Impact of explicit and implicit control sharing on the performance of two-person one-act LQG Nash games*

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A two-person one-act LQG Nash game is considered under three different information structures: explicit control sharing, implicit control sharing and static information. The relations among the corresponding solutions and their impacts on the resulting costs are studies.

1. Introduction

In game problems, the players have certain kinds of information; they make decisions based on this information. We say that there is explicit control sharing (ECS) in a game if a player's information includes the previous control values of other players. Two previous works concerning the impact of ECS on the optimal costs in Nash games were reported in [1] and [2]. In [1] a two-person LQG Nash game was considered where the information structure is partially nested and each player acts once and it was shown (theorem 2 of [1]) that the first player might do better if he reveals his control value to the second player than he could do in a static information structure (SIS). It is known that in Nash games, if there is ECS then in general there exist many solutions [8]. Uchida considered an example of a two-person LQG Nash game [2] where the information is partially nested and each player acts once, and showed that among the nonunique solutions under ECS, one of them is equivalent to the SIS solution. Furthermore, it is claimed in [2] that this SIS solution gives a local minimum of the first player's cost among the linear class of the nonunique solutions. In other words, the first player might do better at least locally in a SIS than if he reveals his control value to the second player. The claim which Uchida did not prove and the result of Ho, Blau and Basar in [1] seem to contradict each other.

In this paper we consider a two-person LQG Nash game where the information is partially nested and each player acts once. We study the impact that the first player, who reveals his control value explicitly and implicitly to the second player, has on the first player's Nash cost. By implicit control sharing (ICS) we mean that player 2 has a noise-corrupted measurement which is affine in the system state and player 1's control. Our aim is to relate the Nash solutions under ECS to those under ICS and give a full view of

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the impact of both ECS and ICS on player 1's Nash performance. The work of [1] and [2] are re-examined and new insights are provided.

The study of the impact of available information on the costs of the players is important since it provides means of creating incentives to the players in order to achieve desirable performance. Although here we deal with a one-act game, it would be interesting to examine whether the main results of the present paper generalize to a multistage problem.

2. Problem formulation

The two-person LQG Nash game considered here is similar to the one considered in [2] and is described as follows: x, w_1 and w_2 are independent Gaussian random variables with zero mean and unit variance. There are two players, denoted by 1 and 2. Their cost functionals are given respectively by J_1 and J_2, where

$$J_1(\gamma_1, \gamma_2) = \mathrm{E}\{q_1(x + b_1u_1 + b_2u_2)^2 + u_1^2 + p_{12}u_2^2\}, \tag{1a}$$

$$J_2(\gamma_1, \gamma_2) = \mathrm{E}\{q_2(x + b_1u_1 + b_2u_2)^2 + p_{21}u_1^2 + u_2^2\}. \tag{1b}$$

q_1, q_2, p_{12} and p_{21} are non-negative real numbers, x is the system state, and u_i is the control variable of player i, $i = 1, 2$. Consider the measurements:

$$y_1 = h_1x + w_1, \tag{2}$$

$$y_2^0 = h_2x + w_2, \tag{3}$$

$$y_2 = y_2^0 + du_1. \tag{4}$$

u_i is chosen as $\gamma_i(\eta_i)$ where $\eta_1 = y_1$ and $\eta_2 \subset \{y_1, y_2, y_2^0, u_1\}$. We will consider the following cases:

Case A: ICS $\eta_2^{A} = \{y_1, y_2\}$;
Case B: ECS $\eta_2^{B} = \{y_1, y_2^0, u_1\}$;
Case C: SIS $\eta_2^{C} = \{y_1, y_2^0\}$.

γ_i is chosen from Γ_i, where Γ_i consists of all measurable functions mapping $\mathbb{R}^{m_i}$ ($m_i = \dim(\eta_i)$) to $\mathbb{R}$ such that $\gamma_i(\eta_i)$ is a second-order random variable. A pair (γ_1^*, γ_2^*) is called a Nash solution to the game if it satisfies the inequalities:

$$J_1(\gamma_1^*, \gamma_2^*) \leq J_1(\gamma_1, \gamma_2^*), \qquad J_2(\gamma_1^*, \gamma_2^*) \leq J_2(\gamma_1^*, \gamma_2), \tag{5}$$

