On the Steady State Solution of a Two-By-Two Dynamic Jamming Game with Cumulative Power Constraints *

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Abstract

The process of jamming can be modelled as a two-person zero-sum non-cooperative dynamic game played between a communicator and a jammer over a number of discrete time instants. The simplest case is when, at each instant, the communicator and jammer randomize their strategies between idleness and transmission. The payoff (throughput) matrix is then two-by-two, with one variable parameter. The payoff function is the average throughput summed over time, to be optimized subject to cumulative power constraints. We find an analytical steady-state solution for this game played over an infinite time duration. Results show that when the throughput parameter is lower than a threshold, the optimal strategies are mixed, and the payoff increment constant; otherwise the strategies are pure, with the payoff increment exhibiting oscillatory behavior.

1 The Dynamic Jamming Game Model

The process of communication jamming can be modelled as a two-person zero-sum non-cooperative game played between a *communicator* and a *jammer*. In [Pen86][PS86], the static case, in which the game is played as a one-shot process, was analyzed. However, in most real situations, jamming is a continuous operation performed over time. Therefore in this paper, we consider the game being played over T discrete time instants, which we shall denote as $1,\ldots,T$. The game is dynamic in the sense that *the information and hence the action of each player at every time instant are influenced by the previous actions of both players.* We focus on the particular case in which the communicator's and jammer's *strategies are characterized by the power levels* of their transmitted signals.

Let X_{t_f} be the *communicator's power level at forward time* t_f, and Y_{t_f} the *jammer's power level at time* t_f, for $t_f = 1,2,\ldots,T$. We assume that X_{t_f} is *randomly distributed over two discrete values* $0, P$ ($P > 0$), and Y_{t_f} over the values $0, J$ ($J > 0$). Associated with each pair (X_{t_f}, Y_{t_f}) is a payoff $f(X_{t_f}, Y_{t_f})$ to the communicator, and $-f(X_{t_f}, Y_{t_f})$ to the jammer.

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Hence we have a 2×2 *payoff matrix*
$$G \triangleq \begin{bmatrix} f(0,0) & f(0,J) \\ f(P,0) & f(P,J) \end{bmatrix}.$$
A reasonable measure of the payoff is the normalized instantaneous throughput, which gives rise to the following *payoff matrix*:

$$G = \begin{bmatrix} 0 & 0 \\ 1 & \alpha \end{bmatrix}, \; 0 \le \alpha \le 1. \tag{1}$$

G has only one variable parameter, α, which is the instantaneous throughput as a function of P and J. We shall call α the *throughput parameter.*

The *payoff function*, which we denote as $\mathcal{J}$, is the time average of the mean throughput $\mathbb{E}[f(X_{t_f}, Y_{t_f})]$, i.e.

$$\mathcal{J} \triangleq \frac{1}{T}\sum_{t_f=1}^{T} \mathbb{E}[f(X_{t_f}, Y_{t_f})]. \tag{2}$$

1.1 Constraints

To account for protection from overheating of the communicating (jamming) transmitter, we assume that the communicator's (jammer's) accumulated power at forward time t_f should not exceed a threshold $\hat{P}$ ($\hat{J}$). This gives rise to the following *cumulative power constraints*:

$$\sum_{\tau=1}^{t_f} \delta_1^{t_f-\tau} X_\tau \le \hat{P}, \quad \sum_{\tau=1}^{t_f} \delta_2^{t_f-\tau} Y_\tau \le \hat{J}, \tag{3}$$

$t_f = 1,2,\ldots,T, \quad 0 < \delta_1, \delta_2 < 1.$

Here δ_1, δ_2 are *thermal memory constants*, $\hat{P}$ is the *communicator's peak power accumulation*, and $\hat{J}$ is the *jammer's peak power accumulation*.

The constraints in (3) are effective only when

$$P \le \hat{P} < \frac{P(1-\delta_1^T)}{(1-\delta_1)}, \quad J \le \hat{J} < \frac{J(1-\delta_2^T)}{(1-\delta_2)} \tag{4}$$

are satisfied, and hence these define the interesting ranges of transmitter parameters.

