Discrete-Time, Linear, Time-Invariant Systems *

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Abstract

This paper deals with the decentralized adaptive control problem of a discrete-time Multi-Input Multi-Output Linear Time-Invariant system, controlled by many controllers having their own information sets, models and objective functions. It is shown that the decentralized one-step-ahead adaptive control scheme with the projection algorithm (for the deterministic case) or the stochastic approximation algorithm (for the stochastic case) will ensure that the behaviour of the closed-loop system with unknown parameters gets closer and closer to that of the closed-loop system with known parameters as time elapses.

1. Introduction

The rapid growth of our society has increased the demands of planning, designing and analyzing a "large scale system" for which a centralized approach requires excessively powerful computing facilities and extremely complex information networks. Therefore, it is not surprising that centralized control systems tend to be replaced by distributed computer controlled systems [1] and many researchers have paid their attention to the various aspects of decentralized control of a large scale system [2-12]. On the other hand, the advent of advanced computer technology has stimulated the development of adaptive control theory , the purpose of which is to find a reasonable method of adjusting the controller parameters in response to changes or uncertainty in process and disturbance dynamics [13-23]. It seems to be interesting to combine the ideas of decentralized control and of adaptive control in order to examine the decentralized control problem of unknown large systems. But it was not a long time ago that there appeared such an attempt as considering a large scale system from the adaptive control viewpoint, and there are not many papers in this direction, some of which are [24], [25], and [26]. In [24], Davison suggested an algorithm to determine the decentralized robust controller for an unknown stable linear time-invariant (LTI) system by performing some experiments. In [25], Ioannou established a decentralized adaptive controller which guarantees the desired stability properties for a class of large scale systems formed by an arbitrary linear interconnection of LTI subsystems with unknown parameters. In [26], Chan applied the one-step-ahead adaptive controller to deal with the multi-controller problem. But, none seems to examined an adaptive game problem, especially for the case where the objectives of several controllers are in conflict and their information is different. The first attempts in this direction seem to be in [27, 28, 29, 30].

In this paper, we shall establish some results of the adaptive control scheme incorporating the one-step-ahead controller with the projection (stochastic approximation) algorithm for the deterministic (stochastic) case which is applied to a discrete-time MIMO LTI system controlled by multiple contollers having their own information, models, and objective functions. There are two special features which distinguish our work from previous ones: (A) Input decentralization by multi-modeling: In most papers dealing with decentralized control, the system to be controlled is modeled as an input-decentralized form in which each subsystem is not affected directly by the other controllers' inputs. Although such an assumption covers many real large systems, it may be very desirable to start with a more general model. In our scheme, the system to be controlled is described by the auto-regressive (AR) model in which each subsystem is affected directly by the inputs from all the controllers. But each controller does not consider the other controllers' inputs in his own subsystem model to predict the corresponding output. This multi-modeling has the input decentralization effect, which appears in the following three aspects: (1) Algorithmical aspect: The real system we are dealing with is essentially a time-invariant system. But, for the model of each controller's subsystem, in which the other controllers' parameters are unmodeled, the parameter estimates are affected by the other controllers' parameters being updated from time to time. Thus, the resulting subsystem appears to be time-varying to its corresponding controller, so that it is reasonable to apply the time-varying variants of the standard parameter estimation algorithm like the least-squares algorithm with exponential data weighting or the projection algorithm. However, it is shown that in the stochastic case, the stochastic approximation algorithm works well. (2) Informational aspect: For the system described by the DAR (deterministic autoregressive) model, the input decentralization sets each controller free from the duty of getting the information about the other controllers' inputs and consequently makes it possible to get some stability result with the observation sharing information pattern. The extension of this feature to the more general ARMA model is quite desirable, but highly nontrivial. (3) Computational aspect: In the MIMO one-step-ahead adaptive controller suggested by Goodwin (p. 204, [23]), a matrix should be "inverted" to get a feedback control law at every instant, whether the parameter estimates are updated by a centralized processor or by several distributed processors. Besides, there is no systematic way of keeping the matrix to be "inverted" to stay away from singularty or near-singularity which might result in a numerical instability problem. Thus, the larger the dimension of our problem is, the more desirable it is to find a way to exploit some special structural feature of our problem for the computational purpose. For the case where the input coefficient matrix of the system equation satisfies a weak coupling type of condition, the input decentralization by multi-modeling excludes the necessity of "inverting" a matrix at each time. The global efficiency of our scheme in the computational aspect is comparable with that of the well-known Jacobi method [31] which can be executed on a parallel processor system to solve a set of simultaneous linear equations. (B) Game Feature: In many cases, the multi-controller problems with no conflict among the different objective functions are considered as a class of game problems. These problems can be somehow transformed into a one-decision maker problem which is equivalent to the original multi-controller problem in the sense that they have the same solution [26]. What is more interesting to us is the game situation where two or more controllers try to control the same output for their own interest. Many researchers have devoted themselves to developing game theory [32-35]. Ikeda [10] has examined a problem that is close to ours, and he established a stabilizing decentralized control law for the overlapping information in the expansion-contraction framework. However, to the author's knowledge, none have addressed such a game

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situation in the context of decentralized adaptive control.

