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ON A CLASS OF DECENTRALIZED DISCRETE-TIME ADAPTIVE CONTROL PROBLEMS†

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ABSTRACT

This paper considers a decentralized adaptive control problem for a class of discrete-time multi-input multi-output deterministic linear systems. It is shown that, under a weak coupling condition, the Projection Algorithm will ensure that the system inputs and outputs remain bounded for all time and that each controller can successfully make the corresponding output tracking error converge to zero.

I. INTRODUCTION

A lot of research has been done on the deterministic adaptive control problem [1], [4], [5]. Almost all the papers in this area deal with the centralized problem with one controller. Very little work has been done on the decentralized scheme with many controllers. Recent work by Chan [2] is one of the few dealing with the adaptive control problem in a decentralized setup and is quite related with the problem studied here.

The problem dealt with in [2] is roughly as follows: For an ARMAX model with a vector input u(t) and a vector output y(t), each component $u_i(t)$ of u(t) is chosen by a controller i whose interest lies in bringing the corresponding component $y_i(t)$ of y(t) to a desired value $y_i^*(t)$, while penalizing a quadratic function of $(u_i(t)-u_i(t-1))$. At time t, each controller i knows the past history of the vectors y(t) and u(t). The reference signals $y_i^*(t)$'s are assumed to be known to every controller. It should be noticed that in [2] all the controllers have exactly the same information, i.e., there is no decentralization of the information and that the decentralized character of the model of [2] lies in the difference among the objectives of the controllers. This problem can be thought of as a Nash game where the players do not have knowledge of the system parameters, but they have the same information. Chan ([2]) considers that each controller estimates the parameters of the overall system using the stochastic approximation algorithm and uses the estimates to calculate his control. Thus, his problem can be cast as a centralized one so that he is able to apply directly the known results pertaining to this case [6] and obtain boundedness of the inputs and outputs and asymptotic tracking by the outputs of the reference signals.

In the present paper, we deal with a different situation. Our model is a DAR model which is simpler than that of [2] in the sense that we do not consider delays in the controls as well as no penalization by each controller of his control. The information known to each controller is the same as in [2], except that each controller doesn't know the past history of u(t). (It should be noted that, if the model of [2] does not have delays in the controls, no knowledge is required of the history of u(t) so that the information availabl to each controller is the same as in this paper.) An important difference between Chan's work [2] and the work presented here is that each controller in our scheme does not estimate all the parameters of the whole systems but only those pertaining to his subsystem, i.e., there is a bona fide decentralization in the calculation of parameter estimates and controls. Another difference lies in the fact that we employ the projection algorithm. Under the assumption of weak coupling, we show that all the inputs and outputs are bounded and asymptotic tracking is achieved.

II. PROBLEM FORMULATION

We consider the decentralized adaptive control of a linear time-invariant finite-dimensional deterministic system described by

$$y(t+1) = \alpha(q^{-1})y(t) + \beta u(t) \tag{1}$$

where

$$\alpha(q^{-1}) = \begin{bmatrix} a_{11}(q^{-1}) & a_{12}(q^{-1}) & \cdots & a_{1N}(q^{-1}) \\ a_{21}(q^{-1}) & a_{22}(q^{-1}) & \cdots & a_{2N}(q^{-1}) \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1}(q^{-1}) & a_{N2}(q^{-1}) & \cdots & a_{NN}(q^{-1}) \end{bmatrix}; \quad \beta = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1N} \\ b_{21} & b_{22} & \cdots & b_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N1} & b_{N2} & \cdots & b_{NN} \end{bmatrix}$$

where:

- u(t), y(t) are the $N \times 1$ output vector.
- $\alpha(q^{-1})$ is an $N \times N$ matrix, each of whose elements i a polynomial denoted by $a_{ij}(q^{-1}) = a_{0,ij} + a_{1,ij}q^{-1} + \dots + a_{n_j,ij}q^{-n_j}$.
- β is a $N \times N$ constant matrix.
- the initial conditions are given.
- q^{-1} denotes the unit delay operator.

