Analysis of an On–Off Jamming Situation as a Dynamic Game

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***Abstract*—The process of communication jamming can be modeled as a two-person zero-sum noncooperative dynamic game played between a communicator (a transmitter–receiver pair) and a jammer. We consider a one-way time-slotted packet radio communication link in the presence of a jammer, where the data rate is fixed and 1) in each slot, the communicator and jammer choose their respective power levels in a random fashion from a zero and a positive value; 2) both players are subject to temporal energy constraints which account for protection of the communicating and jamming transmitters from overheating. The payoff function is the time average of the mean payoff per slot. The game is solved for certain ranges of the players' transmitter parameters. Structures of steady-state solutions to the game are also investigated. The general behavior of the players' strategies and payoff increment is found to depend on a parameter related to the payoff matrix, which we call the payoff parameter, and the transmitters' parameters. When the payoff parameter is lower than a threshold, the optimal steady-state strategies are mixed and the payoff increment constant over time, whereas when it is greater than the threshold, the strategies are pure, and the payoff increment exhibits oscillatory behavior.**

***Index Terms*—Communication jamming, grid solution, noncooperative dynamic game, optimal strategies, temporal energy constraints.**

I. INTRODUCTION

COMMUNICATION jamming is, in reality, a power game between two opponents, a *communicator* (a transmitter–receiver pair) and a *jammer*, each trying to outdo the other by transmitting a signal with a power level greater than that of its adversary. Such a situation can be modeled as a two-person zero-sum *noncooperative game* [1] [2], an idea which was motivated by Shannon's work on the theory of communication [3]. However, the transmission equipment of each contestant has a limitation on its power handling capability, which arises because transmitting at very high power levels over a period of time can cause overheating and consequent thermal breakdown of the equipment. Analysis of such a scenario therefore calls for embedding in the game a suitable model of the thermal limitations of the communicating and jamming transmitters.

We consider the case in which selected communication and jamming strategies are exercised in synchronism over T well-defined time slots, indexed by integers $1, 2, \ldots, T$. In the t_fth slot (t_f denotes the *forward-time index*)[1] the communicator transmits an information-bearing signal with a power level X_{t_f}, and the jammer transmits a jamming signal with a power level Y_{t_f}. It is natural to infer the following. 1) At the beginning of any time slot, there is accumulation of thermal energy in the communicating and the jamming transmitters owing to past transmissions. 2) Over the current slot duration, a fraction of this energy is dissipated, while the remainder adds on to the energy generated by the current slot's transmission. 3) To avoid transmitter failure due to thermal breakdown, the accumulated thermal energy at the end of any slot should not exceed a threshold for either player. This justifies the need for *temporal energy constraints*.

To incorporate this kind of reasoning in a model, let Z_{t_f} represent the accumulated thermal energy in the communicating transmitter at the end of time slot t_f, and assume that a fraction δ_C of this energy has not been dissipated by the end of the following time slot. Assuming that there is no initial accumulated thermal energy, the evolution of the accumulated thermal energy process in the communicating transmitter can be modeled by

$$\begin{aligned} Z_0 &= 0 \\ Z_{t_f} &= \delta_C Z_{t_f-1} + X_{t_f} = \sum_{\eta=0}^{t_f-1} \delta_C^{\eta} X_{t_f-\eta}, \\ & \qquad t_f = 1, \ldots, T;\ 0 < \delta_C < 1 \end{aligned} \tag{1}$$

where δ_C is the *thermal memory constant* of the communicating transmitter. The constraint for survivability of the communication transmitter to the end of slot t_f is simply that

$$Z_{t_f-\eta} \le C_{\max}, \qquad \text{for all } \eta = 0, \ldots, t_f-1;\ t_f = 1, \ldots, T \tag{2}$$

[1] We are using t_f to denote the forward-time index to distinguish it from the *reverse-time index* t which will be used later.

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where $C_{\max}$ is the thermal breakdown threshold in the communicating transmitter. The jammer's accumulated thermal energy is assumed to be governed by a similar equation

$$\begin{aligned} W_0 &= 0 \\ W_{t_f} &= \delta_J W_{t_f-1} + Y_{t_f} = \sum_{\eta=0}^{t_f-1} \delta_J^{\eta} Y_{t_f-\eta}, \\ & \qquad t_f = 1, \ldots, T;\ 0 < \delta_J < 1 \end{aligned} \tag{3}$$

the fraction δ_C being the *thermal memory constant* of the jamming transmitter. To survive to the end of slot t_f, the jammer's equipment must operate with

$$W_{t_f-\eta} \le J_{\max}, \qquad \text{for all } \eta = 0, \ldots, t_f - 1;\ t_f = 1, \ldots, T \tag{4}$$

where $J_{\max}$ is the thermal breakdown threshold in the jamming transmitter. The temporal energy constraints of the communicator and the jammer are thus given by (2) and (4), respectively. These constraints are the elements of our game model which account for the prevention of transmission failure due to thermal breakdown of the players' transmitters and make the game *dynamic*. Models with time-averaged power constraints or energy constraints, which, in our framework, can be interpreted as the nondissipative model with $\delta_C = \delta_J = 1$, have been investigated in [4]–[6].

We assume that in our slotted time epoch of T slots, each slot may or may not contain a packet, depending on whether or not the communicator chooses to transmit a signal. Similarly, the same slot may or may not contain a jamming signal. If the communicator chooses to transmit in a given slot, a fixed level C ($C > 0$) of thermal energy will be released in the communication transmitter. Similarly, a jammer's transmission in a time slot will release a level J ($J > 0$) of energy in the jammer's transmitter. The levels C and J can be scaled to possess the unit of power. The payoff $G(X_{t_f}, Y_{t_f})$ *to the communicator*, as a function of whether or not either of the players are transmitting in time slot t_f, can be described by a simple 2 × 2 matrix[2]

$$\mathbf{G} \triangleq \begin{bmatrix} G(0,0) & G(0,J) \\ G(C,0) & G(C,J) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & \alpha \end{bmatrix}, \qquad 0 < \alpha < 1. \tag{5}$$

When the communicator does not attempt a transmission in slot t_f, i.e., $X_{t_f} = 0$, the payoff for that slot is 0. At the opposite extreme, one unit of payoff is given to the communicator when it transmits and the jammer is off. When both players actively transmit in a time slot, the payoff α will fall between these two extremes. It is plausible that many payoff functions can be normalized in this fashion. We shall call α the *payoff parameter*. A key assumption about the payoff function is that it is additive over a sequence of slotted transmissions. The overall *payoff* is the expected value of the average payoff per slot for activities over a sequence of T time slots. This can be modeled as

$$\mathcal{G} \triangleq \frac{1}{T} \sum_{t_f=1}^{T} \mathbf{E}[G(X_{t_f}, Y_{t_f})]. \tag{6}$$

[2]The communicator's payoff matrix is $\mathbf{G}$. Since the game is zero-sum, the payoff to the jammer is $-G(X_{t_f}, Y_{t_f})$ and the jammer's payoff matrix is $-\mathbf{G}$.

Certainly, a normalized throughput per packet, averaged over channel disturbances, can be viewed as one such additive payoff function $G(X_{t_f}, Y_{t_f})$.

For noncooperative games between a communicator which is designed to communicate over a channel, and a hostile jammer which is designed to jam the communication system, there are a number of papers in the open literature which use different types of payoff functions, such as channel capacity [7], signal-to-noise ratio [8], error probability [9], and mutual information [10]. Games with ensemble average power constraints were dealt with in [11]–[15]. In such work, static games were solved analytically, assuming independent play in each time slot. The use of ensemble averages for average-power-constraint modeling does not take into account the time-varying thermal behavior of the transmitters, and no parameter corresponding to a thermal memory constant exists in these analyses.

Dynamic game models were considered in [4]–[6] and [16]–[20]. Antijamming codes were studied in [6] using a constrained game model with probability of correct decision as the payoff. The performance of the arbitrarily varying channel, which can be interpreted as a model of a channel jammed by an intelligent and unpredictable adversary, was studied in [4], [5], [16]–[18], and [21] from an information theoretic point of view. Significant results on the coding capacities of additive channels were obtained in [22]–[24]. The influence of information in noncooperative games was investigated in [19] using methods of information theory. Maximin and minimax detection problems for signals having temporal power constraints with the payoff as the probability of error of the detector were formulated and solved in [20].

Our work differs from those of the past in that the model is not directly related to channel capacity, but that it pertains to one motivated by simplified communication engineering practice [25]. We first solve a dynamic game over a finite number of slots by using dynamic programming, and then consider the behavior of the optimal strategies as the reverse-time index goes to infinity. The infinite horizon case is also considered. The general behavior of the players' strategies and payoff increment is found to depend on the payoff parameter α and the transmitters' parameters. *When α is lower than a threshold, which is a function of the parameters, the optimal steady-state strategies are mixed and the payoff increment constant over time, whereas when it is greater than the threshold, the strategies are pure, and the payoff increment exhibits oscillatory behavior.* This phenomenon is significant since it is the outcome of the temporal energy constraints which introduce dynamism in the game.

The paper is organized as follows. In Section II, we describe the dynamic jamming game model by expressing the payoff as a function of the strategies and setting up the evolution equation to solve for the optimal strategies using dynamic programming. Section III presents a 2 × 2 grid solution for both the finite horizon and the infinite horizon cases. In Section IV, we analyze the structure of coset-generated grid solutions for the game. Section V provides an example of how the game model can be applied to a communication system which employs binary phase-shift keying (BPSK). The conclusions are given in Section VI.

II. Dynamic Game Model

In our model, the sequence $\{X_{t_f}\}$, $X_{t_f} \in \{0, C\}$ for all $t_f \in \{1, 2, \ldots, T\}$, of *communicator power levels* describes the sequence of communicator's decisions to transmit or not transmit. Similarly, the sequence $\{Y_{t_f}\}$, $Y_{t_f} \in \{0, J\}$ for t_f, of *jammer power levels* describes the sequence of jammer's decisions to jam or not.

The players are subject to the constraints (2) and (4). Let the *operating plane* of the communicator (jammer) be the $C_{\max}/C$ versus δ_C plane ($J_{\max}/J$ versus δ_J plane). In order for the energy constraints to come into play, the *operating regions* of the communicator and the jammer must lie, respectively, within

$$1 \le \frac{C_{\max}}{C} < \frac{1-\delta_C^T}{1-\delta_C} \quad \text{and} \quad 1 \le \frac{J_{\max}}{J} < \frac{1-\delta_J^T}{1-\delta_J}. \tag{7}$$

The *transmitter parameters* $C_{\max}$, δ_C, $J_{\max}$, δ_J are assumed to be known to both players.