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Let Z_{t_f} (W_{t_f}) be a random variable representing the *communicator's (jammer's) past power accumulation at forward time* t_f. Thus from (3), $Z_1 \triangleq 0$, $W_1 \triangleq 0$, and

$$Z_{t_f} \triangleq X_{t_f-1} + \delta_1 Z_{t_f-1}, \quad W_{t_f} \triangleq Y_{t_f-1} + \delta_2 W_{t_f-1}, \tag{5}$$

for $t_f = 2, \ldots, T$.

1.2 Conditional Independence Assumption

We assume that (a) each player has knowledge of the opponent's previous actions, (b) X_1 and Y_1 are independent, and (c) for $t_f = 2, \ldots, T$, X_{t_f} and Y_{t_f} are conditionally independent, given $X_1, \ldots, X_{t_f-1}$, $Y_1, \ldots, Y_{t_f-1}$ (and therefore given Z_{t_f}, W_{t_f}).

Thus we have a *dynamic stochastic game model* with a *Markovian evolution* (see [BO82]), in which (Z_{t_f}, W_{t_f}) is the value of the *state of the system at forward time* t_f. The state equation is given by (5).

1.3 Effect of Constraints

For the communicator, at each $t_f = 2, \ldots, T$, we define the *set of past power accumulations* to be

$$\begin{aligned} \Phi_{t_f} \triangleq \{ z : z = \sum_{\tau=1}^{t_f-1} \delta_1^{t_f-1-\tau} x_\tau, \\ \sum_{\tau=1}^{t'_f} \delta_1^{t'_f-\tau} x_\tau \leq \hat{P},\ t'_f = 1, .., t_f - 1, \\ x_1, \ldots, x_{t_f-1} \in \{0, P\} \}. \end{aligned} \tag{6}$$

Note that $\Phi_1 \triangleq \{0\}$ and $\Phi_2 = \{0, P\}$.

For each $z \in \Phi_1 \cup \cdots \cup \Phi_T$, we define $\mathcal{X}_z$ to be the *communicator's set of allowable power levels at any time instant given that it has accumulated z amount of power in the past.* Thus

$$\mathcal{X}_z \triangleq \{x : x \in \{0, P\}, x + \delta_1 z \leq \hat{P}\}. \tag{7}$$

Similarly, for the jammer, constraints (3) determine the analogous sets Ψ_{t_f} and $\mathcal{Y}_w$.

1.4 The Strategies and the Payoff

Let $p_t(x|z, w)$ ($q_t(y|z, w)$) denote the *probability that the communicator (jammer) selects power level x (y) at reverse time t (i.e., forward time* $T - t$*), given that the communicator and jammer have accumulated z and w amounts of power respectively in the past,* where $x \in \mathcal{X}_z$, $y \in \mathcal{Y}_w$, for each $z \in \Phi_{T-t}$, $w \in \Psi_{T-t}$, $t = 0, \ldots, T-2$. We can then define the *strategy vectors*

$$\mathbf{p}_t(z, w) \triangleq [\cdots p_t(x|z, w) \cdots]', \tag{8a}$$

$$\mathbf{q}_t(z, w) \triangleq [\cdots q_t(y|z, w) \cdots]'. \tag{8b}$$

Let the *initial power selection probabilities* (at reverse time $T - 1$) be

$$p_{T-1}(x) \triangleq \text{Prob}(X_1 = x), \quad x \in \Phi_2 = \{0, P\}, \tag{9a}$$

$$q_{T-1}(y) \triangleq \text{Prob}(Y_1 = y), \quad y \in \Psi_2 = \{0, J\}. \tag{9b}$$

The *initial strategy vector* $\mathbf{p}_{T-1}$ for the communicator can then be defined as

$$\mathbf{p}_{T-1} \triangleq [p_{T-1}(0) \ \ p_{T-1}(P)]'. \tag{10}$$

We define the analogous vector $\mathbf{q}_{T-1}$ for the jammer.