This paper is organized as follows. In Section II, an example is presented to show the motivation for our work. In Sections III and IV, we show that for a certain class of discrete-time MIMO LTI systems described by the AR model, the one-step-ahead controller (with the projection algorithm for the deterministic case and with the stochastic approximation algorithm for the stochastic case) applied in the decentralized framework will ensure the stability and tracking results with the observation sharing information pattern under which each controller does not know the other controllers' inputs. What is different from our previous work [30] is that we consider a wider class of problems which do not necessarily satisfy the weak coupling type of condition (A3) with $r_j = 0$ on the structure of the input coefficient matrix and consequently does include even a game situation with conflict. We have shown that if the closed-loop system with known parameters is stable, and if each controller applies the weighted one-step-ahead adaptive control (with the projection algorithm for the deteministic case and with the stochastic approximation algorithm for the stochastic case), then the closed-loop system with unknown parameters will be stable, and its behavior will get closer and closer to that of the closed-loop system with known parameters.

2. Motivation

2.1. Information Pattern

The operation of the system can be described chronologically as follows.

Generation of initial state at time 0
Observation of outputs $y_i(1)$ for $i = 1$ to N
Estimation of parameters $\hat{\theta}_i(1)$ for $i = 1$ to N
Application of inputs $u_i(1)$ for $i = 1$ to N
Transition of state at time 1

....

Transition of state at time t
Observation of outputs $y_i(t)$ for $i = 1$ to N
Estimation of parameters $\hat{\theta}_i(t)$ for $i = 1$ to N
Application of inputs $u_i(t)$ for $i = 1$ to N
Transition of state at time $t+1$

....

The information pattern of the problem is the specification of the data available as arguments of the estimation model and the control law. We have the following definitions for the three types of information pattern [12] which will be used later.

(a) **Classical information pattern** An information pattern is said to be *classical* if all the controllers receive the same information and have perfect recall, i. e., if at time t, each controller has the information set

$$\{Y_j(t),\ U_j(t) \text{ for } j = 1 \text{ to } N\}. \qquad (2.1)$$

(b) **Delayed sharing information pattern** The *n-step delayed sharing information pattern* is characterized by each controller i having the common data available to every controller at time t

$$\{Y_j(t-n),\ U_j(t-n) \text{ for } j = 1 \text{ to } N\} \qquad (2.2)$$

and the additional data known only to himself at time t, $\{y_i(t), y_i(t-1), \dots, y_i(t-n_1);\ u_i(t-1), \dots, u_i(t-n+1)\}$.

(c) **Observation sharing information pattern** This information pattern is characterized by each controller i having the common data available to every controller at time t, $\{Y_j(t) \text{ for } j = 1 \text{ to } N\}$, and the additional data known only to himself at time t

$$\{U_i(t-1)\}. \qquad (2.3)$$

In other words, under the *observation sharing information pattern*, each controller shares only the observations, but does not know the past histories of the other controllers' inputs.

2.2. Example

For the game situation where multiple controllers with their own information are trying to adaptively control the output of an unknown system, one might ask such questions as:

- Are the parameter estimates convergent or bounded?
- Will the prediction error go to zero as time goes by?
- Under what conditions is the resulting closed-loop system stable?
- How are the inputs, outputs of the closed-loop system with unknown parameters related with those of the closed-loop system with known parameters?
- How is the tracking performance in a non-conflicting case?