In addition, we assume that both the communicator and the jammer have knowledge of their own and their opponent's actions in prior time slots. *It is conceivable that the information about whether the opponent's transmitter has been on or off in previous slots may be available to the transmitters, depending on slot durations and propagation delays, or else the information flow model would have to be adjusted to account for these delays.* Therefore, we have a framework in which: a) X_1 and Y_1 are independent and b) for $t_f = 2, \ldots, T$, X_{t_f} and Y_{t_f} are conditionally independent, given $X_1, \ldots, X_{t_f-1}, Y_1, \ldots, Y_{t_f-1}$ (and therefore, given Z_{t_f-1}, W_{t_f-1}). This is a *dynamic stochastic game model* with a *Markovian evolution* [26], in which X_{t_f} and Y_{t_f} are the *decision variables* of the two players, and the accumulated thermal energy pair (Z_{t_f}, W_{t_f}) is the value of the *state* of the system at the end of slot t_f. The *state equations* that govern the dynamic system are given by (1) and (3).

We further assume that the payoff parameter α [see (5)] is known to both sides. The value of α typically depends on factors not known to either transmitter, e.g., the signal-to-interference ratio in the communication receiver. *Part of the objective of this analysis is to find out how the optimal strategies of the competitors depend on this parameter, as a first step toward approaches to the jamming game without precise knowledge of α.*

The scenario for the game, along with the parameters, is shown in Fig. 1.

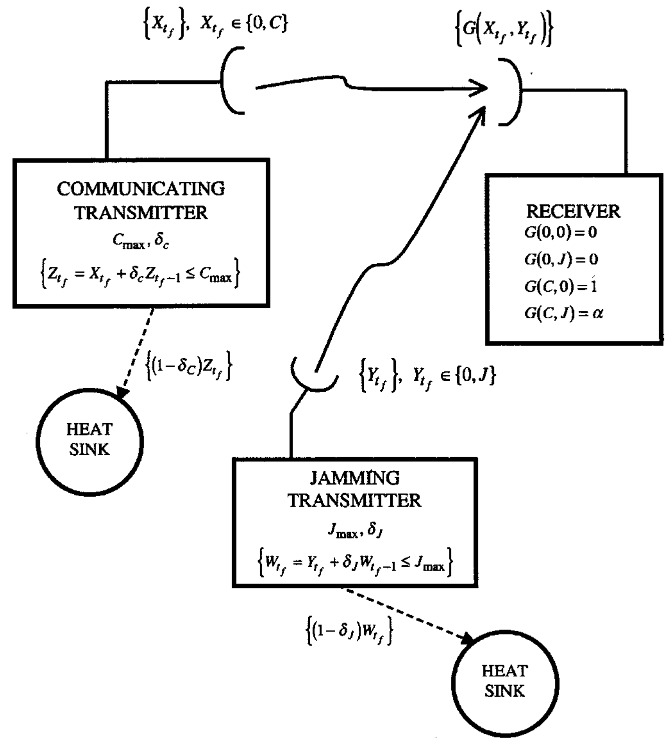

Fig. 1. Communication jamming game environment.

A. Payoff as a Function of the Strategies

Let $t = T - t_f$ denote the *reverse-time index*. From (1) and (3), we find that Z_{T-t} admits only those energies that belong to a set Φ_t defined as

$$\Phi_t \triangleq \left\{ z\colon z = C \sum_{\eta=0}^{T-t-1} \beta_\eta \delta_C^\eta, \quad \beta_0, \ldots, \beta_{T-t-1} \in \{0, 1\} \right\} \cap [0, C_{\max}], \;\; t = 1, \ldots, T \tag{8}$$

and W_{T-t} admits only those energies that belong to an analogous set Ψ_t which can be expressed by replacing C by J, δ_C by δ_J, $C_{\max}$ by $J_{\max}$, z by w in (8). Thus, $\Phi_T = \{0\}$, $\Phi_{T-1} = \{0, C\}$, $\Phi_{T-2} = \{0, \delta_C C, C, (1+\delta_C)C\} \cap [0, C_{\max}]$, and so on. Note that

$$\begin{aligned} &|\Phi_t| \le 2^{T-t}, \quad |\Psi_t| \le 2^{T-t}, \qquad \text{for} \quad t = 1, \ldots, T, \\ &\Phi_T \subset \Phi_{T-1} \subset \cdots \subset \Phi_1 \subseteq [0, C_{\max}], \\ &\Psi_T \subset \Psi_{T-1} \subset \cdots \subset \Psi_1 \subseteq [0, J_{\max}]. \end{aligned} \tag{9}$$

Let $p_t(z, w)$ ($q_t(z, w)$) denote the probability that the communicator (jammer) selects power level C (J) at reverse-time t (i.e., at the beginning of slot $T-t$), given that the communicator and jammer have retained z and w units of energy, respectively, from past transmissions. Then, these selection probabilities or *strategies* can be defined as

$$\begin{aligned} p_t(z, w) &\triangleq \Pr\left(X_{T-t} = C \mid Z_{T-t-1} = z, W_{T-t-1} = w\right) \\ q_t(z, w) &\triangleq \Pr\left(Y_{T-t} = J \mid Z_{T-t-1} = z, W_{T-t-1} = w\right) \end{aligned} \tag{10}$$

where $z \in \Phi_{t+1}$, $w \in \Psi_{t+1}$, $t = 0, \ldots, T-1$.

The payoff $\mathcal{G}$ in (6) can be expressed in terms of a time-reversed sequence $\{S_0, \ldots, S_{T-1}\}$ governed by the equations

$$\begin{aligned} S_0 &\triangleq \mathbf{E}[G(X_T, Y_T) \mid Z_{T-1}, W_{T-1}] \\ S_t &\triangleq \mathbf{E}[\{G(X_{T-t}, Y_{T-t}) + S_{t-1}\} \mid Z_{T-t-1}, W_{T-t-1}], \\ &\qquad \text{for } t = 1, \ldots, T-1 \\ \mathcal{G} &= \frac{S_{T-1}}{T}. \end{aligned} \tag{11}$$

The quantity S_t is thus the backward *accumulated payoff* at the beginning of slot $T-t$ given the past energy accumulations Z_{T-t-1} and W_{T-t-1}. Equation (11) can be rewritten as

$$\begin{aligned} &S_0(z,w)\\ &=[1-p_0(z,w) \quad p_0(z,w)]\begin{bmatrix}0 & 0\\ 1 & \alpha\end{bmatrix}\begin{bmatrix}1-q_0(z,w)\\ q_0(z,w)\end{bmatrix}\\ &=p_0(z,w)[1-(1-\alpha)q_0(z,w)], \qquad z\in\Phi_1;\ w\in\Psi_1 \end{aligned} \tag{12a}$$

$$\begin{aligned} &S_{t+1}(z,w)\\ &=[1-p_{t+1}(z,w) \quad p_{t+1}(z,w)]\\ &\times\begin{bmatrix}S_t(\delta_C z,\delta_J w) & S_t(\delta_C z, J+\delta_J w)\\ 1+S_t(C+\delta_C z,\delta_J w) & \alpha+S_t(C+\delta_C z, J+\delta_J w)\end{bmatrix}\\ &\times\begin{bmatrix}1-q_{t+1}(z,w)\\ q_{t+1}(z,w)\end{bmatrix}, \qquad z\in\Phi_{t+2};\ w\in\Psi_{t+2};\\ &\quad t=0,\ldots,T-2 \quad \text{and} \end{aligned} \tag{12b}$$

$$\mathcal{G}=\frac{S_{T-1}(0,0)}{T} \tag{12c}$$

since there is no initial accumulated energy. The constraints (2) and (4), which preclude transmission when energy accumulations are too high, force the conditions

$$\begin{aligned} p_t(z,w)=0, &\qquad \text{when } z\in\Phi_{t+1}\cap\left(\frac{C_{\max}-C}{\delta_C}, C_{\max}\right]\\ q_t(z,w)=0, &\qquad \text{when } w\in\Phi_{t+1}\cap\left(\frac{J_{\max}-J}{\delta_j}, J_{\max}\right],\\ &\qquad \text{for } t=0,\ldots,T-1. \end{aligned} \tag{13}$$

The *strategy sets* $\mathcal{P}_t$ for the communicator and $\mathcal{Q}_t$ for the jammer at reverse-time t, $t=0,\ldots,T-1$, are defined as

$$\begin{aligned} \mathcal{P}_t &\triangleq \left\{p_t(z,w)U\left(\frac{C_{\max}-C}{\delta_C}-z\right): z\in\Phi_{t+1};\ w\in\Psi_{t+1}\right\}\\ \mathcal{Q}_t &\triangleq \left\{q_t(z,w)U\left(\frac{J_{\max}-J}{\delta_J}-w\right): z\in\Phi_{t+1};\ w\in\Psi_{t+1}\right\} \end{aligned} \tag{14}$$

where $U(\cdot)$ denotes the unit step function, and takes into account the conditions (13). From (9) and (14), the cardinalities of the sets $\mathcal{P}_t$ and $\mathcal{Q}_t$ are given by

$$\begin{aligned} &|\mathcal{P}_t|\le 4^{T-t-1}, \quad |\mathcal{Q}_t|\le 4^{T-t-1}, \quad t=0,\ldots,T-1\\ &\text{implying } \sum_{t=0}^{T-1}|\mathcal{P}_t|\le\frac{4^T-1}{3}, \quad \sum_{t=0}^{T-1}|\mathcal{Q}_t|\le\frac{4^T-1}{3}. \end{aligned} \tag{15}$$

Consider the *finite horizon game*, that is, the case when T is finite. Each nontrivial element of the strategy sets $\mathcal{P}_0,\ldots,\mathcal{P}_{T-1}$, $\mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1}$ is a probability by definition, and therefore belongs to the *compact convex set* $[0,1]$ on the real line. In addition, it is clear from (12) that the payoff $\mathcal{G}$, which can be denoted by $\mathcal{G}(\mathcal{P}_0,\ldots,\mathcal{P}_{T-1};\mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1})$, is affine in each of the nontrivial elements of the strategy sets, and is therefore a *continuous functional* of these elements. Hence

$$\begin{aligned} &\max_{\{\mathcal{P}_0,\ldots,\mathcal{P}_{T-1}\}}\mathcal{G}(\mathcal{P}_0,\ldots,\mathcal{P}_{T-1};\ \mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1})\\ &\min_{\{\mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1}\}}\mathcal{G}(\mathcal{P}_0,\ldots,\mathcal{P}_{T-1};\ \mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1}) \end{aligned}$$

exist.