The payoff $\mathcal{J}$ in (2) can be expressed in terms of the last term of a sequence $\{S_0, S_1, \ldots, S_{T-1}\}$, defined as follows:

$$\begin{aligned} S_0 &= \mathbb{E}_{X_T, Y_T | Z_T, W_T}[f(X_T, Y_T)], & \text{(11a)} \\ S_t &= \mathbb{E}_{X_{T-t}, Y_{T-t} | Z_{T-t}, W_{T-t}} \\ &\quad [f(X_{T-t}, Y_{T-t}) + S_{t-1}], & \text{(11b)} \\ &\quad \text{for } t = 1, \ldots, (T-2), \\ S_{T-1} &= \mathbb{E}_{X_1, Y_1}[f(X_1, Y_1) + S_{T-2}], \text{ and} & \text{(11c)} \\ \mathcal{J} &= \frac{S_{T-1}}{T}. & \text{(11d)} \end{aligned}$$

S_t is thus the *accumulated payoff at reverse time t given the past power accumulations* Z_{T-t} *and* W_{T-t}. Equation (11) can be written in terms of the strategies defined in (8) and (10).

Let $\mathcal{P}_t$ be *the set of all strategy vectors for the communicator at reverse time t, each vector being conditioned upon a past power accumulation* $z \in \Phi_{T-t}$ *and a* $w \in \Psi_{T-t}$*, and having a dimension of* $|\mathcal{X}_z|$, for $t = 0, \ldots, T-2$. Thus

$$\mathcal{P}_t \triangleq \{ \mathbf{p}_t(z, w) : z \in \Phi_{T-t}, w \in \Psi_{T-t} \}. \tag{12}$$

The set $\mathcal{P}_t$ contains $|\Phi_{T-t}|.|\Psi_{T-t}|$ probability vectors.

Similarly, for the jammer, we can define the sets $\mathcal{Q}_0, \ldots, \mathcal{Q}_{T-2}$ of strategy vectors.

The *communicator's strategy set* Γ and the *jammer's strategy set* Δ, given by

$$\Gamma \triangleq \{\mathbf{p}_{T-1}\} \cup \mathcal{P}_{T-2} \cup \cdots \cup \mathcal{P}_0,$$

$$\Delta \triangleq \{\mathbf{q}_{T-1}\} \cup \mathcal{Q}_{T-2} \cup \cdots \cup \mathcal{Q}_0,$$

then determine the payoff $\mathcal{J}$.

1.5 Characterization of the Solution

Solving the dynamic game subject to the cumulative power constraints in (3) reduces to the following problem:

Find Γ^*, Δ^* *such that*

$$\mathcal{J}(\Gamma, \Delta^*) \leq \mathcal{J}(\Gamma^*, \Delta^*) \leq \mathcal{J}(\Gamma^*, \Delta) \quad \forall\ \Gamma, \Delta.$$

Then $\mathcal{J}^* = \mathcal{J}(\Gamma^*, \Delta^*)$ is the *value of the game* and (Γ^*, Δ^*) is the *set of optimal strategies* (see [BG54]).

1.6 The Evolution Equation

When the duration T is finite, dynamic programming (see [Ber76]) can be applied to find $\mathcal{J}^*$ (see (11)). For reverse time $t = 0, 1, 2, \ldots, (T-1)$, we optimize $S_t(z, w)$ with respect to $\mathbf{p}_t(z, w)$ and $\mathbf{q}_t(z, w)$ to obtain $\mathbf{p}_t^*(z, w)$, $\mathbf{q}_t^*(z, w)$ and $S_t^*(z, w)$. Each optimization process involves solving a $|\mathcal{X}_z| \times |\mathcal{Y}_w|$ matrix game, where $z \in \Phi_{T-t}$, $w \in \Psi_{T-t}$.