At time t, each controller i knows $\{Y_i(t), y_i^*(t+1); i = 1 \text{ to } N\}$ and is supposed to apply his control $u_i(t)$ as to minimize

$$J_i(t) = (y_i(t+1) - y_i^*(t+1))^2 \tag{2}$$

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where

$Y_i(t) = \{y_i(t), y_i(t-1), \ldots, y_i(0)\}$

$y_i^*(t+1)$: the desired uniformly bounded output-sequence of the controller i.

The objective function (2) implies that each controller i tries to bring $y_i(t+1)$ to his desired value $y_i^*(t+1)$ one-step-ahead, which would be immediately possible if each controller knew every parameter of the whole system.

We will make the following assumptions:

(A0) The basic structure of the system is known.

(A1) Upperbounds for the orders of the polynomials $a_{ij}(q^{-1})$ are the same for $i = 1$ to N and they are known as n_j.

(A2) The structure of the constant matrix B satisfies the following (weak coupling type) condition:

$|b_{ij}| < |b_{jj}|/(N-1) \quad \forall\, j \neq i$, and

$|b_{jj}| > 0$ for $j = 1$ to N.

(A3) The polynomial $a_{ij}(q^{-1})$'s are unknown, but the parameter b_{ii} is known to controller i $\forall\, i = 1$ to N.

Under the above assumptions, we will show that each controller taking care of only his subsystem (4) can achieve his own objective asymptotically in the sense that

$$\lim_{t\to\infty} |e_i(t)| = \lim_{t\to\infty} |y_i(t) - y_i^*(t)| = 0 \quad \forall\, i = 1 \text{ to } N \quad (3)$$

and that $u_i(t)$, $y_i(t)$ are uniformly bounded.

III. DECENTRALIZED ADAPTIVE CONTROL

Suppose that each controller i regards his subsystems as governed by

$$\begin{aligned} y_i(t+1) &= \alpha_{i1}(a^{-1})y_1(t) + \ldots + \alpha_{iN}(q^{-1})y_N(t) \\ &\quad + b_{ii}u_i(t) + c_{i1}y_1^*(t+1) + \ldots + c_{iN}y_N^*(t+1) \\ &= \phi(t)^T\theta^{(i)} + b_{ii}u_i(t) \end{aligned}$$

where

$$\phi^T(t) = (y_1(t), \ldots, y_1(t-n_1); \ldots; y_N(t), \ldots, y_N(t), \ldots \ldots, y_N(t-n_N);\ y_1^*(t+1), \ldots, y_N^*(t+1)) \quad (5a)$$

$$\theta^{(i)} = (\alpha_{0,i1}, \ldots, \alpha_{n_1,i1}; \ldots; \alpha_{0,iN}, \ldots, \alpha_{n_N,iN};\ c_{i1}, \ldots, c_{iN}) \quad (5b)$$

If controller i knows $\theta^{(i)}$, then he would set

$$y_i^*(t+1) = \phi(t)^T\theta^{(i)} + b_{ii}u_i(t) \quad (6)$$

to obtain his control input

$$u_i(t) = \frac{1}{b_{ii}}\,(y_i^*(t+1) - \phi(t)^T\theta^{(i)}), \quad (7)$$

so that the tracking error would be identically zero. However, since $\theta^{(i)}$ is unknown, we replace (6) by

$$y_i^*(t+1) = \hat{y}_i(t+1) \triangleq \phi(t)^T\hat{\theta}^{(i)}(t) + b_{ii}u_i(t) \quad (8)$$

so that each controller i would apply the control

$$u_i(t) = \frac{1}{b_{ii}}\,(y_i^*(t+1) - \phi(t)^T\hat{\theta}^{(i)}(t)) \quad (9)$$

where the parameter estimate $\hat{\theta}^{(i)}(t)$ is generated by the Projection Algorithm, i.e.,

$$\hat{\theta}^{(i)}(t+1) = \hat{\theta}^{(i)}(t) + \frac{\phi(t)}{1 + \phi(t)^T\phi(t)}\,(y_i(t+1) - \hat{y}_i(t+1)) \quad (10)$$