for all $\gamma_1 \in \Gamma_1$ and $\gamma_2 \in \Gamma_2$. (γ_1^*, γ_2^*) is called a Stackelberg solution with player 1 as the leader if γ_1^* satisfies the following inequality:

$$\sup_{\gamma_2 \in R_2(\gamma_1^*)} J_1(\gamma_1^*, \gamma_2) \leq \sup_{\gamma_2 \in R_2(\gamma_1)} J_1(\gamma_1, \gamma_2), \tag{6a}$$

for every $\gamma_1 \in \Gamma_1$ and $\gamma_2^* \in R_2(\gamma_1^*)$, where $R_2(\gamma_1)$ is defined by

$$R_2(\gamma_1) = \{\gamma_2^0 \in \Gamma_2 \mid J_2(\gamma_1, \gamma_2^0) \leq J_2(\gamma_1, \gamma_2), \quad \forall \gamma_2 \in \Gamma_2\}. \tag{6b}$$

In the case of ECS, the Stackelberg solution is a particular Nash solution. The Nash solutions of cases A, B and C and the Stackelberg solution have been studied in [6]–[8] in a more general framework. The solutions exist under certain nonsingularity conditions. Here we state the solutions and for proofs we refer to [6]–[8]. The impact of ICS and ECS is then considered by comparing the Nash costs $J_1(\gamma_1, \gamma_2)$ of cases A and B (including the Stackelberg cost) with that of Case C.

2.1. *Nash solution of Case A*

Under the condition

$$1 + q_1 b_1^2 + q_2 b_2^2 + \zeta_A (q_1 - p_{12} q_2) b_1 b_2 \neq 0 \tag{7}$$

the unique Nash solution of Case A is given by

$$\gamma_{1A}(y_1) = -\{1 + q_1 b_1^2 + q_2 b_2^2 + \zeta_A (q_1 - p_{12} q_2) b_1 b_2\}^{-1} \{b_1 q_1 + \zeta_A (q_1 - p_{12} q_2) b_2\} \cdot h_1 (1 + h_1^2)^{-1} y_1 , \tag{8a}$$

$$\gamma_{2A}(y_1, y_2) = -(1 + q_2 b_2^2)^{-1} q_2 b_2 \{b_1 \gamma_{1A}(y_1) + [h_1 y_1 + h_2 (y_2 - d\gamma_{1A}(y_1))] \cdot (1 + h_1^2 + h_2^2)^{-1}\} , \tag{8b}$$

where

$$\zeta_A = -(1 + q_2 b_2^2)^{-1} q_2 b_2 h_2 d (1 + h_1^2 + h_2^2)^{-1} . \tag{9}$$

Notice that $(\gamma_{1A}, \gamma_{2A})$ depends on ζ_A, which in turn depends on d. To different d's, corresponds different pairs $(\gamma_{1A}, \gamma_{2A})_d$ provided that (7) holds. Let us call M the class of all these solutions $(\gamma_{1A}, \gamma_{2A})_d$ for varying values of d.

2.2. *Nash solution of Case B (linear class)*

There exist uncountably many Nash solutions for Case B, with the linear ones given by:

$$\gamma_{1B}(y_1) = -\{1 + q_1 b_1^2 + q_2 b_2^2 + \zeta (q_1 - p_{12} q_2) b_1 b_2\}^{-1} \cdot \{b_1 q_1 + \zeta (q_1 - p_{12} q_2) b_2\} \cdot h_1 (1 + h_1^2)^{-1} y_1 , \tag{10a}$$

$$\gamma_{2B}(y_1, y_2^0, u_1) = -(1 + q_2 b_2^2)^{-1} q_2 b_2 \{b_1 \gamma_{1B}(y_1) + [h_1 y_1 + h_2 y_2^0](1 + h_1^2 + h_2^2)^{-1}\} + \zeta (u_1 - \gamma_{1B}(y_1)) , \tag{10b}$$

where ζ is any real number such that

$$1 + q_1 b_1^2 + q_2 b_2^2 + \zeta (q_1 - p_{12} q_2) b_1 b_2 \neq 0 . \tag{11}$$

Let us denote by L the class of all these linear solutions $(\gamma_{1B}, \gamma_{2B})_\zeta$.