B. Existence of a Saddle-Point in the Finite Horizon Game

While playing the game, the communicator assumes the worst case in which the jammer minimizes the payoff over all possible *strategy set sequences* $\{\mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1}\}$, against any sequence that it uses, and chooses a sequence $\{\tilde{\mathcal{P}}_0,\ldots,\tilde{\mathcal{P}}_{T-1}\}$ such that the *maximin* payoff V_L is achieved. Thus

$$\begin{aligned} V_L &= \max_{\{\mathcal{P}_0,\ldots,\mathcal{P}_{T-1}\}}\ \min_{\{\mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1}\}}\\ &\qquad \mathcal{G}(\mathcal{P}_0,\ldots,\mathcal{P}_{T-1};\ \mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1})\\ &= \min_{\{\mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1}\}}\mathcal{G}(\tilde{\mathcal{P}}_0,\ldots,\tilde{\mathcal{P}}_{T-1};\ \mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1}). \end{aligned} \tag{16}$$

On the other hand, the jammer chooses a sequence $\{\tilde{\mathcal{Q}}_0,\ldots,\tilde{\mathcal{Q}}_{T-1}\}$ such that the *minimax* payoff V_U is achieved. This gives

$$\begin{aligned} V_U &= \min_{\{\mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1}\}}\ \max_{\{\mathcal{P}_0,\ldots,\mathcal{P}_{T-1}\}}\\ &\qquad \mathcal{G}(\mathcal{P}_0,\ldots,\mathcal{P}_{T-1};\ \mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1})\\ &= \max_{\{\mathcal{P}_0,\ldots,\mathcal{P}_{T-1}\}}\mathcal{G}(\mathcal{P}_0,\ldots,\mathcal{P}_{T-1};\ \tilde{\mathcal{Q}}_0,\ldots,\tilde{\mathcal{Q}}_{T-1}). \end{aligned} \tag{17}$$

The *minimax theorem* [27] states that $V_L\le V_U$.

A strategy set sequence $\{\tilde{\mathcal{P}}_0,\ldots,\tilde{\mathcal{P}}_{T-1}\}$ satisfying (16) is called an *optimal strategy set sequence* for the communicator, while a sequence $\{\tilde{\mathcal{Q}}_0,\ldots,\tilde{\mathcal{Q}}_{T-1}\}$ satisfying (17) is an optimal strategy set sequence for the jammer. Our objective in solving the game is finding optimal strategy set sequences for the players.

The finite dimensional vector of at most $(2(4^T-1))/3$ elements [see (15)] of the strategy sets $\mathcal{P}_0,\ldots,\mathcal{P}_{T-1}$, $\mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1}$ has some elements which are zeros [due to (13)], and each of the other nontrivial elements belongs to the compact convex set $[0,1]$. The payoff $\mathcal{G}$ is a continuous functional of these nontrivial elements. Therefore, there exists a sequence $\{\mathcal{P}_0^*,\ldots,\mathcal{P}_{T-1}^*\}$ and a sequence $\{\mathcal{Q}_0^*,\ldots,\mathcal{Q}_{T-1}^*\}$ such that [27]

$$\begin{aligned} V_L &\ge \min_{\{\mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1}\}}\mathcal{G}(\mathcal{P}_0^*,\ldots,\mathcal{P}_{T-1}^*;\ \mathcal{Q}_0,\ldots,\mathcal{Q}_{T-1})\\ &\ge \mathcal{G}(\mathcal{P}_0^*,\ldots,\mathcal{P}_{T-1}^*;\ \mathcal{Q}_0^*,\ldots,\mathcal{Q}_{T-1}^*)\\ V_U &\le \max_{\{\mathcal{P}_0,\ldots,\mathcal{P}_{T-1}\}}\mathcal{G}(\mathcal{P}_0,\ldots,\mathcal{P}_{T-1};\ \mathcal{Q}_0^*,\ldots,\mathcal{Q}_{T-1}^*)\\ &\le \mathcal{G}(\mathcal{P}_0^*,\ldots,\mathcal{P}_{T-1}^*;\ \mathcal{Q}_0^*,\ldots,\mathcal{Q}_{T-1}^*). \end{aligned} \tag{18}$$

From the minimax theorem and (18), we get

$$V_L=V_U=\mathcal{G}(\mathcal{P}_0^*,\ldots,\mathcal{P}_{T-1}^*;\ \mathcal{Q}_0^*,\ldots,\mathcal{Q}_{T-1}^*).$$

Therefore, the finite horizon game admits a *saddle-point*, and it is given by strategy sets

$$\mathcal{P}_0^*, \ldots, \mathcal{P}_{T-1}^*, \mathcal{Q}_0^*, \ldots, \mathcal{Q}_{T-1}^*$$

satisfying (18). Since a saddle-point satisfies the optimality conditions (16) and (17), sequences $\{\mathcal{P}_0^*, \ldots, \mathcal{P}_{T-1}^*\}$ and $\{\mathcal{Q}_0^*, \ldots, \mathcal{Q}_{T-1}^*\}$ are optimal strategy set sequences for the communicator and jammer, respectively. The *value* of the game is the quantity $\mathcal{G}(\mathcal{P}_0^*, \ldots, \mathcal{P}_{T-1}^*; \mathcal{Q}_0^*, \ldots, \mathcal{Q}_{T-1}^*)$.

C. The Evolution Equation

For the finite horizon game, a set of optimal strategies can be obtained by applying *dynamic programming* [28] on the accumulated payoff in (12). From (12) and (13), we obtain the *evolution equation*, as shown in (19) at the bottom of the page, for $t = 0, \ldots, T-2$, where value $(\mathbf{H})$ denotes the value of the zero-sum game with payoff matrix $\mathbf{H}$, and $S_t^*(z, w)$ is the *optimum accumulated payoff* at reverse-time t given the past energy accumulations z and w. The *value of the game* is given by

$$\mathcal{G}^* = \frac{1}{T} S_{T-1}^*(0, 0). \tag{20}$$

Equation (19b) is a *reverse-time recursion* in terms of $S_t^*(z, w)$. Each optimization process in the recursion involves solving a matrix game, and this gives the *optimal strategies* $p_t^*(z, w)$ and $q_t^*(z, w)$ starting with

$$\begin{aligned}
&p_0^*(z, w) = 1, \quad q_0^*(z, w) = 1 \quad \text{when} \quad 0 \le z \le \frac{C_{\max} - C}{\delta_C}, \\
&\qquad 0 \le w \le \frac{J_{\max} - J}{\delta_J} \\
&p_0^*(z, w) = 1, \quad q_0^*(z, w) = 0 \quad \text{when} \quad 0 \le z \le \frac{C_{\max} - C}{\delta_C}, \\
&\qquad \frac{J_{\max} - J}{\delta_J} < w \le J_{\max} \\
&p_0^*(z, w) = 0, \quad q_t^*(z, w) = 0 \\
&\qquad \text{when} \quad \frac{C_{\max} - C}{\delta_C} < z \le C_{\max}, \\
&\qquad 0 \le w \le \frac{J_{\max} - J}{\delta_J} \\
&p_0^*(z, w) = 0, \quad q_0^*(z, w) = 0 \\
&\qquad \text{when} \quad \frac{C_{\max} - C}{\delta_C} < z \le C_{\max}, \\
&\qquad \frac{J_{\max} - J}{\delta_J} < w \le J_{\max}.
\end{aligned} \tag{21}$$

$$S_0^*(z, w) = \begin{cases} \text{value}\left(\begin{bmatrix} 0 & 0 \\ 1 & \alpha \end{bmatrix}\right) = \alpha, & \text{if } 0 \le z \le \dfrac{C_{\max} - C}{\delta_C};\ 0 \le w \le \dfrac{J_{\max} - J}{\delta_J} \\ \max(0, 1) = 1, & \text{if } 0 \le z \le \dfrac{C_{\max} - C}{\delta_C};\ \dfrac{J_{\max} - J}{\delta_J} < w \le J_{\max} \\ \min(0, 0) = 0, & \text{if } \dfrac{C_{\max} - C}{\delta_C} < z \le C_{\max};\ 0 \le w \le \dfrac{J_{\max} - J}{\delta_J} \\ 0, & \text{if } \dfrac{C_{\max} - C}{\delta_C} < z \le C_{\max};\ \dfrac{J_{\max} - J}{\delta_J} < w \le J_{\max} \end{cases} \tag{19a}$$

$$S_{t+1}^*(z, w) = \begin{cases} \text{value}\left(\begin{bmatrix} S_t^*(\delta_C z, \delta_J w) & S_t^*(\delta_C z, J + \delta_J w) \\ 1 + S_t^*(C + \delta_C z, \delta_J w) & \alpha + S_t^*(C + \delta_C z, J + \delta_J w) \end{bmatrix}\right), & \text{if } 0 \le z \le \dfrac{C_{\max} - C}{\delta_C}; \\ & \quad 0 \le w \le \dfrac{J_{\max} - J}{\delta_J} \\ \max(S_t^*(\delta_C z, \delta_J w), 1 + S_t^*(C + \delta_C z, \delta_J w)), & \text{if } 0 \le z \le \dfrac{C_{\max} - C}{\delta_C}; \\ & \quad \dfrac{J_{\max} - J}{\delta_J} < w \le J_{\max} \\ \min(S_t^*(\delta_C z, \delta_J w), S_t^*(\delta_C z, J + \delta_J w)), & \text{if } \dfrac{C_{\max} - C}{\delta_C} < z \le C_{\max}; \\ & \quad 0 \le w \le \dfrac{J_{\max} - J}{\delta_J} \\ S_t^*(\delta_C z, \delta_J w), & \text{if } \dfrac{C_{\max} - C}{\delta_C} < z \le C_{\max}; \\ & \quad \dfrac{J_{\max} - J}{\delta_J} < w \le J_{\max}. \end{cases} \tag{19b}$$

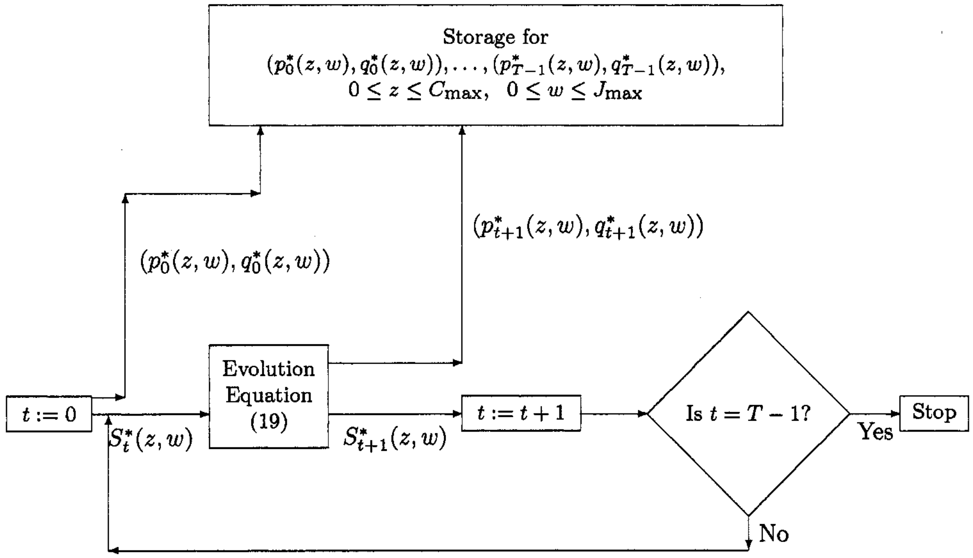

Fig. 2. Roadmap of the computation of optimal strategies.