In terms of the optimum payoffs, equation (11) gives rise to the following evolution equations:

$$S_0^*(z,w) = \max_{\mathbf{p}_0(z,w)} \min_{\mathbf{q}_0(z,w)} \sum_{x\in\mathcal{X}_z} \sum_{y\in\mathcal{Y}_w} p_0(x|z,w) q_0(y|z,w) f(x,y) \quad (13)$$

$z \in \Phi_T,\ w \in \Psi_T,$

$$S_{t+1}^*(z,w) = \max_{\mathbf{p}_{t+1}(z,w)} \min_{\mathbf{q}_{t+1}(z,w)} \sum_{x\in\mathcal{X}_z} \sum_{y\in\mathcal{Y}_w} p_{t+1}(x|z,w) q_{t+1}(y|z,w) [f(x,y) + S_t^*(x+\delta_1 z, y+\delta_2 w)] \quad (14)$$

$z \in \Phi_{T-t-1},\ w \in \Psi_{T-t-1}$, for $t = 0, 1, \ldots, (T-2)$.
From (7) it is clear that

$$\mathcal{X}_z = \begin{cases} \{0, P\} & \text{if } 0 \le z \le \frac{\hat{P}-P}{\delta_1}, \\ \{0\} & \text{if } \frac{\hat{P}-P}{\delta_1} < z \le \hat{P}. \end{cases} \quad (15)$$

We have an analogous expression for $\mathcal{Y}_w$.

2 Simplification of the Problem

2.1 The Payoff Function

From the structures of $\mathcal{X}_z$ (see (15)) and $\mathcal{Y}_w$, it can be shown that when the conditions

$$\frac{1}{(1-\delta_1^2)} < \frac{\hat{P}}{P} < (1+\delta_1), \quad \frac{1}{(1-\delta_2^2)} < \frac{\hat{J}}{J} < (1+\delta_2) \quad (16)$$

are satisfied, a 2×2 *grid solution* to (14) exists in which $S_t^*(z,w)$ is a *four-valued function* defined as follows:

$$S_t^*(z,w) = \begin{cases} S_{11}^t & \text{if } 0 \le z \le \frac{\hat{P}-P}{\delta_1}, & 0 \le w \le \frac{\hat{J}-J}{\delta_2} \\ S_{12}^t & \text{if } 0 \le z \le \frac{\hat{P}-P}{\delta_1}, & \frac{\hat{J}-J}{\delta_2} < w \le \hat{J} \\ S_{21}^t & \text{if } \frac{\hat{P}-P}{\delta_1} < z \le \hat{P}, & 0 \le w \le \frac{\hat{J}-J}{\delta_2} \\ S_{22}^t & \text{if } \frac{\hat{P}-P}{\delta_1} < z \le \hat{P}, & \frac{\hat{J}-J}{\delta_2} < w \le \hat{J}. \end{cases} \quad (17)$$

for $t = 1, \ldots, (T-1)$. From (13) the solution at reverse time $t = 0$ is found to be

$$S_{11}^0 = \alpha, \ S_{12}^0 = 1, \ S_{21}^0 = S_{22}^0 = 0. \quad (18)$$

The inequalities in (16) determine the *operating regions of the players* in the "$\frac{\hat{P}}{P}$ *versus* δ_1" and "$\frac{\hat{J}}{J}$ *versus* δ_2" planes respectively, for which this form of solution exists.