Let us introduce an example. Consider a system described by

$$y(t+1) = a\,y(t) + b_1 u_1(t) + b_2 u_2(t);\ y(0) \text{ given} \qquad (2.4)$$

where $b_1 = b_2 = 1$ is known, $y(t)$ is a scalar, and $u_i(t)$ is exerted by controller i to minimize

$$J_i(t) = (y(t+1) - y_i^*(t+1))^2 + r_i u_i(t)^2;\ r_i \geq 0. \qquad (2.5)$$

Case 1 *Known parameter and Classical information*

If both controllers know the value of the parameter 'a' and have the common information set at time t, $\{y(k),\ u_1(k-1), u_2(k-1) \text{ for } k = 1 \text{ to } t \text{ and } y(0)\}$, they will exert their control inputs as

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} 1+r_1 & 1 \\ 1 & 1+r_2 \end{bmatrix}^{-1} \begin{bmatrix} y_1^*(t+1) - ay(t) \\ y_2^*(t+1) - ay(t) \end{bmatrix} \qquad (2.6)$$

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \frac{1}{(1+r_1)(1+r_2)-1} \times \begin{bmatrix} -ar_2 y(t) + (1+r_2)y_1^*(t+1) - y_2^*(t+1) \\ -ar_1 y(t) + (1+r_1)y_2^*(t+1) - y_1^*(t+1) \end{bmatrix}$$

and the resulting closed-loop system will be

$$y(t+1) = A_c y(t) + d_1 y_1^*(t+1) + d_2 y_2^*(t+1) \qquad (2.7)$$

where

$$A_c = a\frac{r_1 r_2}{(1+r_1)(1+r_2)-1} \qquad (2.8a)$$

$$d_1 = \frac{r_2}{(1+r_1)(1+r_2)-1} \qquad (2.8b)$$

$$d_2 = \frac{r_1}{(1+r_1)(1+r_2)-1} \qquad (2.8c)$$

Remark 1 Notice that if each controller does not penalize the input in his objective function, the problem becomes singular because for $r_1 = r_2 = 0$, we have $(1+r_1)(1+r_2) - 1 = 0$. The interpretation of this observation is that if each controller is allowed to make his input as large as he wants, he may be selfish enough to apply even infinitely large input to counterbalance the other controller's influence on the output for achieving his own objective, especially in the case where their objectives are conflicting. Hence, the existence of the penalizing-input term on the objective functions may be appreciated for playing a role as a mediator to make a compromise possible between the two controllers by leading them to self-constraint.

Now, throughout the following 2 cases, we assume

(i) $y_1^*(t+1) = y_2^*(t+1) = 0\ \forall t \geq 0$,

(ii) $|A_c| = \left|a\frac{r_1 r_2}{(1+r_1)(1+r_2)-1}\right| < 1$,

which implies that the output reference signals are constants at zero level and the closed-loop system with known parameter is asymptotically stable.

Case 2 *Unknown parameter and Classical information*[1]

If each controller does not know the parameter 'a', but knows r_1, r_2 and has the same information as in *case 1*, he has his model to predict the output as

$$\hat{y}(t+1) = \hat{\theta}(t)y(t) + u_1(t) + u_2(t) \qquad (2.9)$$

and, according to his objective function with $\hat{y}(t+1)$ instead of $y(t+1)$, he applies the input computed by

$$u_1(t) = -\frac{r_2}{(1+r_1)(1+r_2)-1}\hat{\theta}(t)y(t) \qquad (2.10a)$$

$$u_2(t) = -\frac{r_1}{(1+r_1)(1+r_2)-1}\hat{\theta}(t)y(t) \qquad (2.10b)$$

Then, the resulting closed-loop system will be

$$y(t+1) = \left\{a - \frac{r_1\hat{\theta}(t) + r_2\hat{\theta}(t)}{(1+r_1)(1+r_2)-1}\right\}y(t)$$

$$y(t+1) = \left\{A_c - \frac{r_1\tilde{\theta}(t) + r_2\tilde{\theta}(t)}{(1+r_1)(1+r_2)-1}\right\}y(t) \qquad (2.11)$$

Suppose each controller uses the projection algorithm to estimate the parameter 'a' as

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{y(t-1)}{1+y(t-1)^2} \qquad (2.12)$$
$$\{y(t) - \hat{\theta}(t-1)y(t-1) - u_1(t-1) - u_2(t-1)\}$$

which results in the parameter estimation error equation

$$\tilde{\theta}(t) = M_1(t-1)\tilde{\theta}(t-1) \qquad (2.13)$$

$$\tilde{\theta}(t) = \hat{\theta}(t) - a;\ M_1(t) = 1 - \Delta(t);\ \Delta(t) = \frac{y(t)^2}{1+y(t)^2}.$$