Note that in (21), when $(C_{\max}-C)/\delta_C < z \le C_{\max}, 0 \le w \le (J_{\max} - J)/\delta_J$, the communicator cannot transmit owing to the constraints, implying $p_0^*(z, w) = 0$. The jammer, although having the option of transmitting with any nonzero probability and still maintaining $S_0^*(z, w) = 0$, chooses to remain idle because it has no signal to jam. Therefore, $q_0^*(z, w) = 0$.

It can be shown from constraints (2) and (4) and the condition (13) that for all $t = 1, \ldots, T-1$, $S_t^*(z, w)$ *is nonincreasing with increase in* z *and nondecreasing with increase in* w, and that

$$\begin{aligned} p_t^*(z, w) &= 1, \quad q_t^*(z, w) = 0 \\ &\quad \text{when} \quad 0 \le z \le \frac{C_{\max} - C}{\delta_C}, \\ &\quad \frac{J_{\max} - J}{\delta_J} < w \le J_{\max} \\ p_t^*(z, w) &= 0, \quad q_t^*(z, w) = 0 \\ &\quad \text{when} \quad \frac{C_{\max} - C}{\delta_C} < z \le C_{\max}, \\ &\quad 0 \le w \le J_{\max}. \end{aligned} \tag{22}$$

Therefore, the only unknown optimal strategies are $p_t^*(z, w)$ and $q_t^*(z, w)$, when $0 \le z \le (C_{\max} - C)/\delta_C$, $0 \le w \le (J_{\max} - J)/\delta_J$, $t = 1, \ldots, T-1$. A roadmap of the computation of optimal strategies is shown in Fig. 2.

III. A 2× 2 Grid Solution Under Certain Operating Conditions

In the evolution equation (19), the communicator's past energy accumulation z at reverse-time $t+1$ has two images at t

$$\begin{aligned} &\delta_C z, && \text{when} \quad X_{T-t-1} = 0 \\ &C + \delta_C z, && \text{when} \quad X_{T-t-1} = C. \end{aligned} \tag{23}$$

The mapping of the intervals $[0, (C_{\max} - C)/\delta_C]$, $((C_{\max} - C)/\delta_C, C_{\max}]$ on the z-axis from $t+1$ to t is therefore given by

$$\begin{aligned} \left[0, \frac{C_{\max} - C}{\delta_C}\right] &\longrightarrow [0, C_{\max} - C], [C, C_{\max}] \\ \left(\frac{C_{\max} - C}{\delta_C}, C_{\max}\right] &\longrightarrow (C_{\max} - C, \delta_C C_{\max}], \\ &\qquad (C_{\max}, C + \delta_C C_{\max}]. \end{aligned} \tag{24}$$

Since $0 \le z \le C_{\max}$ for all t, the interval $(C_{\max}, C + \delta_C C_{\max}]$ need not be considered. Also note that $[0, C_{\max} - C] \subset [0, (C_{\max} - C)/\delta_C]$. Now if the condition

$$\delta_C C_{\max} \le \frac{C_{\max} - C}{\delta_C} < C \tag{25}$$

is satisfied, then we have

$$\begin{aligned} [C, C_{\max}] &\subset ((C_{\max} - C)/\delta_C, C_{\max}] \\ (C_{\max} - C, \delta_C C_{\max}] &\subset [0, (C_{\max} - C)/\delta_C] \end{aligned}$$

and the interval mapping for z from $t+1$ to t becomes

$$\begin{aligned} \left[0, \frac{C_{\max} - C}{\delta_C}\right] &\longrightarrow \left[0, \frac{C_{\max} - C}{\delta_C}\right], \\ &\qquad \left(\frac{C_{\max} - C}{\delta_C}, C_{\max}\right] \\ \left(\frac{C_{\max} - C}{\delta_C}, C_{\max}\right] &\longrightarrow \left[0, \frac{C_{\max} - C}{\delta_C}\right]. \end{aligned} \tag{26}$$

The condition (25) for the communicator can be rewritten as

$$\frac{\left(\dfrac{C_{\max} - C}{\delta_C}\right)}{\delta_C} \ge C_{\max}, \quad \frac{\left(\dfrac{C_{\max} - C}{\delta_C} - C\right)}{\delta_C} < 0$$

$$\text{or, alternatively, as} \quad \frac{1}{1 - \delta_C^2} \le \frac{C_{\max}}{C} < 1 + \delta_C. \tag{27}$$

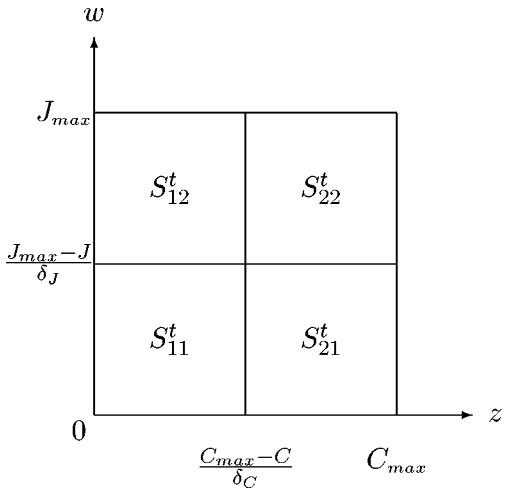

Fig. 3. Optimum accumulated payoff for a 2×2 grid solution.

This specifies the operating condition of the communicator for the optimum accumulated payoff to have *two intervals* $[0, ((C_{\max}-C)/\delta_C)]$, $((C_{\max}-C)/\delta_C, C_{\max}]$ or simply *one critical point* $(C_{\max}-C)/\delta_C$ on the z-axis for all t. We shall call (27) the *communicator's one-critical-point region*. It is indicated in Fig. 8, region "1." It corresponds to the condition that the communicator may pick any sequence of "transmit/no transmit" choices which does not include two consecutive transmits.

In addition to (27), when the jammer's parameters also satisfy the analogous condition

$$\frac{1}{1-\delta_J^2} \leq \frac{J_{\max}}{J} < 1+\delta_J \tag{28}$$

which is the *jammer's one-critical-point-region*, $S_t^*(z, w)$ in (19) has a 2×2 *grid structure* on the (z, w) plane for all t (see Fig. 3), and can be defined as

$$S_t^*(z,w) \triangleq \begin{cases} S_{11}^t, & \text{if } 0 \leq z \leq \dfrac{C_{\max}-C}{\delta_C}; \\ & \quad 0 \leq w \leq \dfrac{J_{\max}-J}{\delta_J} \\ S_{12}^t, & \text{if } 0 \leq z \leq \dfrac{C_{\max}-C}{\delta_C}; \\ & \quad \dfrac{J_{\max}-J}{\delta_J} < w \leq J_{\max} \\ S_{21}^t, & \text{if } \dfrac{C_{\max}-C}{\delta_C} < z \leq C_{\max}; \\ & \quad 0 \leq w \leq \dfrac{J_{\max}-J}{\delta_J} \\ S_{22}^t, & \text{if } \dfrac{C_{\max}-C}{\delta_C} < z \leq C_{\max}; \\ & \quad \dfrac{J_{\max}-J}{\delta_J} < w \leq J_{\max} \end{cases} \tag{29}$$

for $t = 0, \ldots, T-1$. We shall call a solution corresponding to (29) a 2×2 *grid solution*, the *operating conditions* for which are given by (27) and (28).

From (19a) and (29), the solution of the evolution equation at $t = 0$ simplifies to

$$S_{11}^0 = \alpha, \quad S_{12}^0 = 1, \quad S_{21}^0 = S_{22}^0 = 0. \tag{30}$$

In accordance with the 2×2 grid structure of $S_t^*(z, w)$ in (29), the communicator's optimal strategy $p_t^*(z, w)$ and the jammer's optimal strategy $q_t^*(z, w)$ are denoted as

$$p_{ij}^t, \, q_{ij}^t, \qquad \text{for } i=1, 2; \;\; j=1, 2; \;\; t=0, \ldots, T-1.$$

From (21) and (22), we have

$$\begin{aligned} &p_{12}^t = 1, \quad q_{12}^t = 0, \quad p_{21}^t = q_{21}^t = 0, \quad p_{22}^t = q_{22}^t = 0, \\ &\qquad t = 0, \ldots, T-1 \end{aligned} \tag{31}$$

and $p_{11}^0 = q_{11}^0 = 1$. Therefore, we just need to solve for the optimal strategies p_{11}^t and q_{11}^t for $t = 1, \ldots, T-1$.

Substituting the payoff function of (29) in (19b), and using (22), we obtain the following system of equations [29]:

$$S_{11}^{t+1} = \text{value}\left(\begin{bmatrix} S_{11}^t & S_{12}^t \\ 1+S_{21}^t & \alpha+S_{22}^t \end{bmatrix}\right) \tag{32a}$$

$$S_{12}^{t+1} = 1 + S_{21}^t \tag{32b}$$

$$S_{21}^{t+1} = S_{22}^{t+1} = S_{11}^t. \tag{32c}$$

The reverse-time initial conditions are given by (30).

A. Solution to the Finite Horizon Problem

When the duration of play or horizon length T is finite, the evolution equation (32) can be solved to obtain the optimal strategies p_{11}^t, q_{11}^t for $t = 1, \ldots, T-1$. From (20), the value of the game is $\mathcal{G}^* = (1/T)S_{11}^{T-1}$. We will formulate the evolution equation in terms of the payoff increments to obtain the solution.