Note that, using (1) and (8) with (9), we have

$$\begin{aligned} e_i(t+1) &\triangleq y_i(t+1) - \hat{y}_i(t+1) \\ &= a_{i1}(q^{-1})y_1(t) + \ldots + a_{iN}(q^{-1})y_N(t) + b_{ii}u(t) \\ &\quad + \sum_{k\neq i} b_{ik}u_k(t) - \hat{\alpha}_{i1}(q^{-1})y_1(t) - \ldots - \hat{\alpha}_{iN}(q^{-1})y_N(t) \\ &\quad - b_{ii}u_i(t) - \hat{c}_{i1}y_1^*(t+1) - \ldots - \hat{c}_{iN}y_N^*(t+1) \\ &= -[\hat{\alpha}_{i1}(q^{-1}) - \{a_{i1}(q^{-1}) - \sum_{k\neq i}^{N} h_{ik}\hat{\alpha}_{k1}(q^{-1})\}]y_1(t) \\ &\quad - \ldots - [\alpha_{iN}(q^{-1}) - \{a_{iN}(q^{-1}) - \sum_{k\neq i}^{N} h_{ik}\hat{\alpha}_{kN}(q^{-1})\}]y_N(t) \\ &\quad - \{\hat{c}_{ii} - \sum_{k\neq i}(-h_{ik})\hat{c}_{ki}\}y_i^*(t+1) - \sum_{j\neq i}\hat{c}_{ij} \\ &\quad - \sum_{k\neq i}(-h_{ik})\hat{c}_{kj} - h_{ij}\}y_j^*(t+1) \end{aligned} \quad (11)$$

where

$$h_{ik} \triangleq b_{ik}/b_{kk} \quad (12)$$

Now, let us define $N \times \{\sum_{k=1}^{N}(n_k+1) + N\}$ parameter values by the following sets of simultaneous linear equations for $j = 1$ to N.

$$\left.\begin{aligned} \alpha_{1j}(q^{-1}) &= a_{1j}(q^{-1}) - \sum_{k\neq 1}^{N} h_{1k}\alpha_{kj}^o(q^{-1}) \\ &\vdots \\ \alpha_{Nj}^o(q^{-1}) &= a_{Nj}(q^{-1}) - \sum_{k\neq N}^{N} h_{NK}\alpha_{kj}^o(q^{-1}) \end{aligned}\right\} \quad (12a)$$

$$\left.\begin{aligned} c_{1j}^o &= h_{1j}\bar{\delta}_{1j} - \sum_{k\neq 1}^{N} h_{1k}c_{kj}^o \\ &\vdots \\ c_{Nj}^o &= h_{Nj}\bar{\delta}_{Nj} - \sum_{k\neq N}^{N} h_{Nk}c_{kj}^o \end{aligned}\right\} \quad (12b)$$

where

$$\alpha_{ij}^o(q^{-1}) = \alpha_{0,ij}^o + \alpha_{1,ij}^o q^{-1} + \ldots + \alpha_{n_j,ij}^o q^{-n_j}$$

$$\bar{\delta}_{ij} = 1 - \delta_{ij} = \begin{cases} 1 & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}.$$

Also, we define the parameter estimation error

$$\tilde{\alpha}_{ij}(q^{-1}) = \hat{\alpha}_{ij}(q^{-1}) - \alpha^{o}_{ij}(q^{-1}) \quad (13a)$$

$$\tilde{c}_{ij} = \hat{c}_{ij} - c^{o}_{ij} \quad (13b)$$

Notice that from the parameter estimates $\hat{\alpha}_{ij}(q^{-1})$ and $\hat{c}_{ij}$, the time index has been omitted.

Then, letting

$$\hat{\alpha}_{kj}(q^{-1}) = \alpha^{o}_{kj}(q^{-1}) + \tilde{\alpha}_{kj}(q^{-1})$$

$$\hat{c}_{kj} = c^{o}_{kj} + \tilde{c}_{kj}$$

we can rewrite (11) as

$$e_i(t+1) = y_i(t+1) - \hat{y}_i(t+1) = y_i(t+1) - y^{*}_i(t+1)$$
$$= -\phi(t)^T[\tilde{\theta}^{(i)}(t) + \sum_{k\neq i}^{N} h_{ik}\tilde{\theta}^{(k)}(t)] \quad (14)$$