Case 3 *Unknown parameter and Observation sharing information*

Suppose each controller knows $\{y(k); k = 0 \text{ to } t\}$ and neither has the knowledge of the other 'a' nor the information about the other controller's input so that he has his own model to predict the output based on his information as

$$\hat{y}_i(t+1) = \hat{\theta}_i(t)y(t) + u_i(t) \qquad (2.14)$$

where he thinks of the system as governed only by his input and applies the input computed by

$$u_i(t) = -\frac{\hat{\theta}_i(t)}{1+r_i}y(t) \qquad (2.15)$$

Then the closed-loop system will be

$$\begin{aligned} y(t+1) &= a_c(t)y(t) \qquad (2.16)\\ &= \left(a - \frac{\hat{\theta}_1(t)}{1+r_1} - \frac{\hat{\theta}_2(t)}{1-r_2}\right)y(t)\\ &= \left(A_c - \frac{\tilde{\theta}_1(t)}{1+r_1} - \frac{\tilde{\theta}_2(t)}{1+r_2}\right)y(t)\end{aligned}$$

where

$$A_c = a - \frac{\theta_1^\circ}{1+r_1} - \frac{\theta_2^\circ}{1+r_2} = \frac{ar_1r_2}{(1+r_1)(1+r_2)-1} \qquad (2.17)$$

$$\text{with } |A_c| < 1,\ \theta_1^\circ = \frac{ar_2(1+r_1)}{(1+r_1)(1+r_2)-1},$$

$$\theta_2^\circ = \frac{ar_1(1+r_2)}{(1+r_1)(1+r_2)-1},\ \tilde{\theta}_i(t) = \hat{\theta}_i(t) - \theta_i^\circ.$$

[1]Notice that the symbols $\hat{}$ (head) and $\tilde{}$ (tilde) are used to stand for the estimate and estimation error that are obviously functions of time, whence containing the time index, though not explicitly.

Suppose each controller uses the modified least-squares algorithm with exponential data weighting to generate the parameter estimate as

$$\hat{\theta}_i(t+1) = \hat{\theta}_i(t) + \frac{y(t)}{\lambda^{t+1} + \sum_{s=0}^{t}\lambda^{t-s}y(s)^2}(y(t+1) - \hat{y}_i(t+1)) \qquad (2.18)$$

with $0 < \lambda < 1$ as an exponential data weighting factor. Then it is readily shown that the parameter estimation error being affected by the other controller's estimation error evolves as

$$\tilde{\theta}(t+1) = M_2(t)\tilde{\theta}(t) \qquad (2.19)$$

where

$$\tilde{\theta}(t)^T = \tilde{\theta}_1(t), \tilde{\theta}_2(t)$$

$$M_2(t) \doteq \begin{bmatrix} 1-\Delta(t) & -\frac{\Delta(t)}{1+r_2} \\ -\frac{\Delta(t)}{1+r_2} & 1-\Delta(t) \end{bmatrix}$$

$$\Delta(t) = \frac{y(t)^2}{\lambda^{t+1} + \sum_{s=0}^{t}\lambda^{t-s}y(s)^2}.$$

Remark 2

(i) The projection algorithm applied to Case 3 also would bring the similar result only with the difference

$$\Delta(t) = \frac{y(t)^2}{1+y(t)^2}$$

(ii) Notice that, for $M(t) = M_1(t)$ (Case 2) or $M_2(t)$ (Case 3),

$$\|M(t)\| \le 1 \text{ and}$$
$$\|M(t)\| = 1 \text{ if and only if } M(t) = I, \text{ i. e., } \Delta(t) = 0$$

Thus, the transition matrix $M(t)$ in the parameter estimation error equation is always non-expansive and moreover, as long as $y(t)$ and so $\Delta(t)$ does not vanish, it is contractive, so that the closed-loop gain $A_c(t)$ of the system with unknown parameters gets closer and closer to that of the system with known parameters as time elapses. Therefore, it is not difficult to see that the convergence of the parameter estimate to some limit and also the closed-loop stability with unknown parameters are guaranteed by the closed-loop stability with known parameters.

(iii) For a sufficiently small positive value of the exponential weighting factor λ, the modified least-squares estimation algorithm brings the convergence of the parameter estimate to the true parameter with the exception of two cases:

- $y(t)$ happens to be zero at some instant;
- by any chance, the closed-loop gain $A_c(t)$ is convergent to zero.

It is implied that if the exponential data weighting factor $\lambda > 0$ is smaller than the absolute value of the closed-loop gain A_c with known parameter, at least the local convergence of the parameter estimate to the true parameter is achieved.

Remark 3 Although the above example is very simple and not all of the ideas carry over to a more general case, it provides us with useful intuition about the more general case.