The *increment of the optimum payoff* S_{ij}^t, $i, j \in \{1,2\}$, in going from t to $t+1$, is bounded and lies in [0, 1], since each element of the payoff matrix $\mathbf{G}$ [see (5)] lies in $[0, 1]$. We denote this increment by λ_{ij}^t. Thus

$$\lambda_{ij}^t \triangleq S_{ij}^{t+1} - S_{ij}^t, \qquad i=1, 2; \;\; j=1, 2; \;\; t=0, \ldots, T-2$$

and the condition $0 \leq \lambda_{ij}^t \leq 1$ holds.

Substituting $S_{ij}^{t+1} = S_{ij}^t + \lambda_{ij}^t$ in (32) and eliminating S_{11}^t, S_{12}^t, and S_{22}^t, (32a) simplifies to

$$\begin{aligned} \lambda_{11}^{t+2} = \text{value}\left(\begin{bmatrix} 0 & 1-\lambda_{11}^{t+1}-\lambda_{11}^t \\ 1-\lambda_{11}^{t+1} & \alpha-\lambda_{11}^{t+1} \end{bmatrix}\right), \\ t = 0, \ldots, T-4. \end{aligned} \tag{33}$$

This can be rewritten as a *second-order nonlinear difference equation*

$$\begin{aligned} \lambda_{11}^{t+2} &= \frac{(1-\lambda_{11}^{t+1}-\lambda_{11}^t)(1-\lambda_{11}^{t+1})}{2-\alpha-\lambda_{11}^{t+1}-\lambda_{11}^t}, \qquad \text{if } \lambda_{11}^t < 1-\alpha \\ &= \alpha - \lambda_{11}^{t+1}, \qquad \text{if } \lambda_{11}^t \geq 1-\alpha; \;\; t=0, \ldots, T-4 \end{aligned} \tag{34}$$

having initial conditions

$$\begin{aligned}\lambda_{11}^0 &= \frac{1-\alpha}{2}\\ \lambda_{11}^1 &= \frac{1+\alpha}{6}, \qquad \text{for} \quad 0<\alpha<\frac{1}{2}\\ &= \frac{3\alpha-1}{2}, \qquad \text{for} \quad \frac{1}{2}\le\alpha<1.\end{aligned} \tag{35}$$

In addition, from (33), the optimal strategies p_{11}^t, q_{11}^t are governed by the equations

$$\begin{aligned}(p_{11}^{t+3}, q_{11}^{t+3}) &= \left(\frac{1-\lambda_{11}^{t+1}-\lambda_{11}^t}{2-\alpha-\lambda_{11}^{t+1}-\lambda_{11}^t}, \frac{1-\lambda_{11}^{t+1}}{2-\alpha-\lambda_{11}^{t+1}-\lambda_{11}^t}\right),\\ &\quad \text{if } \lambda_{11}^t<1-\alpha\\ &= (1,\,1), \qquad \text{if } \lambda_{11}^t\ge 1-\alpha; \ \ t=0,\,\ldots,\,T-4\end{aligned} \tag{36}$$

with

$$\begin{aligned}p_{11}^1 = q_{11}^1 &= \frac{1}{2}\\ (p_{11}^2,\, q_{11}^2) &= \left(\frac{1}{3}, \frac{1+\alpha}{3(1-\alpha)}\right), \qquad \text{if } 0<\alpha<\frac{1}{2}\\ &= (1,\,1), \qquad \text{if } \frac{1}{2}\le\alpha<1.\end{aligned} \tag{37}$$

When $\lambda_{11}^t = 1-\alpha$ in (33), we have

$$\lambda_{11}^{t+2} = \text{ value } \left(\begin{bmatrix} 0 & \alpha-\lambda_{11}^{t+1}\\ 1-\lambda_{11}^{t+1} & \alpha-\lambda_{11}^{t+1}\end{bmatrix}\right) = \alpha-\lambda_{11}^{t+1}$$

and it is clear that $q_{11}^{t+3} = 1$. However, any $p_{11}^{t+3} \in [0,\,1]$ will give the value $\alpha-\lambda_{11}^{t+1}$. In our scenario, the communicator would rather transmit with probability 1, and therefore we choose $p_{11}^{t+3}=1$.

To investigate the behavior of the optimal strategies p_{11}^t and q_{11}^t as $t\to\infty$, we analyze difference equation (34). The following proposition establishes the behavior of λ_{11}^t for $t\to\infty$.

Proposition 1: When $t\to\infty$, the solution λ_{11}^t of the difference equation (34)

$$\begin{aligned}&\text{converges to } \lambda = \frac{(5-\alpha)-\sqrt{(9-\alpha)(1-\alpha)}}{8},\\ &\quad \text{if } 0<\alpha\le\frac{2}{3}\\ &\text{oscillates between } (1-\alpha) \text{ and } (2\alpha-1)\\ &\text{with a period of 2 if } \frac{2}{3}<\alpha<1\end{aligned} \tag{38}$$

for all initial conditions $\lambda_{11}^0, \lambda_{11}^1 \in \mathcal{R}$.

The proposition can be proved by first finding the local Lyapunov exponents of (34), and then applying the multiplicative ergodic theorem of Oseledec to obtain the global Lyapunov exponents [30]. The proof is given in [31].

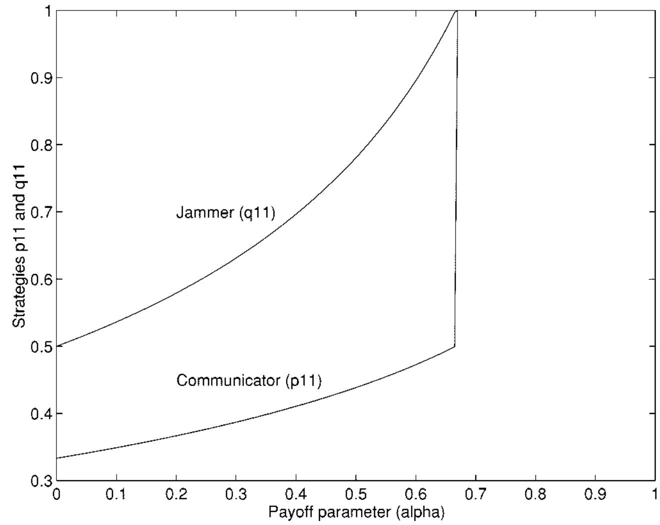

Fig. 4. Steady-state strategies p_{11}, q_{11} versus payoff parameter α in 2×2 grid solution.

Proposition 1 and (36) imply that the finite horizon optimal strategies p_{11}^t and q_{11}^t converge to *steady-state strategies*

$$\begin{aligned}p_{11} &= \lim_{t\to\infty} p_{11}^t\\ &= \frac{1-2\lambda}{2(1-\lambda)-\alpha}\\ &= \frac{(3-\alpha)-\sqrt{(1-\alpha)(9-\alpha)}}{2\alpha}, \qquad \text{if } 0<\alpha<\frac{2}{3}\\ &= 1, \qquad \text{if } \frac{2}{3}\le\alpha<1\\ q_{11} &= \lim_{t\to\infty} q_{11}^t\\ &= \frac{1-\lambda}{2(1-\lambda)-\alpha}\\ &= \frac{-(1-\alpha)+\sqrt{(1-\alpha)(9-\alpha)}}{4(1-\alpha)}, \qquad \text{if } 0<\alpha<\frac{2}{3}\\ &= 1, \qquad \text{if } \frac{2}{3}\le\alpha<1.\end{aligned} \tag{39}$$

A plot of p_{11} and q_{11} versus α is shown in Fig. 4. For $0<\alpha<2/3$, the strategies are mixed, and for $2/3\le\alpha<1$, the strategies are pure. In the mixed strategy zone, both p_{11} and q_{11} increase with increase in α, but $p_{11}<q_{11}$.

When $t\to\infty$, the *payoff increment profile* (profile of λ_{ij}^t) on the $(z,\,w)$ plane is a constant λ for $0<\alpha\le 2/3$, but oscillates between two patterns as shown in Fig. 5(a) and (b) for $2/3<\alpha<1$.

B. The Infinite Horizon Game

We treat the infinite horizon problem as the limit of the finite horizon case with horizon length T as $T\to\infty$, provided the payoff $\mathcal{G}$ remains bounded and the optimal strategy sequences $\{p_{11}^{T-t_f}\}$, $\{q_{11}^{T-t_f}\}$ converge to well-defined limits for every finite forward-time index t_f.

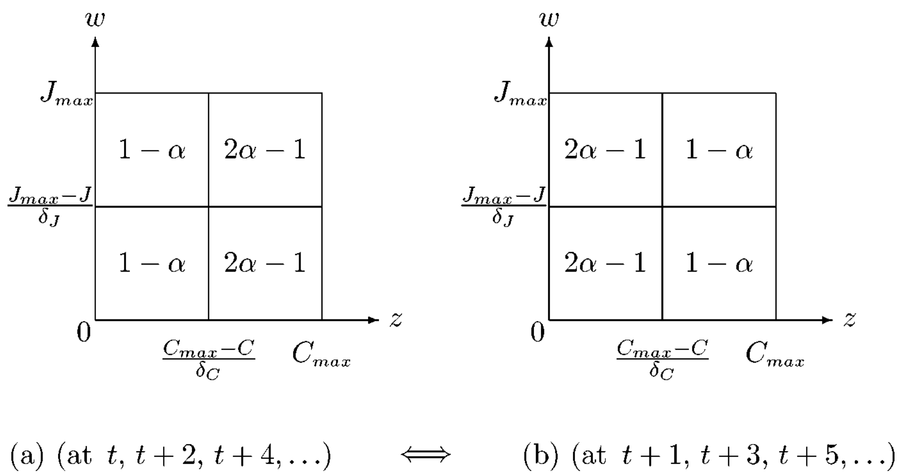

Fig. 5. Oscillatory behavior of payoff increment profile when $t \to \infty$ for $2/3 < \alpha < 1$.

We find from (5) that the payoff $\mathcal{G}$ in (6) satisfies $0 \le \mathcal{G} \le 1$ and is therefore bounded. Also, the sequence

$$\left\{ \sum_{t_f=1}^{T} E[G(X_{t_f}, Y_{t_f})] \right\}_{T=1}^{\infty}$$

is nondecreasing for increasing T and the Tth term of the sequence lies in $[0, T]$. Therefore, the limit

$$\mathcal{G}_\infty \triangleq \lim_{T\to\infty} \frac{1}{T} \sum_{t_f=1}^{T} E[G(X_{t_f}, Y_{t_f})]$$

exists and lies in $[0, 1]$ for any choice of strategies. We call $\mathcal{G}_\infty$ the *payoff of the infinite horizon game*.