where

$$\hat{\theta}^{(i)}(t)^T = (\hat{\alpha}_{0,i1}(t),\ldots,\hat{\alpha}_{n_1,i1}(t);\ldots;\hat{\alpha}_{0,iN}(t),\ldots$$
$$\ldots,\hat{\alpha}_{n_N,iN}(t);\hat{c}_{i1}(t),\ldots,\hat{c}_{iN}(t)) \quad (15a)$$

$$\theta_o^{(i)T} = (\alpha^{o}_{0,i1},\ldots,\alpha^{o}_{n_1,i1};\ldots;\alpha^{o}_{0,iN},\ldots,\alpha^{o}_{n_N,iN};$$
$$c^{o}_{i1},\ldots,c^{o}_{iN}) \quad (15b)$$

$$\tilde{\theta}^{(i)T}(t) = (\tilde{\alpha}_{0,i1}(t),\ldots,\tilde{\alpha}_{n_1,i1}(t);\ldots;\tilde{\alpha}_{0,iN}(t),\ldots$$
$$\ldots,\tilde{\alpha}_{n_N,iN}(t);\tilde{c}_{i1}(t),\ldots,\tilde{c}_{iN}(t)) \quad (15c)$$

For the above set-up we have the following theorem:

Theorem 1. For the system (1) subject to the assumptions (A0) - (A3), if each controller at time t knows the information $\{Y_i(t), y^{*}_i(t+1);\ 1 \leq i \leq N\}$ and applies the Projection Algorithm (10) for his parameter estimation and (9) for his control input computation, then

(a) $\|\tilde{\theta}(t+1)\| \leq \|\tilde{\theta}(t)\| \leq \|\tilde{\theta}(0)\| \quad \forall\ t \geq 0$

(b) $\lim_{t\to\infty} \dfrac{\varepsilon(t)}{(1+\phi(t)^T\phi(t))^{1/2}} = 0$

(c) $\lim_{t\to\infty} \dfrac{e(t)}{(1+\phi(t)^T\phi(t))^{1/2}} = 0$

where

$$e(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \\ \vdots \\ e_N(t) \end{bmatrix} = \begin{bmatrix} y_1(t) - y^{*}_1(t) \\ y_2(t) - y^{*}_2(t) \\ \vdots \\ y_N(t) - y^{*}_N(t) \end{bmatrix};$$

$$\varepsilon(t) = \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \vdots \\ \varepsilon_N(t) \end{bmatrix} = \begin{bmatrix} \phi(t)^T\tilde{\theta}^{(1)}(t) \\ \phi(t)^T\tilde{\theta}^{(2)}(t) \\ \vdots \\ \phi(t)^T\tilde{\theta}^{(N)}(t) \end{bmatrix};$$

$$\tilde{\theta}(t) = \begin{bmatrix} \tilde{\theta}^{(1)}(t) \\ \tilde{\theta}^{(2)}(t) \\ \vdots \\ \tilde{\theta}^{(N)}(t) \end{bmatrix}$$

(d) $\{y(t)\}$ and $\{u(t)\}$ are uniformly bounded for all time.

(e) Each output asymptotically tracks the corresponding controller's desired output sequence, i.e.,

$$\lim_{t\to\infty} |y_i(t) - y^{*}_i(t)| = 0 \quad \forall\ i = 1 \text{ to } N.$$

Proof. (a),(b): Subtracting $\theta_o^{(i)}$ from both sides of (10) and using (14), we obtain a set of N equations in the parameter estimation error:

$$\begin{bmatrix} \tilde{\theta}^{(1)}(t+1) \\ \tilde{\theta}^{(2)}(t+1) \\ \vdots \\ \tilde{\theta}^{(N)}(t+1) \end{bmatrix} = \begin{bmatrix} \tilde{\theta}^{(1)}(t) \\ \tilde{\theta}^{(2)}(t) \\ \vdots \\ \tilde{\theta}^{(N)}(t) \end{bmatrix} - \frac{\phi(t)}{1+\phi(t)^T\phi(t)} \begin{bmatrix} 1 & h_{12} \cdots h_{1N} \\ h_{21} & 1 \cdots h_{2N} \\ \vdots & \ddots \\ h_{N1} & h_{N2} \cdots 1 \end{bmatrix} \begin{bmatrix} \phi(t)^T\tilde{\theta}^{(1)}(t) \\ \phi(t)^T\tilde{\theta}^{(2)}(t) \\ \vdots \\ \phi(t)^T\tilde{\theta}^{(N)}(t) \end{bmatrix} \quad (16)$$