3. Deterministic Case

3.1. Problem Formulation

We deal with the situation where multiple controllers are trying to achieve their own objectives on the output of an LTI finite-dimensional deterministic system described by

$$y(t+1) = A(q^{-1})y(t) + B\,u(t), \text{ with the initial condition } x_0. \qquad (3.1)$$

At time t, each controller i is supposed to have his own information, objective function and predictor model as follows.

Information set

$$\{Y_j(t), Y_j^*(t+1);\ j = 1 \text{ to } N\} \text{ and } \{U_i(t-1)\}. \quad (3.2)$$

This implies that each controller knows the past histories of all the outputs and output reference signals, but does not know the past histories of the other controllers' inputs.

Objective function

$$J_i(t) = \{y_i(t+1) - y_i^*(t+1)\}^2 + r_i u_i(t)^2 \text{ with } r_i > 0 \quad (3.3)$$

This implies that each controller is trying to make the corresponding output $y_i(t)$ tracking his output reference signal $y_i^*(t)$ which is assumed to be uniformly bounded.

Predictor model

$$\hat{y}_i(t+1) = \phi_i(t)^T \theta_i(t) + b_{ii} u_i(t) \quad (3.4)$$

where

$$\phi_1(t) = \phi_2(t) = \ldots = \phi_N(t) = \phi(t) \quad (3.5a)$$

$$\phi(t) = [\underline{y}_1(t)^T, \underline{y}_2^T, \ldots \underline{y}_N(t)^T;\ y_1^*(t+1), y_2^*(t+1), \ldots, y_N^*(t+1)]^T \quad (3.5b)$$

$$\theta_i(t) = [\underline{\hat{\alpha}}_{i2}(t)^T, \ldots, \underline{\hat{\alpha}}_{iN}(t)^T;\ \underline{\hat{d}_{i1}}(t), \hat{d}_{i2}(t), \ldots, \hat{d}_{iN}(t)]^T \quad (3.5c)$$

b_{ii} is assumed to be known to controller i.

This implies that each controller regards his subsystem as controlled only by his own input, so that he does not model the other controllers' inputs in his subsystem model to predict his output.

Based on this formulation, each controller uses the following decentralized adaptive control scheme:

Controller structure

$$u_i(t) = \frac{b_{ii}}{b_{ii}^2 + r_i}\{y_i^*(t+1) - \phi(t)^T \theta_i(t)\} \quad (3.6)$$

(This implies that each controller does not need to solve a set of N linear equations for obtaining this output.)

Parameter estimator with projection algorithm

$$\theta_i(t+1) = \theta_i(t) + \frac{\phi(t)}{1 + \phi(t)^T\phi(t)}\{y_i(t+1) - \hat{y}_i(t+1)\} \quad (3.7)$$

Let

$$\theta_i^\circ = [\underline{\alpha}_{i1}^{\circ T}, \underline{\alpha}_{i2}^{\circ T}, \ldots, \underline{\alpha}_{iN}^{\circ T};\ d_{i1}^\circ, d_{i2}^\circ, \ldots, d_{iN}^\circ]^T \quad (3.8)$$

$$\tilde{\theta}_i(t) = \theta_i(t) - \theta_i^\circ \quad (3.9)$$

$$e_i(t) = -\phi(t)^T \tilde{\theta}_i(t). \quad (3.10)$$

Then, from (3.1), (3.4), (3.6), and (3.10) with (2.1) and (2.2), we get

$$\begin{aligned} e_i(t+1) &= y_i(t+1) - \hat{y}_i(t+1) \\ &= \sum_{j=1}^{N} a_{ij}(q^{-1}) y_j(t) + \sum_{k=1}^{N} b_{ik} u_k(t) \\ &\quad - \sum_{j=1}^{N} \hat{\alpha}_{ij}(q^{-1}) y_j(t) - b_{ii} u_i(t) - \sum_{j=1}^{N} \hat{d}_{ij} y_j^*(t+1) \\ &= -\sum_{j=1}^{N}[\hat{\alpha}_{ij}(q^{-1}) - \{a_{ij}(q^{-1}) - \sum_{k\neq 1}^{N} h_{ik}\hat{\alpha}_{kj}(q^{-1})\}] y_j(t) \\ &\quad - \sum_{j=1}^{N}[\hat{d}_{ij} - \{\delta_{ij} h_{ij} - \sum_{k\neq 1}^{N} h_{ik}\hat{d}_{kj}\}] y_j^*(t+1) \\ &= -\phi(t)^T[\tilde{\theta}_i(t) + \sum_{k\neq i}^{N} h_{ik}\tilde{\theta}_k(t)] \end{aligned}$$

so that

$$e(t+1) = \mathcal{H}e(t) \quad (3.11)$$

Also, substituting (3.6) into (3.1), we obtain the closed-loop system equation

$$y(t+1) = A_c(q^{-1})y(t) + Gy^*(t+1) + He(t) \quad (3.12)$$

which can be rewritten in the state-space representation form as

$$\overline{y}(t+1) = A_a\overline{y}(t) + G_a y^*(t+1) + H_a e(t). \quad (3.13)$$

3.2. Assumptions

(A0) The basic structure of the system is known

(A1) An upperbound n for the order of the polynomial matrix $A(q^{-1})$ in the system equation (3.1) is known.