2.3. *Stackelberg solution of Case B*

The Stackelberg solution with player 1 as the leader is denoted by $(\gamma_{1S}, \gamma_{2S})$ and is the following:

$$(\gamma_{1S}, \gamma_{2S}) = (\gamma_{1B}, \gamma_{2B})_{\zeta = \zeta_S} , \tag{12}$$

where

$$\zeta_S = -(1 + q_2 b_2^2)^{-1} q_2 b_1 b_2 . \tag{13}$$

2.4. *Nash solution of Case C*

The Nash solution of Case C is a special one of Case A with $\zeta_A = 0$ in (8). It is also a special one of Case B with $\zeta = 0$ in (10). Notice that (7) and (11) are satisfied when $\zeta_A = \zeta = 0$.

3. An ordering of the Nash solutions

Two things are done in this section. First, it is shown that to every ICS Nash pair (with some particular d in (3)), there corresponds an ECS Nash pair (with some particular ζ) such that the two pairs represent the same random variable pair, and vice versa. Secondly, we make an ordering of all the Nash pairs in L and M.

Consider the Nash pair (10), with

$$\zeta = \zeta_A = -(1 + q_2 b_2^2)^{-1} q_2 b_2 h_2 d(1 + h_1^2 + h_2^2)^{-1} . \tag{14}$$

(Notice that (11) is satisfied.) A simple calculation shows that $(\gamma_{1A}, \gamma_{2A})$ and $(\gamma_{1B}, \gamma_{2B})$ are equivalent in the sense that they represent the same random variable pair, or equivalently the same function of the random variables x, w_1 and w_2. Hence we write

$$(\gamma_{1A}, \gamma_{2A})_d = (\gamma_{1B}, \gamma_{2B})_{\zeta=\zeta_A} . \tag{15}$$

Furthermore, if $q_2 h_2 \neq 0$, then to each Nash pair in L with some particular ζ, there corresponds a Nash pair in M with some particular d in (3), and vice versa. We thus have the following lemma.

Lemma 1. *If $q_2 h_2 \neq 0$ then there is a one-to-one correspondence between M and L where*

$$M \overset{\Delta}{=} \{(\gamma_{1A}, \gamma_{2A})_d \mid d \in R \text{ such that (7) holds}\} , \tag{16}$$

$$L \overset{\Delta}{=} \{(\gamma_{1B}, \gamma_{2B})_\zeta \mid \zeta \in R \text{ such that (11) holds}\} , \tag{17}$$

and the correspondence is given by (15).

Substituting $(\gamma_{1B}, \gamma_{2B})_\zeta$ of (10) into (1a), we obtain $J_1(\gamma_{1B}, \gamma_{2B})$ as a differentiable function with respect to (w.r.t.) ζ except possibly at some singular point g where

$$1 + q_1 b_1^2 + q_2 b_2^2 + g(q_1 - p_{12} q_2) b_1 b_2 = 0 . \tag{18}$$

By carrying out some calculations, we obtain the following derivative:

$$\frac{\mathrm{d}}{\mathrm{d}\zeta} J_1(\gamma_{1B}, \gamma_{2B}) = \left\{ \frac{b_2^2 h_1^2 (q_1 - p_{12} q_2)^2 (1 + q_2 b_2^2)}{(1 + h_1^2)[1 + q_1 b_1^2 + q_2 b_2^2 + \zeta(q_1 - p_{12} q_2) b_1 b_2]^3} \right\} \cdot (\zeta - \zeta_S) . \tag{19}$$

We assume, without loss of generality, that $(q_1 - p_{12} q_2) b_1 b_2$ is positive-valued (if not then either the singular point g does not exist or Fig. 1 is flipped over the J_1 axis and looks like Fig. 2), in which case:

$$\frac{\mathrm{d}}{\mathrm{d}\zeta} J_1(\gamma_{1B}, \gamma_{2B}) \begin{cases} >0, & \text{if } \zeta > \zeta_S \text{ or } \zeta < g , \\ =0 & \text{if } \zeta = \zeta_S, \\ <0 & \text{if } g < \zeta < \zeta_S . \end{cases} \tag{20}$$

Since $\lim_{\zeta \to +\infty} J_1(\gamma_{1B}, \gamma_{2B}) = \lim_{\zeta \to -\infty} J_1(\gamma_{1B}, \gamma_{2B})$ exists, and $\lim_{\zeta \to +\infty} J_1(\gamma_{1B}, \gamma_{2B}) > J_1(\gamma_{1B}, \gamma_{2B})_{\zeta=\zeta_S}$, the value $J_1(\gamma_{1B}, \gamma_{2B})$ versus ζ can be plotted as in Fig. 1. $J_1(\gamma_{1A}, \gamma_{2A})$ versus d is then plotted in Fig. 2 in view of Lemma 1. Figs. 1 and 2 give a full view of the impact of ECS and ICS on player 1's Nash performance. Several important features of this impact are summarized in the following theorem.

