Stochastic Optimization Topics

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1. SCOPE AND MATERIAL OF THE COURSE

In this course we intend to cover some basic ideas which serve as useful background as well as motivating applications, in the area of stochastic optimization. Stochastic optimization deals with models and methods for making decisions when several things, ex. parameters or part of the structure, are unknown or measured with noise. Among the many topics to be discussed we choose some that are of relevance to applications close to Control and Game Theory.

Dynamic Programming. The basic ideas of the Dynamic Programming Algorithm are presented and applied to the Dynamic Stochastic Control Problem in the Discrete Time framework. Material from the first three chapters of ref.1 is presented with emphasis on demonstrating through applications the essential points. The Linear Quadratic Case with Noisy Measurements is studied in order to clarify the Separation Principle. This is the subject that was popularized by R.Bellman and is the basic tool for deriving optimal decisions in a stochastic dynamic set-up.

Markovian Learning. The basic models were introduced by biologists in trying to model how an animal learns to choose between two decisions or modes of behavior, one of which is good (say socially acceptable) and the other one is bad. The good choice gives satisfaction (reward) with high probability, whereas the bad one results in punishment with high probability. Nonetheless it is possible for a good action to result in punishment as well as a bad action to result in reward. Thus one learns by updating the probabilities of choosing the good or bad action and this results in Markovian Learning models. The connections with classical adaptive control are intimate. But in the adaptive control case we usually consider as unknowns the values of some parameters which we try to identify as time goes by, in order to use them in our control law and we do no think in terms of rewards and penalties. On the other hand, in a repeated matrix game context, not knowing the costs and actions of the other player's results in taking actions whose outcomes we can think of as rewards or punishments, and the models of Markovian Learning, as developed by Biologists, seem more natural to use. Thus, many learning processes in repeated matrix games benefit directly by using Markovian Learning .Of course dynamic games which are described by difference or differential equations with unknown parameters benefit directly from the adaptive control methodologies where we also have an underlying difference or differential equation. We follow the first two chapters of ref.2 in presenting motivating and analyzing the problem. The classical work of refs 3and 4 provide historical motivation and theoretical results. Applications are discussed.

Stochastic Approximation. Here we can think of trying to minimize a function and using Steepest Descent or Newton's method, but a small error occurs in calculating derivatives or function values. These errors could be due to approximation errors ,or due to the fact that the function we want to minimize is not precisely known. Such is the case for example in many classical identification problems and that is why these methods serve in motivating and analyzing algorithms, for identification. Some of the key ideas of the pioneer papers of refs 5 and 6 are motivated and presented. The book of ref. 7 presents a wide collection of classical results. Applications in Estimation Theory and Identification are discussed.

Stochastic Stability. In both the Markovian Learning and in the Stochastic Approximation problems, one ends up studying the convergence of an iterative algorithm that aims at improving some actions or estimates. This updating is actually a stochastic dynamic equation. Thus we are led to studying equilibria and stability of equilibria, ie their attraction, region of attraction and rapidity of convergence. In the deterministic case the Lyapunov technique is the tool used. In the presence of noise, the Lyapunov function is replaced by a Supermartingale. The pioneer work of ref.8 where the notion of Lyapunov Function is extended for Stochastic Dynamical Equations via the Supermartingale idea, is discussed. It provides a unifying framework for studying the convergence of both Markovian Learning and of Stochastic Approximation The chapter 2 of ref. 9 is followed for presenting in a very concise, simple and general way the basic results and tools needed.

2. USEFULNESS OF THE COURSE, BACKGROUND NEEDED

The aim of this course is to familiarize beginning graduate students and practicing engineers with some very useful techniques, as to be able to use them without becoming experts in several mathematical technicalities. Some knowledge of very basic ideas in systems, optimization, control and probability acquired during undergraduate work is sufficient. Emphasis will be placed on the qualitative use of ideas for introducing and explaining models, methods and results. Students and current and future practitioners in Control, Estimation/Identification, Game Theory and Operations Research will find it very useful.

Remark on the references: 1-13 provide some basic references which are of either of historical and/or of didactic value. 14-35 refer to work of the instructor, where the ideas and tools presented here have been used in a variety of contexts and applications.

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