Since it has already been found from Proposition 1 and (36) that the optimal strategy sequences $\{p_{11}^t\}$ and $\{q_{11}^t\}$ converge to well-defined limits p_{11} and q_{11}, respectively, when the reverse-time index $t \to \infty$, we have, for every finite t_f

$$\lim_{T\to\infty} p_{11}^{T-t_f} = p_{11}, \quad \lim_{T\to\infty} q_{11}^{T-t_f} = q_{11}$$

where p_{11} and q_{11} are given by (39).

It is also clear that strategies p_{11} and q_{11} constitute the stationary solution of the evolution equation (32), that is, they are the *optimal stationary strategies* of the game as $T \to \infty$. The value of the game for the infinite horizon case is therefore given by [see (12c), (20), and (38)]

$$\begin{aligned} \mathcal{G}_\infty^* &= \frac{(5-\alpha) - \sqrt{(9-\alpha)(1-\alpha)}}{8}, \qquad \text{if } 0 < \alpha \le \frac{2}{3} \\ &= \frac{(1-\alpha) + (2\alpha - 1)}{2} \\ &= \frac{\alpha}{2}, \qquad \text{if } \frac{2}{3} < \alpha < 1. \end{aligned} \tag{40}$$

A plot of $\mathcal{G}_\infty^*$ versus α is shown in Fig. 6. In the figure, the region $0 < \alpha \le 2/3$ corresponds to the constant payoff increment profile, while the region $2/3 < \alpha < 1$ corresponds to the oscillatory profile as stated in Proposition 1. As result, when $\alpha \in (0, 2/3]$, the value of the game is simply the payoff increment $\lambda = ((5-\alpha) - \sqrt{(9-\alpha)(1-\alpha)})/8$. On the other hand, when $\alpha \in (2/3, 1)$, since the period of oscillation is 2, the value is $(\lambda_{11}^t + \lambda_{11}^{t+1})/2 = ((1-\alpha) + (2\alpha - 1))/2 = \alpha/2$.

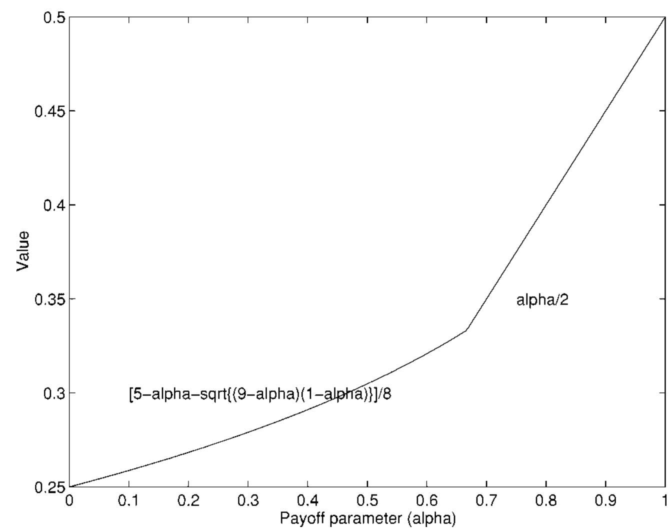

Fig. 6. Value of infinite horizon game versus payoff parameter α.

IV. COSET-GENERATED $M \times N$ GRID SOLUTIONS

In the 2×2 grid solution considered so far, the optimum accumulated payoff $S_t^*(z, w)$ has one critical point $(C_{\max} - C)/\delta_C$ on the z-axis, and another critical point $(J_{\max} - J)/\delta_J$ on the w-axis for all t. From the evolution equation (19), we find that in general $S_t^*(z, w)$ has a grid structure on the (z, w) plane for all t, the boundaries of the grids being determined by *critical points*. Thus, $S_0^*(z, w)$ has one critical point on either axis, and, as t increases, the number of critical points on the respective axes may or may not increase depending on the *operating points* $(\delta_C, C_{\max}/C)$, $(\delta_J, J_{\max}/J)$.

Define an operator $\mathcal{Z}$ which maps the communicator's past energy accumulation z at reverse-time t to that at $t+1$ as

$$\mathcal{Z}(z) \triangleq \begin{cases} \dfrac{z}{\delta_C}, & \text{if } 0 \le z \le C \\ \dfrac{z - C}{\delta_C}, & \text{if } C < z \le C_{\max}. \end{cases} \tag{41}$$

An analogous operator $\mathcal{W}$ which maps the jammer's past energy accumulation w at t to that at $t+1$ can be expressed by replacing C by J, δ_C by δ_J, $C_{\max}$ by $J_{\max}$, z by w in (41). Let $\mathcal{U}_C(t)$

denote the communicator's *critical point set* at reverse-time t, and $\mathcal{U}_J(t)$ the jammer's. Then

$$\mathcal{U}_C(0) = \left\{\frac{C_{\max} - C}{\delta_C}\right\} \cap [0,\, C_{\max}) = \left\{\frac{C_{\max} - C}{\delta_C}\right\}$$
$$\mathcal{U}_J(0) = \left\{\frac{J_{\max} - J}{\delta_J}\right\} \cap [0,\, J_{\max}) = \left\{\frac{J_{\max} - J}{\delta_J}\right\} \quad (42)$$

since, from (7), we have

$$1 \le \frac{C_{\max}}{C} < \frac{1}{1-\delta_C}, \quad 1 \le \frac{J_{\max}}{J} < \frac{1}{1-\delta_J}. \quad (43)$$

The backward recursions that describe the communicator's and jammer's critical point set computations are given by

$$\mathcal{U}_C(t+1) = \{\mathcal{Z}(\mathcal{U}_C(t)) \cup \mathcal{U}_C(t)\} \cap [0,\, C_{\max})$$
$$\mathcal{U}_J(t+1) = \{\mathcal{W}(\mathcal{U}_J(t)) \cup \mathcal{U}_J(t)\} \cap [0,\, J_{\max}) \quad (44)$$

for $t = 0,\, 1,\, 2, \ldots$. Therefore, at every reverse-time step, the number of critical points on the z-axis or w-axis can at most double. If we have the condition

$$\frac{\mathcal{U}_C(0)}{\delta_C} \ge C_{\max}, \quad \frac{\mathcal{U}_C(0) - C}{\delta_C} < 0 \quad (45)$$

which is the same as (27), then $\mathcal{U}_C(t) = \{(C_{\max} - C)/\delta_C\}$ for all t. Similarly, the condition

$$\frac{\mathcal{U}_J(0)}{\delta_J} \ge J_{\max}, \quad \frac{\mathcal{U}_J(0) - J}{\delta_J} < 0 \quad (46)$$

guarantees that $\mathcal{U}_J(t) = \{(J_{\max} - J)/\delta_J\}$ for all t. Conditions (45) and (46) specify the operating conditions for a 2 × 2 grid solution, which have already been found earlier [see (27) and (28)].

We are interested in finding operating conditions for which there are at most $M-1$ critical points of $S_t^*(z,\, w)$ on the z-axis and $N-1$ critical points on the w-axis for all t, that is, $S_t^*(z,\, w)$ *has an* $M \times N$ *grid structure*. Say, for some reverse-time η, $S_\eta^*(z,\, w)$ has critical points $a_1, \ldots, a_{M-1}$ ($a_1 < \cdots < a_{M-1}$) on the z-axis lying in $[0, C_{\max})$, and critical points $b_1, \ldots, b_{N-1}$ ($b_1 < \cdots < b_{N-1}$) on the w-axis lying in $[0, J_{\max})$, where $M,\, N \in \{2,\, 3,\, 4,\, 5,\, \ldots\}$. Since $\mathcal{U}_C(0) \subseteq \mathcal{U}_C(\eta)$ and $\mathcal{U}_J(0) \subseteq \mathcal{U}_J(\eta)$ [see (42) and (44)], assume

$$a_k = \frac{C_{\max} - C}{\delta_C}, \quad b_l = \frac{J_{\max} - J}{\delta_J}, \quad \text{for some } k \in \{1,\, \ldots,\, M-1\}, \quad l \in \{1,\, \ldots,\, N-1\}. \quad (47)$$

Let us force the conditions that

$$\mathcal{U}_C(t) = \{a_1,\, \ldots,\, a_{M-1}\} \quad \text{and}$$
$$\mathcal{U}_J(t) = \{b_1,\, \ldots,\, b_{N-1}\}, \quad \text{for all } t \ge \eta. \quad (48)$$

This implies that $S_t^*(z,\, w)$ has an $M \times N$ grid structure on the $(z,\, w)$ plane $\forall\, t \ge \eta$. For $t < \eta$, the elements of $\mathcal{U}_C(t)$ and $\mathcal{U}_J(t)$ which do not belong to $\{a_1,\, \ldots,\, a_{M-1}\}$ and $\{b_1,\, \ldots,\, b_{N-1}\}$, respectively, are treated as *redundant critical points*. Thus, an $M \times N$ grid structure of $S_t^*(z,\, w)$ can be generalized for all $t \ge 0$. Also, let

$$a_0 \triangleq 0, \quad a_M \triangleq C_{\max}, \quad b_0 \triangleq 0, \quad b_N \triangleq J_{\max}.$$

If we define M disjoint intervals along the z-axis as

$$\mathcal{A}_1 \triangleq [a_0,\, a_1], \quad \mathcal{A}_i \triangleq (a_{i-1},\, a_i], \qquad i = 2,\, \ldots,\, M$$

and N disjoint intervals along the w-axis as

$$\mathcal{B}_1 \triangleq [b_0,\, b_1], \quad \mathcal{B}_j \triangleq (b_{j-1},\, b_j], \qquad j = 2,\, \ldots,\, N$$

then $S_t^*(z,\, w)$ has an $M \times N$ grid structure given by

$$S_t^*(z,\, w) = S_{ij}^t, \qquad \text{if } (z,\, w) \in \mathcal{A}_i \times \mathcal{B}_j;\ \ i = 1,\, \ldots,\, M; \ \ j = 1,\, \ldots,\, N;\ \ \text{for } t = 0,\, \ldots,\, T-1. \quad (49)$$

One way of obtaining the communicator's critical points $a_1,\, \ldots,\, a_{M-1}$ satisfying (48) is to consider the situation when, for some $r \in \{1,\, \ldots,\, M-1\}$, critical point a_r satisfies

$$\frac{a_r}{\delta_C} \ge C_{\max}, \quad \frac{a_r - C}{\delta_C} < 0. \quad (50)$$

Then (48) implies

$$\{a_k\} \cup \left\{\frac{a_1}{\delta_C}, \ldots, \frac{a_{r-1}}{\delta_C}\right\} \cup \left\{\frac{a_{r+1} - C}{\delta_C}, \ldots, \frac{a_{M-1} - C}{\delta_C}\right\} = \{a_1,\, \ldots,\, a_{M-1}\}$$

where a_k is given by (47), and this results in the *critical point generation system* [32]

$$a_k = a_{c_1} = \mathcal{Z}(a_M) = \mathcal{Z}(C_{\max})$$
$$a_{c_i} = \mathcal{Z}(a_{c_{i-1}}) = \mathcal{Z}^i(a_M), \qquad i = 2,\, \ldots,\, M-2$$
$$a_r = a_{c_{M-1}} = \mathcal{Z}(a_{c_{M-2}}) = \mathcal{Z}^{M-1}(a_M) \quad (51)$$

where $[c_1,\, \ldots,\, c_{M-1}]$ is some permutation of $[1,\, \ldots,\, M-1]$ satisfying $c_1 = k$, $c_{M-1} = r$, for which the above form exists. We shall call (51) the *relevant form* of the critical point generation system. The vector $[c_1,\, \ldots,\, c_{M-1}]$ shall be called the *critical point generation index vector*. If a system can be written in the form of (51), we say that it has a *relevant solution* $(a_1,\, \ldots,\, a_{M-1})$.