Thus, we have

$$\begin{bmatrix} \tilde{\theta}^{(1)}(t+1) \\ \tilde{\theta}^{(2)}(t+1) \\ \vdots \\ \tilde{\theta}^{(N)}(t+1) \end{bmatrix}^T + \begin{bmatrix} \tilde{\theta}^{(1)}(t) \\ \tilde{\theta}^{(2)}(t) \\ \vdots \\ \tilde{\theta}^{(1)}(t) \end{bmatrix}^T = 2 \begin{bmatrix} \tilde{\theta}^{(1)}(t) \\ \tilde{\theta}^{(2)}(t) \\ \vdots \\ \tilde{\theta}^{(N)}(t) \end{bmatrix}^T$$

$$- \frac{1}{1+\phi(t)^T\phi(t)}$$

$$\begin{bmatrix}\phi(t)^T\tilde{\theta}^{(1)}(t)\\ \phi(t)^T\tilde{\theta}^{(2)}(t)\\ \vdots\\ \phi(t)^T\tilde{\theta}^{(N)}(t)\end{bmatrix}^T \begin{bmatrix}1 & h_{21}\cdots h_{N1}\\ h_{12} & 1\ \cdots h_{N2}\\ \vdots & \vdots\ \ddots\ \vdots\\ h_{1N} & h_{2N}\cdots 1\end{bmatrix} \begin{bmatrix}\phi(t)^T & 0 & \cdots & 0\\ 0 & \phi(t)^T & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \phi(t)^T\end{bmatrix} \tag{17}$$

and

$$\begin{bmatrix}\tilde{\theta}^{(1)}(t+1)\\ \tilde{\theta}^{(2)}(t+1)\\ \vdots\\ \tilde{\theta}^{(N)}(t+1)\end{bmatrix} - \begin{bmatrix}\tilde{\theta}^{(1)}(t)\\ \tilde{\theta}^{(2)}(t)\\ \vdots\\ \tilde{\theta}^{(N)}(t)\end{bmatrix}$$

$$= -\frac{1}{1+\phi(t)^T\phi(t)} \begin{bmatrix}\phi(t) & 0 & \cdots & 0\\ 0 & \phi(t) & \cdots & 0\\ \vdots & & & \\ 0 & 0 & \cdots & \phi(t)\end{bmatrix}$$

$$\begin{bmatrix}1 & h_{12} & \cdots & h_{1N}\\ h_{21} & 1 & \cdots & h_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ h_{N1} & h_{N2} & \cdots & 1\end{bmatrix} \begin{bmatrix}\phi(t)^T\tilde{\theta}^{(1)}(t)\\ \phi(t)^T\tilde{\theta}^{(2)}(t)\\ \vdots\\ \phi(t)^T\tilde{\theta}^{(N)}(t)\end{bmatrix} \tag{18}$$

so that premultiplying (18) by (17) gives

$$\|\tilde{\theta}(t+1)\|^2 - \|\tilde{\theta}(t)\|^2 = \frac{\varepsilon^T(t)}{(1+\phi(t)^T\phi(t))^{1/2}}\, At\, \frac{\varepsilon(t)}{(1+\phi(t)^T\phi(t))^{1/2}} \tag{19}$$

where, with $\mu_t = \dfrac{\phi(t)^T\phi(t)}{1+\phi(t)^T\phi(t)}$,

$$[At]_{ij} = \begin{cases} 2 - {}_t(1 + \sum_{k\neq i}^{N} h_{ki}^2) & \text{for } i = j\\ 2h_{ij} - {}_t(h_{ij} + h_{ji} + \sum_{k\neq i,j}^{N} h_{ki}h_{kj}) & \text{for } i \neq j\end{cases}$$