2.2 Optimal Strategies

Let

$$p_t^*(P|z,w) = \begin{cases} p_{11}^t & \text{if } 0 \le z \le \frac{\hat{P}-P}{\delta_1}, \ 0 \le w \le \frac{\hat{J}-J}{\delta_2} \\ p_{12}^t & \text{if } 0 \le z \le \frac{\hat{P}-P}{\delta_1}, \ \frac{\hat{J}-J}{\delta_2} < w \le \hat{J} \\ p_{21}^t & \text{if } \frac{\hat{P}-P}{\delta_1} < z \le \hat{P}, \ 0 \le w \le \frac{\hat{J}-J}{\delta_2} \\ p_{22}^t & \text{if } \frac{\hat{P}-P}{\delta_1} < z \le \hat{P}, \ \frac{\hat{J}-J}{\delta_2} < w \le \hat{J}. \end{cases} \quad (19)$$

Similarly we can define q_{ij}^t's corresponding to $q_t^*(J|z,w)$. The p_{ij}^t's and q_{ij}^t's are the *optimum probabilities of transmission at reverse time t*. Using (8) and (15), we can show that

$$p_{22}^t = p_{21}^t = 0, \quad q_{22}^t = q_{12}^t = 0.$$

2.3 The Simplified Evolution Equation

The quantities z and w are measures of the *communicator's and jammer's power accumulations.* As z increases, the communicator transmits less frequently (i.e., with lower probability), and the throughput decreases. Therefore, for any reverse time index t, $S_t^*(z,w)$ *decreases as z increases.* On the other hand, an increase in w causes the jammer to transmit less frequently, which increases the throughput. Thus $S_t^*(z,w)$ *increases as w increases.*

Substitution of the payoff function of (17) in (14) results in four equations, one for each region of the (z,w) plane. The quantity S_{11}^{t+1} is the value of a 2×2 game whose payoff matrix depends on the S_{ij}^t's. In the other three regions, we need to solve a 2×1, a 1×2 and a 1×1 game, all of which are trivial. When we take into account the fact that $S_t^*(z,w)$ decreases (increases) with increase in z (w), we obtain the conditions

$$S_{12}^t \ge S_{11}^t \ge S_{21}^t = S_{22}^t, \quad S_{11}^t \le (1+S_{21}^t),$$

and (14) simplifies to

$$S_{11}^{t+1} = \text{value}\left(\begin{bmatrix} S_{11}^t & S_{12}^t \\ 1+S_{21}^t & \alpha+S_{22}^t \end{bmatrix}\right) \quad (20a)$$

$$S_{12}^{t+1} = \max(S_{11}^t, 1+S_{21}^t) = 1+S_{21}^t \quad (20b)$$

$$S_{21}^{t+1} = \min(S_{11}^t, S_{12}^t) = S_{11}^t \quad (20c)$$

$$S_{22}^{t+1} = S_{11}^t \quad (20d)$$

Also from (19)

$$\max(S_{11}^t, 1+S_{21}^t) = 1+S_{21}^t \implies p_{12}^t = 1,$$
$$\min(S_{11}^t, S_{12}^t) = S_{11}^t \implies q_{21}^t = 0. \quad (21)$$

3 The Steady State Solution

For a given finite time duration T, the evolution equation (20) can be solved using the initial conditions in (18) to obtain the optimal strategies p_{11}^t, q_{11}^t for $t = 1, \ldots, (T-1)$. The limiting behavior of the optimal strategies when the reverse time index goes to infinity is of particular interest, since it gives us a better analytical feel of the problem. So we attempt to solve the evolution equation for $S_t^*(z,w)$ when $t \to \infty$.

3.1 The Payoff Increments

From (14), it is clear that the *optimum payoff* S_{ij}^t ($i, j \in \{1,2\}$) increases as t increases, but its *increment* (in going from t to $(t+1)$) is bounded and lies in [0,1], since each element of the payoff matrix (see (1)) lies in [0,1].

Let

$$\lambda_{ij}^t \triangleq S_{ij}^{t+1} - S_{ij}^t, \quad i = 1,2, \quad j = 1,2, \quad t = 0,1,2,\cdots \quad (22)$$

The condition $0 \leq \lambda_{ij}^t \leq 1$ holds.