Let a full cyclotomic coset mod $(2^M - 1)$ be written as an M-tuple $(\nu_1,\, \ldots,\, \nu_M)$, where $\nu_1 < \cdots < \nu_M$. The coset can also be written as $(\nu_M,\, \nu_{c_1},\, \ldots,\, \nu_{c_{M-1}})$, where the *coset generation index vector* $[c_1,\, \ldots,\, c_{M-1}]$ is some permutation of $[1,\, \ldots,\, M-1]$ for which we obtain the *coset generation system*

$$\nu_{c_1} = 2\nu_M \bmod (2^M - 1)$$
$$\nu_{c_i} = 2\nu_{c_{i-1}} \bmod (2^M - 1), \qquad i = 2,\, \ldots,\, M-1 \quad (52)$$

which has the same form as (51).

For a given M, let the operator $\mathcal{T}_M$ operating on $\nu \in \{1,\, \ldots,\, 2^M - 2\}$ be defined as

$$\mathcal{T}_M(\nu) \triangleq 2\nu \bmod \left(2^M - 1\right). \quad (53)$$

Comparing (51) and (52), we find that the following isomorphisms hold for each generation index vector $[c_1,\, \ldots,\, c_{M-1}]$:

$$(\nu_M,\, \nu_{c_1},\, \ldots,\, \nu_{c_{M-1}}) \longleftrightarrow (C_{\max},\, a_{c_1},\, \ldots,\, a_{c_{M-1}})$$
$$\mathcal{T}_M \longleftrightarrow \mathcal{Z}. \quad (54)$$

Thus, we conclude that the number of *coset-generated relevant solutions* of the critical point generation system for any natural

$M=2$			
coset (ν_2, ν_{c_1})	critical point generation system	critical point set $\{a_1\}$	one-critical-point region
$(2,1)$	$a_2 = C_{max}$ $a_1 = \frac{a_2-C}{\delta_C}$	$\{\frac{C_{max}-C}{\delta_C}\}$	$\frac{1}{1-\delta_C^2} \le \frac{C_{max}}{C} < 1+\delta_C$

$M=3$			
coset $(\nu_3, \nu_{c_1}, \nu_{c_2})$	critical point generation system	critical point set $\{a_1, a_2\}$	two-critical-point region
$(4,1,2)$	$a_3 = C_{max}$ $a_1 = \frac{a_3-C}{\delta_C}$ $a_2 = \frac{a_1}{\delta_C}$	$\{\frac{C_{max}-C}{\delta_C}, \frac{C_{max}-C}{\delta_C^2}\}$	$\frac{1}{1-\delta_C^3} \le \frac{C_{max}}{C} < 1+\delta_C^2$
$(6,5,3)$	$a_3 = C_{max}$ $a_2 = \frac{a_3-C}{\delta_C}$ $a_1 = \frac{a_2-C}{\delta_C}$	$\{\frac{C_{max}-C(1+\delta_C)}{\delta_C^2}, \frac{C_{max}-C}{\delta_C}\}$	$\frac{1+\delta_C}{1-\delta_C^3} \le \frac{C_{max}}{C} < 1+\delta_C+\delta_C^2$

$M=4$			
coset $(\nu_4, \nu_{c_1}, \nu_{c_2}, \nu_{c_3})$	critical point generation system	critical point set $\{a_1, a_2, a_3\}$	three-critical-point region
$(8,1,2,4)$	$a_4 = C_{max}$ $a_1 = \frac{a_4-C}{\delta_C}$ $a_2 = \frac{a_1}{\delta_C}$ $a_3 = \frac{a_2}{\delta_C}$	$\{\frac{C_{max}-C}{\delta_C}, \frac{C_{max}-C}{\delta_C^2}, \frac{C_{max}-C}{\delta_C^3}\}$	$\frac{1}{1-\delta_C^4} \le \frac{C_{max}}{C} < 1+\delta_C^3$
$(12,9,3,6)$	$a_4 = C_{max}$ $a_3 = \frac{a_4-C}{\delta_C}$ $a_1 = \frac{a_3-C}{\delta_C}$ $a_2 = \frac{a_1}{\delta_C}$	$\{\frac{C_{max}-C(1+\delta_C)}{\delta_C^2}, \frac{C_{max}-C(1+\delta_C)}{\delta_C^3}, \frac{C_{max}-C}{\delta_C}\}$	$\frac{1+\delta_C}{1-\delta_C^4} \le \frac{C_{max}}{C} < 1+\delta_C+\delta_C^3$
$(14,13,11,7)$	$a_4 = C_{max}$ $a_3 = \frac{a_4-C}{\delta_C}$ $a_2 = \frac{a_3-C}{\delta_C}$ $a_1 = \frac{a_2-C}{\delta_C}$	$\{\frac{C_{max}-C(1+\delta_C+\delta_C^2)}{\delta_C^3}, \frac{C_{max}-C(1+\delta_C)}{\delta_C^2}, \frac{C_{max}-C}{\delta_C}\}$	$\frac{1+\delta_C+\delta_C^2}{1-\delta_C^4} \le \frac{C_{max}}{C} < 1+\delta_C+\delta_C^2+\delta_C^3$

Fig. 7. Tabulation of each coset, its critical point generation system, the critical point set, and the corresponding coset-generated $(M-1)$-critical-point region for $M = 2, 3, 4$.

number $M \geq 2$ equals the number $h(M)$ of full cyclotomic cosets mod $(2^M - 1)$ given by [33]

$$h(M) = \frac{1}{M} \sum_{d|M} \mu(d) 2^{M/d}$$

where μ is the *Möbius function of number theory*.

The *communicator's coset-generated* $(M-1)$*-critical-point region* for which $a_1, \ldots, a_{M-1}$ are the *coset-generated critical points* of $S_t^*(z, w)$ on the z-axis for all t is given by (50). The *jammer's coset-generated* $(N-1)$*-critical-point region* for which $b_1, \ldots, b_{N-1}$ are the coset-generated critical points on the w-axis for all t can be found in an analogous way.

For $M = 2, 3, 4$, the possible sets of $M-1$ coset-generated critical points $a_1, \ldots, a_{M-1}$ and each of the $h(M)$ coset-generated $(M-1)$-critical-point regions are shown in Fig. 7. Fig. 8 shows a plot of these coset-generated critical-point regions on the $(\delta_C, C_{\max}/C)$ plane (operating plane of communicator). Thus, the number of one-critical-point regions is $h(2) = 1$, the number of two-critical-point regions is $h(3) = 2$, and the number of three-critical-point regions is $h(4) = 3$. All these regions are disjoint, are bounded by the curves $C_{\max}/C = 1/(1-\delta_C)$, $C_{\max}/C = 1/\delta_C$, $C_{\max}/C = 1$, and lie within

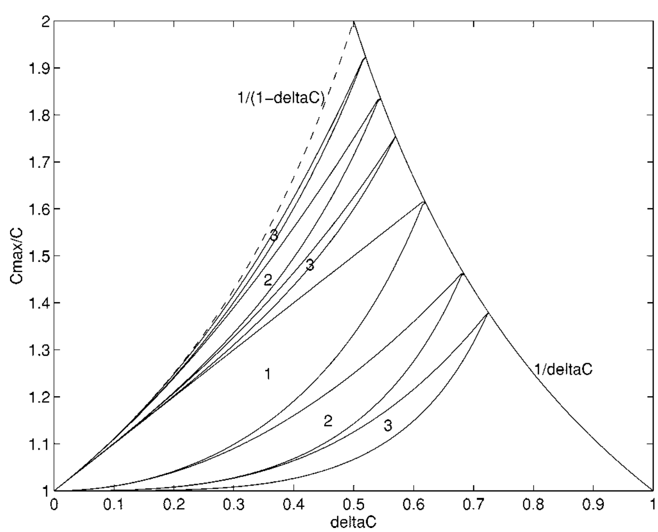

Fig. 8. Commmunicator's coset-generated critical-point regions for $(M-1) = 1, 2, 3$.

the region given by (43). For operating points outside these regions, coset-generated grid solutions do not exist.

For given M and N, the game has $h(M)h(N)$ different *coset-generated* $M \times N$ *grid solutions*. For a solution with index vector $[c_1, \ldots, c_{M-1}]$ of the communicator's coset $(\nu_1, \ldots, \nu_M)$ and $[c'_1, \ldots, c'_{N-1}]$ of the jammer's coset $(\nu'_1, \ldots, \nu'_N)$, the *coset-generated operating conditions* are [using (50)]

$$\frac{a_{c_{M-1}}}{\delta_C} \geq C_{\max}, \quad \frac{a_{c_{M-1}} - C}{\delta_C} < 0$$
$$\frac{b_{c'_{N-1}}}{\delta_J} \geq J_{\max}, \quad \frac{b_{c'_{N-1}} - J}{\delta_J} < 0.$$

The point $a_k = (C_{\max} - C)/\delta_C$ on the z-axis (for the communicator) given by (47) and (51) corresponds to the coset element ν_k, which can be shown to be the *largest odd element* of the coset $(\nu_1, \ldots, \nu_M)$. Similarly, the point $b_l = (J_{\max} - J)/\delta_J$ (for the jammer) corresponds to the coset element ν'_l, the largest odd element of the coset $(\nu'_1, \ldots, \nu'_N)$. The procedure for formulation of the evolution equation using coset generation index vectors has been presented in [34]. A general way of expressing analytically the behavior of optimal strategies as $t \to \infty$ has not yet been found.