Therefore, in order to show that

- $\|\tilde{\theta}(t)\|^2$ is a (bounded) nonincreasing (real-valued) function; and
- $\dfrac{\varepsilon(t)}{(1+\phi(t)^T\phi(t))^{1/2}} \neq 0$ implies $\|\tilde{\theta}(t+1)\|^2 - \|\tilde{\theta}(t)\|^2 < 0$, i.e.,

$\|\tilde{\theta}(t+1)\|^2 - \|\tilde{\theta}(t)\|^2 = 0$ implies

$$\frac{\varepsilon(t)}{(1+\phi(t)^T\phi(t))^{1/2}} = 0$$

so that (a), (b) are finally achieved, we have only to prove that the matrix $[A_t + A_t^T]$ is positive definite.

Note that, with $|h_{ij}| < 1/(N-1)\ \forall\ i \neq j$ (Assumption (A2)),

$$\text{i) } [A_t + A_t^T]_{ii} = 4 - 2\mu_t(1 + \sum_{k\neq i}^{N} h_{ki}^2)$$
$$\geq 4 - 2\mu_t\{1 + (N-1)/(N-1)^2\}$$
$$= 4 - 2\mu_t\{1 + (N-1)^{-1}\}$$

$$\text{ii) } |[A_t + A_t^T]_{ij}| = |2(h_{ij} + h_{ji}) - 2\mu_t(h_{ij} + h_{ji} + \sum_{k\neq i,j}^{N} h_{ki}h_{kj})|$$
$$= |2(1-\mu_t)(h_{ij} + h_{ji}) - 2\mu_t \sum_{k\neq i,j} h_{ki}h_{kj}|$$
$$< 4(1-\mu_t)/(N-1) + 2\mu_t(N-2)/(N-1)^2$$
$$= (N-1)^{-1}[4(1-\mu_t) + 2\mu_t\{1 - (N-1)^{-1}\}]$$
$$= (N-1)^{-1}[4 - 2\mu_t\{1 + (N-1)^{-1}\}]$$

Comparing the above two inequalities, we get

$$[A_t + A_t^T]_{ii} > \sum_{j\neq i}^{N} |[A_t + A_t^T]_{ij}|$$

which implies positive diagonal dominancy of $[A_t + A_t^T]$ and so positive-definiteness of the matrix $[A_t + A_t^T]$.

(c) It immediately follows from (b), by noting that (14) can be rewritten as

$$e(t) = H\varepsilon(t) = \begin{bmatrix}1 & h_{12} & \cdots & h_{1N}\\ h_{21} & 1 & \cdots & h_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ h_{N1} & h_{N2} & \cdots & 1\end{bmatrix} \varepsilon(t)$$

where the matrix H is positive diagonal dominant and so nonsingular.

(d), (e). Noting that the assumptions (A0) to (A3) and the part (c) of this theorem ensure that all the assumptions 6.3.M(1) to 6.3.M(4) and the precondition (6.3.134) of Theorem 6.3-3 in [3] are satisfied. the result follows immediately from the proof of Theorem 6.3-3 [3].

VI. CONCLUSIONS

This paper deals with a decentralized adaptive control problem using the Projection Algorithm for parameter estimation and one-step-ahead control for control purposes. It is shown that, for a certain class of discrete-time multi-input multi-output deterministic linear system and under a weak coupling type condition, both the input and the output are bounded for all time and each output asymptotically tracks the corresponding controller's desired output sequence.

Even though the adaptive control scheme developed here is not decentralized in the information aspect, it still deserves the adjective "decentralized," because each controller has only to take care of his subsystem. Our result suggests how and under what conditions some large scale centralized adaptive control problems can be decentralized so that one can take advantage of parallelism to decrease the complexity and save a considerable amount of time and effort for computation. One might appreciate the global efficiency of this scheme by recalling how powerfully the parallelism of the Jacobi iteration can be exploited for the computational purpose to find the inverse of a large matrix with a dominant diagonal.

Although the problem studied here is a simple DAR model without delays in the control, it is of importance since it is one of the first attempts to deal with decentralized adaptive scheme. Study of more general models with different parameter estimation schemes such as least square algorithm, etc., as well as simulation studies for comparison are obviously needed before the area of decentralized adaptive control can achieve a degree of maturity.

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