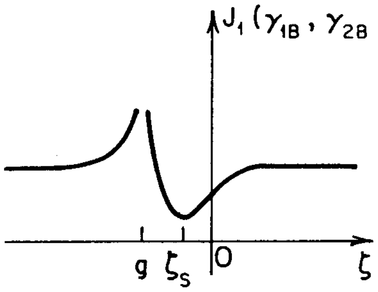

Fig. 1. Impact of ECS: $J_1(\gamma_{1B}, \gamma_{2B})$ as a function of ζ where ζ_S denotes the Stackelberg solution and 0 denotes the SIS Nash solution.

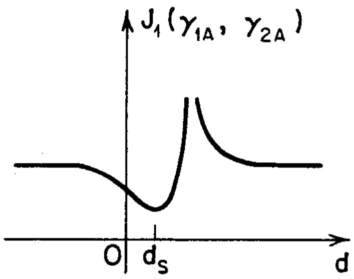

Fig. 2. Impact of ICS: $J_1(\gamma_{1A}, \gamma_{2A})$ as a function of d where $d_S = (1 + h_1^2 + h_2^2)b_1/h_2$ and 0 denotes the SIS Nash solution.

Theorem 1.
(i) *In L, the set of uncountably many linear Nash solutions under ECS, the unique local and global minimum of* J_1 *is given by* $(\gamma_{1B}, \gamma_{2B})_{\zeta=\zeta_S}$ *which is the Stackelberg solution.*
(ii) *Under ECS, player* 1 *can do better than under SIS if*

$$\zeta \in [\zeta_S, 0).$$

(iii) *Under ICS, player* 1 *can do better than under SIS if*

$$d \in (0, (1 + h_1^2 + h_2^2)b_1/h_2].$$

Remark 1. This theorem shows that Uchida's claim, namely that the SIS solution is a local minimum of J_1 in L, remark 3.3(i) of [2], is false.

Remark 2. This theorem indicates that the Stackelberg solution is more beneficial to player 1 as should be expected in general than all the other Nash solutions under ECS and SIS. It is not difficult to see that the Nash solution under ECS considered in theorem 2 of [1] is actually a Stackelberg solution.

Remark 3. This theorem and Fig. 1 give a general description of the impact of ECS on J_1 which includes the result of theorem 2 of [1] as one particular impact out of uncountable ones.

Remark 4. The parameter d in (3) can be regarded as a measure of the strength with which player 1 communicates his control implicitly to player 2. It can be regarded also as an incentive mechanism in a leader–follower situation, e.g. if the leader cannot communicate his control value to the follower free from noise, then by designing $d = (1 + h_1^2 + h_2^2)b_1/h_2$ in (3) and playing Nash (ICS), the leader can expect the same performance as in a Stackelberg game where the follower has perfect knowledge of the leader's control value.

4. Comments

In this section we give comments concerning the impact of ECS on J_1. In the first part we explain part (i) of theorem 1, i.e. why a local minimum of J_1 among L is given by the Stackelberg solution instead of the SIS solution as claimed by Uchida. In the second part we explain part (ii) of theorem 1, i.e. why player 1 can do better in a continuous range of ζ under ECS than under SIS.