Substituting $S_{ij}^{t+1} = S_{ij}^t + \lambda_{ij}^t$ in (20) and eliminating S_{11}^t, S_{12}^t and S_{22}^t, the equations simplify to

$$\lambda_{11}^t + \lambda_{22}^t \;=\; \text{value}\left(\begin{bmatrix} \lambda_{22}^t & 1-\lambda_{12}^t \\ 1 & \alpha \end{bmatrix}\right). \qquad (23)$$

Note that $S_{21}^t = S_{22}^t$ implies $\lambda_{21}^t = \lambda_{22}^t$.

If we *assume that the optimal strategies exhibit steady state behavior* as $t \to \infty$, i.e.,

$$\lim_{t\to\infty} p_{11}^t = p_{11}\,, \quad \lim_{t\to\infty} q_{11}^t = q_{11}\,,$$

exist, then from (20), we get

$$\lambda_{11}^{t+1} \;=\; \text{value}\left(\begin{bmatrix} \lambda_{11}^t & \lambda_{12}^t \\ \lambda_{22}^t & \lambda_{22}^t \end{bmatrix}\right) \qquad (24a)$$

$$\lambda_{12}^{t+1} \;=\; \lambda_{22}^t \qquad (24b)$$

$$\lambda_{22}^{t+1} \;=\; \lambda_{11}^t\,. \qquad (24c)$$

3.2 Case 1: Mixed Strategies

Assume that the strategies p_{11} and q_{11} are mixed, i.e. $0 < p_{11}, q_{11} < 1$. We can show from (24) that this implies

$$\lambda_{11}^t = \lambda_{12}^t = \lambda_{21}^t = \lambda_{22}^t \triangleq \lambda \quad \forall t \quad \text{when } t \to \infty. \qquad (25)$$

Equation (23) then gives

$$\lambda \;=\; \frac{(5-\alpha) - \sqrt{(9-\alpha)(1-\alpha)}}{8}. \qquad (26)$$

We see that λ increases as α increases. It can be shown that (25) holds true only when $(1-\lambda) \geq \alpha$.

When $(1-\lambda) = \alpha$, substitution in (26) gives $\alpha = \frac{2}{3}$, $\lambda = \frac{1}{3}$. Therefore, *(25) holds true for* $0 \leq \alpha \leq \frac{2}{3}$ *only*. In this range, λ increases from $\frac{1}{4}$ at $\alpha = 0$ to $\frac{1}{3}$ at $\alpha = \frac{2}{3}$.

The *steady state mixed strategies* are given by

$$p_{11} = \frac{(1-2\lambda}{2(1-\lambda)-\alpha}\,, \quad q_{11} = \frac{(1-\lambda}{2(1-\lambda)-\alpha}\,, \qquad (27)$$

where λ is given by (26). Thus p_{11} and q_{11} are functions of α.

3.3 Case 2: Pure Strategies

When we assume that the strategies p_{11} and q_{11} are pure, they take values 0 or 1 only. The range of α for this case is $\frac{2}{3} < \alpha \leq 1$. It can be shown that only $p_{11} = q_{11} = 1$ may be possible.

Assuming $p_{11} = q_{11} = 1$, we get the following result:

When $t \to \infty$, if, for any reverse-time index t

$$\lambda_{11}^t = \lambda_{12}^t = \lambda_1\,, \quad \lambda_{22}^t = \lambda_{21}^t = \lambda_2\,,$$

then, at the next reverse-time index $(t+1)$, we have

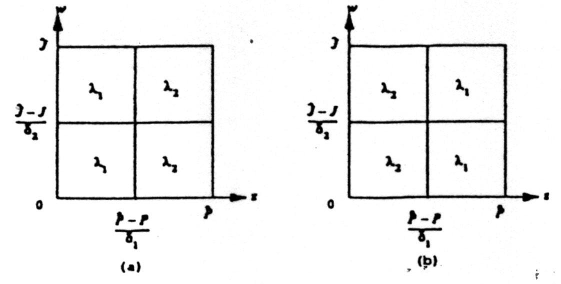

Figure 1: Oscillatory Behavior of Payoff Increment in Steady State

$$\lambda_{11}^{t+1} = \lambda_{12}^{t+1} = \lambda_2\,, \quad \lambda_{22}^{t+1} = \lambda_{21}^{t+1} = \lambda_1\,.$$

The λ_{ij}^t plots on the (z, w) plane are shown in Fig 1. The plot *oscillates* between the pattern in Fig 1a and that in Fig 1b. The *period of oscillation* in terms of the number of reverse time instants is 2.