(A2) Every parameter $a_{ij}(q^{-1})$, b_{ij} is unknown, except that b_{ii} is known to controller i.

(A3) Every parameter of the input coefficient matrix B in the system equation (3.1) together with the weighting coefficient r_i's on the input term of the objective function (3.3) satisfies the weak coupling type of condition

$$|h_{ij}| < \frac{1}{N-1}\ \forall\, i \neq j \text{ where } h_{ij} = \frac{b_{ij}b_{jj}}{b_{jj}^2 + r_j}$$

(A4) The closed-loop system with known parameters is asymptotically stable in the sense that Det $[I = q^{-1}A_c(q^{-1})]$ has all its zeros inside the unit circle (u. c.), or equivalently the matrix A_a has all its eigenvalues inside the unit circle, where A_a was defined in Section 2.1.

3.3. Results

Proposition 1 The N-player discrete-time dynamic game described above with known parameters admits a (unique) set of feedback Nash equilibrium strategies given by

$$u_i^\circ(t) = \frac{b_{ii}}{b_{ii}^2 + r_i}(y_i^*(t+1) - \phi(t)^T\theta_i^\circ)\ \forall\, i = 1 \text{ to } N. \quad (3.14)$$

Proof: For this proof, as well as those of the result to follow, see [36]. Let us define

$$y^\circ(t+1) = A_c(q^{-1})y^\circ(t) + Gy^*(t+1), \quad (3.15)$$

with the initial condition x_0.

Then, $y^\circ(t)$ is the output vector sequence which would have been obtained if every controller had known all the parameters so that controller i had applied $u_i^\circ(t)$ from the beginning and $\{u^\circ(t)\}$, $\{y^\circ(t)\}$ can easily be shown to be uniformly bounded by the assumption (A4). We have the following result.

Theorem 3.1 *For the system (3.1) subject to the assumptions (A0)-(A4), the decentralized weighted one-step-ahead adaptive control scheme (3.6), (3.7) yields*

(a) $\|\tilde{\theta}(t+1)\|_J \leq \|\tilde{\theta}(0)\|\ \forall\, t \geq 0$.

(b) $\lim_{t\to\infty} \frac{e(t)}{(1+\phi(t)^T)}\phi(t)^{\frac{1}{2}} = 0$.

(c) $\lim_{t\to\infty} e(t) = 0$.

(d) $\lim_{t\to\infty} e(t) = 0$.

(e) $\{\phi(t)\}$ is uniformly bounded and so are $\{y(t)\}$, $\{u(t)\}$.

(f) $\lim_{t\to\infty}\{u_i(t) - u_i^\circ(t)\} = 0\ \forall\, i = 1$ to N.

(g) $\lim_{t\to\infty}\{y_i(t) - y_i^\circ(t)\} = 0\ \forall\, i = 1$ to N.

Let us now consider a special case where the input term is not penalized in the objective function, i. e. , $r_i = 0$. In this case, we have a smaller class of problems for which our decentralized one-step-ahead adaptive control scheme ensures the closed-loop stability, since the assumption (A3) becomes $|b_{ij}/b_{jj}| < 1/(N-1)$, which does not cover the game situation where two or more controllers are trying to control the same output in their own way. But, we have relatively stronger result asymptotic tracking performance.

Theorem 3.2 *For the system (3.1) subject to the assumptions (A0)-(A3), the decentralized one-step-ahead adaptive control scheme (3.6), (3.7) with $r_i = 0$ for all $i = 1$ to N achieves asymptotic tracking in addition to the results of Theorem 3.1;* $\lim_{t\to\infty} |y_i(t) - y_i^*(t)| = 0 \ \forall i = 1$ *to N.*

4. Stochastic Case

4.1. Problem Formulation

We deal with the situation where multiple controllers are trying to achieve their own objectives on the output of a LTI finite-dimensional stochastic system described by $y(t+1) = A(q^{-1})y(t) + Bu(t) + v(t+1)$ with the initial condition x_o. Let $\mathcal{F}_t$ be the σ-subalgebra generated by the observations up to and including time t. $\mathcal{F}_o$ includes the initial condition information. Each component of the noise vector sequence $\{v(t)\}$ is taken to be a real stochastic process defined on a probability space $(\Omega, \mathcal{F}, P)$ and adapted to the sequence of increasing σ-subalgebras $\{\mathcal{F}_t;\ t = 0, 1, 2, \ldots\}$. The following assumptions are made on the process $\{v(t)\}$.