Since $J_i(\gamma_1, \gamma_2)$ is quadratic in γ_j, $i, j = 1, 2$, $J_i(\gamma_1, \gamma_2)$ is differentiable w.r.t. γ_j. Furthermore, γ_2 is differentiable w.r.t. γ_1 since we take it to be linear in γ_1. By definition of a Nash solution (5), we have:

$$\left.\frac{\partial J_2(\gamma_1, \gamma_2)}{\partial \gamma_2}\right|_{(\gamma_1, \gamma_2)\in L} = 0, \tag{21}$$

$$\left.\frac{\mathrm{d}J_1(\gamma_1, \gamma_2)}{\mathrm{d}\gamma_1}\right|_{(\gamma_1, \gamma_2)\in L} = \left.\frac{\partial J_1(\gamma_1, \gamma_2)}{\partial \gamma_1}\right|_{(\gamma_1, \gamma_2)\in L} + \left.\frac{\partial J_1(\gamma_1, \gamma_2)}{\partial \gamma_2}\cdot\frac{\partial \gamma_2}{\partial \gamma_1}\right|_{(\gamma_1, \gamma_2)\in L} = 0. \tag{22}$$

For every $(\gamma_1, \gamma_2) \in L$, the derivative of $J_1(\gamma_1, \gamma_2)$ w.r.t. ζ can be written as:

$$\frac{\mathrm{d}J_1(\gamma_1, \gamma_2)}{\mathrm{d}\zeta} = \frac{\partial J_1(\gamma_1, \gamma_2)}{\partial \gamma_1}\cdot\frac{\partial \gamma_1}{\partial \zeta} + \frac{\partial J_1(\gamma_1, \gamma_2)}{\partial \gamma_2}\left(\frac{\partial \gamma_2}{\partial \zeta} + \frac{\partial \gamma_2}{\partial \gamma_1}\cdot\frac{\partial \gamma_1}{\partial \zeta}\right) \tag{23a}$$

$$= \left(\frac{\partial J_1(\gamma_1, \gamma_2)}{\partial \gamma_1} + \frac{\partial J_1(\gamma_1, \gamma_2)}{\partial \gamma_2}\cdot\frac{\partial \gamma_2}{\partial \gamma_1}\right)\cdot\frac{\partial \gamma_1}{\partial \zeta} + \frac{\partial J_1(\gamma_1, \gamma_2)}{\partial \gamma_2}\cdot\frac{\partial \gamma_2}{\partial \zeta} \tag{23b}$$

$$= \frac{\partial J_1(\gamma_1, \gamma_2)}{\partial \gamma_2}\cdot\frac{\partial \gamma_2}{\partial \zeta}, \tag{23c}$$

where we use (22) to obtain (23c). Let ζ^* correspond to the pair $(\gamma_{1B}, \gamma_{2B})_{\zeta=\zeta^*}$ which achieves a local minimum of J_1 in L, then

$$\left.\frac{\mathrm{d}J_1(\gamma_1, \gamma_2)}{\mathrm{d}\zeta}\right|_{\zeta^*} = 0. \tag{24}$$

Since $\partial\gamma_2/\partial\zeta|_{\zeta^*} \neq 0$ in general, (23c) and (24) imply

$$\left.\frac{\partial J_1(\gamma_1, \gamma_2)}{\partial \gamma_2}\right|_{\zeta^*} = 0. \tag{25}$$

(22) and (25) in turn imply

$$\left.\frac{\partial J_1(\gamma_1, \gamma_2)}{\partial \gamma_1}\right|_{\zeta^*} = 0. \tag{26}$$

Equations (21), (25) and (26) simply mean that with the constraint (21), J_1 is optimized w.r.t. both γ_1 and γ_2 at ζ^*. By a well-known theorem in optimization theory (p. 224 of [3]), equations (21), (25) and (26) are equivalent to saying that the following equations hold:

$$\frac{\partial}{\partial \gamma_i}[J_1(\gamma_1, \gamma_2) + \lambda G(\gamma_1, \gamma_2)]|_{\zeta^*} = 0, \quad i = 1, 2, \tag{27}$$

where

$$G(\gamma_1, \gamma_2) = \left.\frac{\partial J_2(\gamma_1, \gamma_2)}{\partial \gamma_2}\right|_{(\gamma_1, \gamma_2)\in L} = 0, \tag{28}$$

and λ is a Lagrange multiplier. Equations (27) and (28) are the first-order necessary condition for the

Stackelberg solution (6) to hold [4]. Since J_1 and J_2 are convex in γ_1 and γ_2, the first-order necessary condition is also a sufficient condition for ζ^* to be a Stackelberg solution.