The next step is to find the steady state payoff increments λ_1 and λ_2 in terms of α. At reverse-time t, let $\lambda_{11}^t = \lambda_{12}^t = \lambda_1\,,\ \lambda_{22}^t = \lambda_2$. It can be shown that we can have

$$\lambda_1 = (1-\alpha)\,, \quad \lambda_2 = (2\alpha - 1) \qquad (28a)$$

$$\text{or} \quad \lambda_1 = (2\alpha - 1)\,, \quad \lambda_2 = (1-\alpha). \qquad (28b)$$

Note that *(28) holds true for* $\frac{2}{3} < \alpha \leq 1$ *only*. The *steady state pure strategies* are given by

$$p_{11} = 1\,, \quad q_{11} = 1\,. \qquad (29)$$

Here the strategies do not vary with variation in α as long as α lies in the range $(\frac{2}{3}, 1]$.

3.4 Results

The optimum steady state transmission probabilities (p_{ij}'s and q_{ij}'s) tell us how the strategies of the players vary with changes in the amounts of power accumulated (z and w) at any time instant as the game progresses. We find that when the communicator's power accumulation (z) exceeds $\frac{\hat{P}-P}{\delta_1}$, it is forced to remain idle (i.e. $p_{21} = p_{22} = 0$), because if it does not, the present power accumulation $x + \delta_1 z$ will exceed $\hat{P}$, violating the power constraint. Since the jammer has nothing to jam in this situation, it remains idle too (i.e. $q_{21} = q_{22} = 0$). When the jammer's power accumulation (w) exceeds $\frac{\hat{J}-J}{\delta_2}$, but the communicator's does not, the jammer remains idle ($q_{12} = 0$) to keep the present power accumulation $y + \delta_2 w$ below $\hat{J}$, while the communicator takes advantage of the situation by transmitting with 100% probability ($p_{12} = 1$) and achieving success.

When z and w satisfy $0 \leq z \leq \frac{\hat{P}-P}{\delta_1}$ and $0 \leq w \leq \frac{\hat{J}-J}{\delta_2}$, the communicator and jammer transmit with probabilities p_{11} and q_{11} respectively. These probabilities define the strategies of both players, which depend upon α.

Fig 2 is a plot of p_{11} and q_{11} versus α. *For* $0 \le \alpha \le \frac{2}{3}$ *the strategies are mixed, i.e.* $0 < p_{11}, q_{11} < 1$. The probabilities increase as α increases. *For* $\frac{2}{3} < \alpha \le 1$ *the strategies are pure, i.e.* $p_{11} = q_{11} = 1$. Note that in the mixed strategy zone, $p_{11} < q_{11}$, implying that the communicator has to transmit less frequently compared to the jammer to attain optimum throughput. As a result, there is a jump in the p_{11} curve at $\alpha = \frac{2}{3}$ from $\frac{1}{2}$ to 1. The q_{11} curve is continuous.

The *steady state payoff increment* is a single quantity λ (see (25)) when $0 \le \alpha \le \frac{2}{3}$, and oscillates between two quantities λ_1 and λ_2 (see (28)) when $\frac{2}{3} < \alpha \le 1$. A plot of these quantities is shown in Fig 3. As α increases, λ increases from $\frac{1}{4}$ at $\alpha = 0$ to $\frac{1}{3}$ at $\alpha = \frac{2}{3}$, after which it *splits* into two quantities $\lambda_1 = (1 - \alpha)$ and $\lambda_2 = (2\alpha - 1)$. One decreases to 0 at $\alpha = 1$ whereas the other increases to 1.