There exist other kinds of $M \times N$ grid solutions in which one or both of the players' critical points are not generated by cosets. For example, if we choose

$$C_{\max}/C = (1 + \delta_C^2 - \delta_C^3)/(1 - \delta_C^3)$$

and δ_C is not too large, then there are three critical points

$$\delta_C^2 C/(1 - \delta_C^3),\ \delta_C C/(1 - \delta_C^3),\ C/(1 - \delta_C^3)$$

on the z-axis. As an illustration, consider $\delta_C = 2/3, C_{\max}/C = 31/19$. The critical points $a_1,\ a_2,\ a_3$ on the z-axis are generated as

$$a_4 = \frac{31}{19} C \longrightarrow a_2 = \frac{18}{19} C \longrightarrow a_3 = \frac{27}{19} C \longrightarrow$$
$$a_1 = \frac{12}{19} C \longrightarrow a_2 = \frac{18}{19} C.$$

We will not consider such investigations here.

V. An Example of BPSK Signaling

Consider a situation in which the transmitter communicates with a coherent receiver over an AWGN channel by employing BPSK signaling with carrier frequency f_c. The jammer tries to jam the receiver's signal by injecting additional noise into the receiver. When symbol i $(i = 0, 1)$ is transmitted, the received signal over the t_fth symbol interval (time slot) of duration T_s is given by

$$r(\tau) = \sqrt{2X_{t_f}}\ \cos(2\pi f_c \tau + i\pi) + n(\tau) + n_J(\tau),$$
$$(t_f - 1)T_s \leq \tau < t_f T_s;\ \ t_f = 1, \ldots, T \qquad (55)$$

where X_{t_f} is the communicating signal power, $n(\tau)$ the additive channel noise, and $n_J(\tau)$ the additive jamming noise. The noises $n(\tau)$ and $n_J(\tau)$ are assumed to be independent zero-mean white Gaussian random processes with two-sided power spectral densities (PSD's) $N_0/2$ and $Y_{t_f}/(2B)$, respectively, Y_{t_f} being the jamming signal power and B the channel bandwidth.

Since the PSD of the total noise $n(\tau) + n_J(\tau)$ is $(N_0/2) + (Y_{t_f}/(2B))$, the symbol-error probability for the receiver is given by

$$P_e(X_{t_f}, Y_{t_f}) = Q\left(\sqrt{\frac{2X_{t_f}T_s}{N_0 + \dfrac{Y_{t_f}}{B}}}\right) = Q\left(\sqrt{\frac{2BT_s X_{t_f}}{N_0 B + Y_{t_f}}}\right). \qquad (56)$$

The transmitter–receiver link can be viewed as a binary symmetric channel, and therefore a reasonable measure of the communicator's performance is the channel capacity, which (in bits/symbol) is given by

$$\mathcal{C}_{\text{cap}}(X_{t_f}, Y_{t_f}) = 1 + P_e(X_{t_f}, Y_{t_f}) \log_2 P_e(X_{t_f}, Y_{t_f})$$
$$+ [1 - P_e(X_{t_f}, Y_{t_f})]$$
$$\times \log_2[1 - P_e(X_{t_f}, Y_{t_f})]. \qquad (57)$$

Since $\mathcal{C}_{\text{cap}}(0, 0) = \mathcal{C}_{\text{cap}}(0, J) = 0$, we can define the payoff to the communicator as

$$G(X_{t_f}, Y_{t_f}) = \frac{\mathcal{C}_{\text{cap}}(X_{t_f}, Y_{t_f})}{\mathcal{C}_{\text{cap}}(C, 0)} \qquad (58)$$

which is nondecreasing with an increase in X_{t_f} and a decrease in Y_{t_f}, and satisfies the conditions

$$G(0, 0) = G(0, J) = 0, \quad G(C, 0) = 1,$$

$$\alpha = G(C, J) = \frac{\left\{\begin{array}{l} 1 \\ +Q\left(\sqrt{\dfrac{2BT_sC}{N_0B + J}}\right) \\ \quad \times \log_2 Q\left(\sqrt{\dfrac{2BT_sC}{N_0B + J}}\right) \\ +\left[1 - Q\left(\sqrt{\dfrac{2BT_sC}{N_0B + J}}\right)\right] \\ \quad \times \log_2\left[1 - Q\left(\sqrt{\dfrac{2BT_sC}{N_0B + J}}\right)\right] \end{array}\right\}}{\left\{\begin{array}{l} 1 \\ +Q\left(\sqrt{\dfrac{2BT_sC}{N_0B}}\right) \\ \quad \times \log_2 Q\left(\sqrt{\dfrac{2BT_sC}{N_0B}}\right) \\ +\left[1 - Q\left(\sqrt{\dfrac{2BT_sC}{N_0B}}\right)\right] \\ \quad \times \log_2\left[1 - Q\left(\sqrt{\dfrac{2BT_sC}{N_0B}}\right)\right] \end{array}\right\}} \qquad (59)$$

corresponding to the payoff matrix $\mathbf{G}$ in (5).

To compare a randomized power game situation with a fixed power scheme, consider the following two cases. 1) The communicator and jammer randomize their power levels over $\{0, C\}$ and $\{0, J\}$, respectively, and the conditions (27) and (28) for a 2 × 2 grid solution hold. 2) The communicator and jammer use fixed power levels $\overline{C}$ and $\overline{J}$, respectively, satisfying $\overline{C} \leq C_{\max}(1 - \delta_C)$ and $\overline{J} \leq J_{\max}(1 - \delta_J)$ owing to the temporal energy constraints.

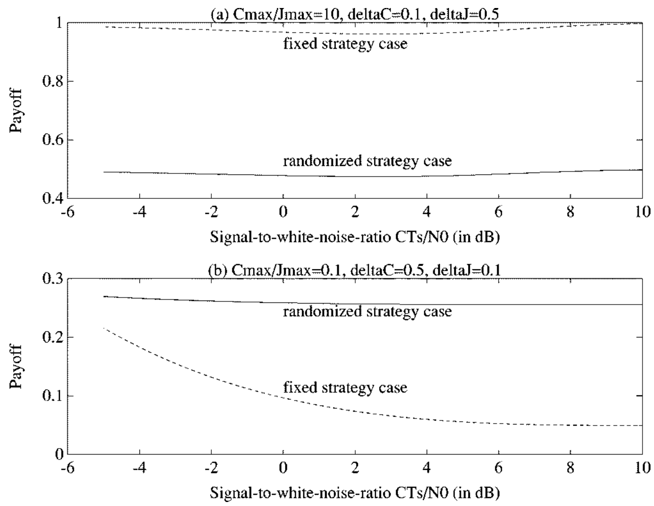

Fig. 9. Payoff versus signal-to-white-noise-ratio CT_s/N_0 (in decibels) for randomized and fixed strategy cases with channel bandwidth $B = 1/T_s$ and BPSK signaling when (a) $C_{\max} = 10J_{\max}$, $\delta_C = 0.1$, $\delta_J = 0.5$ and (b) $C_{\max} = 0.1J_{\max}$, $\delta_C = 0.5$, $\delta_J = 0.1$.

In case 1), let

$$C = \frac{C_{\max}}{2}\left(\frac{1}{1+\delta_C} + 1 - \delta_C^2\right)$$
$$J = \frac{J_{\max}}{2}\left(\frac{1}{1+\delta_J} + 1 - \delta_J^2\right) \tag{60}$$

while in case 2) we assume that each player transmits at the maximum allowable power level to prevent the opponent from taking any advantage, implying

$$\overline{C} = C_{\max}(1-\delta_C), \quad \overline{J} = J_{\max}(1-\delta_J). \tag{61}$$

When the communicator and jammer operate over a long period of time (T, the number of time slots, is large), the optimum payoff in case 1) is the value $\mathcal{G}^*_\infty$ of the infinite horizon game given by (40) with α as in (59), while in case 2), the payoff is simply $G(\overline{C}, \overline{J})$, where $G(.,.)$ is given by (58). With $B = 1/T_s$, plots of the payoffs for the randomized strategy case 1) and the fixed strategy case 2) versus the signal-to-white-noise-ratio CT_s/N_0 are shown in Fig. 9. The plots reveal that when the communicator's transmitter is more powerful than the jammer's [characterized by $C_{\max} > J_{\max}$, $\delta_C < \delta_J$ as in Fig. 9(a)], the payoff is higher for the fixed strategy case. However, when the jammer's transmitter is more powerful than the communicator's [characterized by $C_{\max} < J_{\max}$, $\delta_C > \delta_J$ as in Fig. 9(b)], the randomized strategy case gives a higher payoff. Therefore, a communicator with a powerful transmitter is better off by transmitting at a fixed power level, since this compels the weak jammer to do the same. On the other hand, when the jammer's transmitter is strong and the communicator's is weak, the communicator should use a randomized transmission scheme; this also forces the jammer to randomize its transmission.

VI. Conclusions

The main finding is that under certain operating conditions, the dynamic jamming game which we have considered admits steady-state optimal strategies that are mixed when α is lower than a threshold, but pure when it is higher. The mixed strategies give rise to a constant payoff increment profile on the energy accumulation plane [(z, w) plane], while the pure strategies result in an oscillatory profile, except for the fact that at the threshold the strategies are pure but the payoff increment profile is a constant. We have also shown how some grid solutions can be obtained from cyclotomic cosets. An example of a typical communication scenario comparing the use of randomized strategies with that of fixed ones is also presented.

The oscillatory behavior of the payoff increment and the role of the threshold for its appearance are very interesting for both the practical communication jamming problem as well as the game-theoretic problem perse. In infinite time games, people usually consider stationary nonoscillatory behavior, whereas our study indicates that oscillatory ones should be a legitimate object of study. Actually, other game applications, such as battle of species models, lead to similar oscillatory behaviors as time increases.

It is also to be noted that two assumptions in the model we have considered are: 1) both players have precise knowledge of the payoff parameter α and the transmitters' parameters; 2) each player obtains correct feedback information about the other's past actions. A situation in which the knowledge of α and the transmitters' parameters is not precise and the feedback information is not always correct calls for an adaptive game model, and is an interesting and relevant topic for further research.

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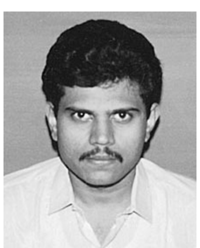

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