It is remarkable that under ECS, although $J_1(\gamma_{1B}, \gamma_{2B})$ depends on the statistics of the observation noise, its ordering for different ζ is independent of the noise, as we can see from (20). Thus, in order to explain the ordering of $J_1(\gamma_{1B}, \gamma_{2B})$ for different ζ, one need consider only the deterministic game. In Section 2, if there is no observation noise in (2)–(4), then for any given u_1 the optimal value of u_2 minimizing J_2 is determined uniquely by

$$\gamma_2(x, u_1) = -(1 + q_2 b_2^2)^{-1} q_2 b_2 (b_1 u_1 + x) . \tag{29}$$

The locus of such points (u_1, u_2) given by (29) for all $u_1 \in R$ is called the reaction curve of player 2. The reaction curve of player 1 is similarly determined. Equicost contours of J_1 and J_2 and the reaction curves of both players are plotted in Fig. 3 for some particular values for the parameters of the game. The Nash solutions of Case B given by (10) now reduces to:

$$\gamma_{1B}(x) = -\{1 + q_1 b_1^2 + q_2 b_2^2 + \zeta(q_1 - p_{12} q_2) b_1 b_2\}^{-1} \{q_1 b_1 + \zeta(q_1 - p_{12} q_2) b_2\} x , \tag{30a}$$

$$\gamma_{2B}(x, u_1) = -(1 + q_2 b_2^2)^{-1} q_2 b_2 \{b_1 \gamma_{1B}(x) + x\} + \zeta(u_1 - \gamma_{1B}(x)) , \tag{30b}$$

for all $\zeta \in R$ such that (11) holds. Notice that at each solution point of (30), the value of u_2 given by (30b) is equal to that determined by the strategy

$$\gamma_2(x, u_1) = -(1 + q_2 b_2^2)^{-1} q_2 b_2 (b_1 u_1 + x) . \tag{31}$$

Equation (31) is the same as (29), which means that all the Nash solution pairs $(\gamma_{1B}, \gamma_{2B})_\zeta$ are on R_2, the reaction curve of player 2. Furthermore, since $\{u_1\}$ given by (30a) for all $\zeta \in R$ such that (11) holds, is the real line, we conclude that R_2 comprises all the linear Nash solutions of Case B. Point C in Fig. 3 represents $(\gamma_{1C}, \gamma_{2C}) = (\gamma_{1B}, \gamma_{2B})_{\zeta=0}$, the SIS solution where R_1 and R_2 intersect. Point S represents $(\gamma_{1S}, \gamma_{2S}) = (\gamma_{1B}, \gamma_{2B})_{\zeta=\zeta_S}$, the Stackelberg solution, where R_2 is tangent to the contour of J_1 [1]. Fig. 3 shows clearly that point S gives a global minimum of J_1 on R_2 and point C is by no means a local minimum of J_1 on R_2. All the points between C and S on R_2 yield lower cost of J_1 than point C. Finally,

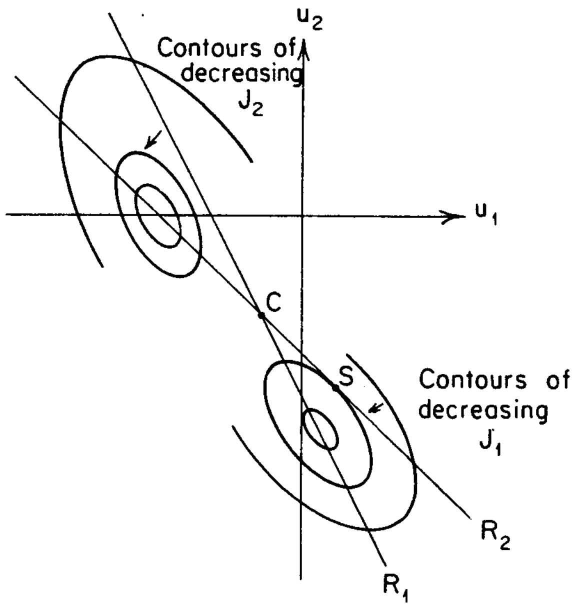

Fig. 3. Illustration of the impact of ECS on J_1. R_2: reaction curve of player 2; R_1: reaction curve of player 1.

we notice that while Fig. 3 provides an exact illustration of the impact of control sharing on J_1, it does not give a general description of the impact on J_2. For example, while Fig. 3 indicates that point S results in higher cost of J_2 than point C, things might go the other way around. For more detail we refer to [5].

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