(V1) $E\{v(t+1)|\mathcal{F}_t\} = 0$ a. s.

(V2) $E\{v(t+1)v(t+1)^T|\mathcal{F}_t\} = \Gamma$ with $\mathrm{tr}\,\Gamma < \infty$ a. s.

(V3) $\lim_{T\to\infty} \sup \frac{1}{T}\sum_{t=1}^{T} \|v(t)\|^2 < \infty$ a. s.

At time t, each controller i is supposed to have his own information, objective function and predictor model as below.

Information set

$$\{Y_j(t),\ Y_j^*(t+1);\ j = 1 \text{ to } N\} \text{ and } \{U_i(t-1)\} \quad (4.1)$$

Objective function

$$J_i(t) = E\{(y_i(t+1) - y_i^*(t+1))^2 + r_i u_i(t)^2|\mathcal{F}_t\}. \quad (4.2)$$

Predictor model

$$\hat{y}_i(t+1) = \phi(t)^T\hat{\theta}_i(t) + b_{ii}u_i(t) \quad (4.3)$$

where $\phi(t)$, $\hat{\theta}_i(t)$ are as defined by (3.5). Based on this formulation, each controller uses the following decentralized adaptive control scheme:

Controller structure

$$u_i(t) = \frac{b_{ii}}{b_{ii}^2 + r_i}\{y_i^*(t+1) - \phi(t)^T\hat{\theta}_i(t)\}\ \forall i = 1 \text{ to } N. \quad (4.4)$$

Parameter estimator with stochastic approximation algorithm

$$\hat{\theta}_i(t+1) = \hat{\theta}_i(t) + \frac{\overline{a}_i}{\gamma(t)}\phi(t)(y_i(t+1) - \hat{y}_i(t+1));\ \overline{a}_i > 0\ \forall i = 1 \text{ to } N. \quad (4.5)$$

$$\gamma(t) = \gamma(t-1) + \phi(t)^T\phi(t);\ \gamma(-1) = \gamma_o > 0. \quad (4.6)$$

Subtracting $(\hat{y}(t+1) + v(t+1))$ from both sides of (4.1) yields

$$e(t+1) = v(t+1) = A(q^{-1})y(t) + Bu(t) - \hat{y}(t+1) \quad (4.7)$$

where $e(t+1) = y(t+1) - \hat{y}(t+1)$. Let

$$z(t) = e(t-1) - v(t+1) \quad (4.8a)$$

$$e_i(t) = -\phi(t)^T\tilde{\theta}_i(t) \quad (4.8b)$$

where $\tilde{\theta}_i(t)$ is defined by (3.9) with (2.1) and (2.2). Then, by the same argument as deriving (3.11) and from (4.8) together with (4.4), (4.5) and (4.9), we can get

$$z(t) = \mathcal{H}e(t)$$

$$e(t) = \mathcal{H}^{-1}z(t) \quad (4.9)$$

where the invertibility of the matrix $\mathcal{H}$ is guaranteed by the assumption (A3). Also, substituting (4.5) into (4.1) gives the closed-loop system equation

$$\begin{aligned} y(t+1) &= A_c(q^{-1})y(t) + Gy^*(t+1) + He(t) + v(t+1) \quad (4.10) \\ &= A_c(q^{-1})y(t) + Gy^*(t+1) + H\mathcal{H}^{-1}z(t) + v(t+1) \end{aligned}$$

which can be rewritten in the state-space representation form as

$$\overline{y}(t+1) = A_a\overline{y}(t) + G_a y^*(t+1) + H_a e(t) + i_a v(t+1). \quad (4.11)$$

4.2. Assumptions and Results

In addition to the noise assumptions (V1)-(V3) and the system assumptions (A0)-(A4), we can make one more assumption:

(A5) The scalar gains $\overline{a}_i$'s of the error forcing term in the parameter estimation algorithm (4.6) are chosen so that $M > (1+\rho)I$ for an arbitrarily small $\rho > 0$, where

$$M = \mathcal{H}^{-1}\overline{A}^{-1} + \overline{A}^{-T}\mathcal{H}^{-T} \text{ with } \mathcal{H}, \overline{A} \text{ invertible} \quad (4.12a)$$

$$[\overline{A}]_{ij} = \begin{cases} \overline{a}_i > 0 & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases} \quad (4.12b)$$

As shown in [36], Proposition 3.1 also holds for the stochastic case. Thus we can define $u^o(t)$, $y^o(t)$ in the same way as in Section 3.3:

$$u_i^o(t) = \frac{b_{ii}}{b_{ii}^2 + r_i}(y_i^*(t+1) - \phi(t)^T\theta_i^o). \quad (4.13)$$

$$y^o(t+1) = A_c(q^{-1})y^o(t) + Gy^*(t+1) + v(t+1) \quad (4.14)$$
with the initial condition x_o.

Then we have the following result.

Theorem 4.1 *For the system (4.1) subject to the assumptions (V1)-(V3) and (A0)-(A5), the decentralized weighted minimum variance adaptive control scheme (4.5)-(4.7) yields*

(a) $\lim_{T\to\infty} \sum_{t=1}^{T} \frac{1}{\gamma(t)}\|z(t)\|^2 < \infty$ a. s.

(b) $\lim_{T\to\infty} \frac{T}{\gamma(T)}\frac{1}{T}\sum_{t=1}^{T} \|z(t)\|^2 < 0$ a. s.

(c) $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \|z(t)\|^2 < 0$ a. s.

(d) $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \|e(t)\|^2 < 0$ a. s.

(e) $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} E\{(y_i(t+1) - \hat{y}_i(t+1))^2|\mathcal{F}_t\} = \Gamma_{ii}$ a. s. $\forall i = 1$ to N.

(f) $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \|u(t) - u^o(t)\|^2 = 0$ a. s.

(g) $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \|y(t) - y^o(t)\|^2 = 0$ a. s.

(h) $\lim_{T\to\infty} \sup \frac{1}{T}\sum_{t=1}^{T} \|y(t)\|^2 < \infty$ a. s.

(i) $\lim_{T\to\infty} \sup \frac{1}{T}\sum_{t=1}^{T} \|u(t)\|^2 < \infty$ a. s.

Also, with $r_i = 0$ for all $i = 1$ to N, we have the counterpart of Theorem 3.2 for the stochastic case:

Theorem 4.2 *For the system (4.1) subject to the assumptions (V1)-(V3) and (A0)-(A3) and (A5), the decentralized minimum variance adaptive control scheme (4.5), (4.6) with $r_i = 0$ for all $i = 1$ to N yields*

(a) $\lim_{T\to\infty} \sup \frac{1}{T}\sum_{t=1}^{T} \|y(t)\|^2 < \infty$ a. s.

(b) $\lim_{T\to\infty} \sup \frac{1}{T}\sum_{t=1}^{T} \|u(t)\|^2 < \infty$ a. s.

(c) $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} E\{(y_i(t+1) - y_i^*(t+1))^2|\mathcal{F}_t\} = \Gamma_{ii}$ a. s. $\forall\ i = 1$ to N.

5. Conclusions

In this paper, we have established a decentralized adaptive control scheme which yields the closed-loop stability for a class of systems described by the AR model in a game situation where the objectives of several controllers are in conflict. As a by-product, our results suggest how and under what conditions a centralized adaptive control problem of a large scale system can be decentralized in such a way that distributed parallel computation can be executed to decrease the informational complexity and save on computational resources. One might appreciate the global efficiency of our scheme by recalling how powerfully the parallelism of the Jacobi iteration can be exploited for the computational purpose to find the inverse of a large-dimensional matrix with a dominant diagonal.

It is interesting to note that in a situation of conflict, the decentralized one-step-ahead control without penalization of the input can not make a compromise among the controllers who have no hesitation in exerting — even infinitely — big inputs in order to be dominant over others and to achieve their own objectives. The decentralized one-step-ahead control with penalization of the input can lead the controllers to self-restraint and mutual concession towards a compromise, while causing the non-exact tracking as a negative effect.

The results can be easily extended to the case where each controller has more than one output to control, which yields a relaxation of the weak coupling condition as a positive effect and an increase in the dimension of subproblems as a negative effect.

We have tried to extend our results to the ARMA model, but succeeded only with the one-step-delayed sharing information pattern under which each controller knows not only all the previous outputs, but also the past history of other controllers' inputs, while the computer simulation results obtained under observation sharing information pattern are as good as those obtained with classical information pattern under which all the information is commonly available to every controller.

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