4 Conclusion

When the throughput parameter α is low, the communicator has a greater need to transmit in order to maximize the payoff. The cumulative power constraint prevents him from transmission at full power with 100% probability at all times, as a result he uses a randomized mixed strategy so that he can transmit most of the time. To counteract the communicator, the jammer uses a mixed strategy too. The payoff increment, which is a measure of the average increase in throughput, also remains constant at all times in steady state because of the possible presence of the same mixed strategy at all times. On the other hand, when the throughput parameter is high (closer to 1), the communicator need not bother so much about transmitting at all times with mixed strategies. So he uses a pure strategy at one time, but this forces him into idleness at the next time instant, to satisfy the power constraint. Again, at the next time instant, he transmits with probability 1, and the phenomenon is repeated. To counteract the communicator, the jammer does the same thing. As a result, the payoff increment oscillates between a high value and a low value. This explains the mathematical results obtained.

It must also be noted that the 2×2 grid structure of the optimum accumulated payoff (see (17)) is not the only possible one that yields a steady state solution in terms of payoff increments and optimal strategies. It covers only small portions in the $\frac{\dot{P}}{P}$ *versus* δ_1 and $\frac{\dot{J}}{J}$ *versus* δ_2 planes (see (16)). In this paper we have discussed the simplest of the infinitely many steady state solutions of this game.

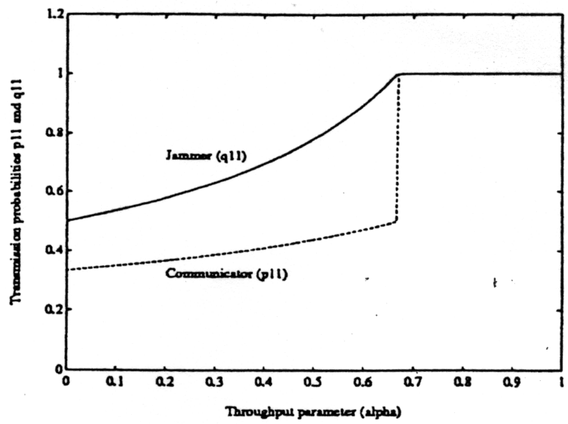

Figure 2: Steady State Transmission Probability versus Throughput Parameter α

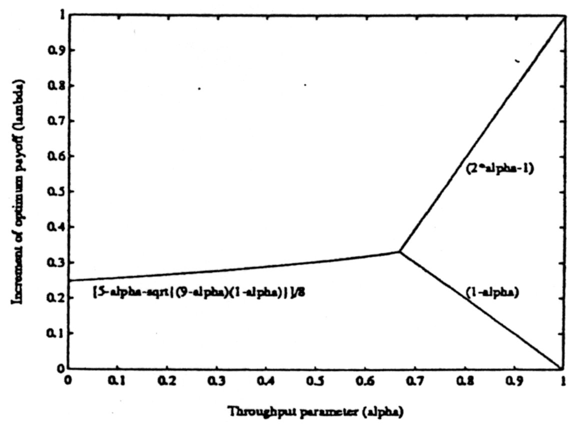

Figure 3: Steady State Payoff Increment versus Throughput Parameter α

References

[Ber76] D. P. Bertsekas. *Dynamic Programming and Stochastic Control.* Academic Press, 1976.

[BG54] D. Blackwell and M. A. Girshick. *Theory of Games and Statistical Decisions.* Dover Pub., 1954.

[BO82] T. Basar and G. J. Olsder. *Dynamic Non-Cooperative Game Theory.* Academic Press, 1982.

[Pen86] Wei-Chung Peng. *Some Communication Jamming Games.* PhD thesis, University of Southern California, 1986.

[PS86] Wei-Chung Peng and Robert A. Scholtz. A Study of Discrete Link Jamming Games. In *MILCOM Record, Vol. 1*, pages 2.6.1–2.6.10, 1986.